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Class	10th	Chapter	02
Subject	Mathematics	Board	Punjab

Chapter 2 (Quadratic Equations and Inequalities)

EXERCISE 2.1

Q1. Write the following quadratic equations in standard form:

- **(i)** $3x - 1 = 2x^2$
  - Rearranging terms to one side: $2x^2 - 3x + 1 = 0$
- **(ii)** $2x(x + 1) = 4(2x + 3)$
  - Expanding both sides: $2x^2 + 2x = 8x + 12$
  - Bringing all terms to the left: $2x^2 + 2x - 8x - 12 = 0 \Rightarrow 2x^2 - 6x - 12 = 0$
  - Dividing by 2: $x^2 - 3x - 6 = 0$
- **(iii)** $2x^2 - 4x = 4x + 7$
  - Rearranging terms: $2x^2 - 4x - 4x - 7 = 0 \Rightarrow 2x^2 - 8x - 7 = 0$
- **(iv)** $4(3x - 2) = 9x^2$
  - Expanding and rearranging: $12x - 8 = 9x^2 \Rightarrow 9x^2 - 12x + 8 = 0$
- **(v)** $2x + \frac{1}{x} = 5 - \frac{1}{x}$, $(x \neq 0)$
  - Multiplying by x: $2x^2 + 1 = 5x - 1 \Rightarrow 2x^2 - 5x + 2 = 0$
- **(vi)** $\frac{6x+6}{20-x} = \frac{1}{x}$, $(x \neq 0,20)$
  - Cross-multiplying: $x(6x + 6) = 20 - x \Rightarrow 6x^2 + 6x + x - 20 = 0 \Rightarrow 6x^2 + 7x - 20 = 0$

Q2. Solve the following quadratic equations by factorization method:

- **(i)** $x^2 - x - 6 = 0$
  - Factorization: $x^2 - 3x + 2x - 6 = 0 \Rightarrow x(x - 3) + 2(x - 3) = 0 \Rightarrow (x - 3)(x + 2) = 0$
  - **Solution Set:** $\{-2,3\}$
- **(ii)** $x^2 + 3x - 28 = 0$

- Factorization: $x^2 + 7x - 4x - 28 = 0 \Rightarrow x(x+7) - 4(x+7) = 0 \Rightarrow (x+7)(x-4) = 0$
- **Solution Set:** $\{-7, 4\}$

- **(iii)** $6x^2 + 13x - 5 = 0$
  - Factorization: $6x^2 + 15x - 2x - 5 = 0 \Rightarrow 3x(2x+5) - 1(2x+5) = 0 \Rightarrow (3x-1)(2x+5) = 0$
  - **Solution Set:** $\left\{-\frac{5}{2}, \frac{1}{3}\right\}$
- **(iv)** $x^2 - \frac{3}{2}x = \frac{9}{2}$
  - Multiply by 2: $2x^2 - 3x = 9 \Rightarrow 2x^2 - 3x - 9 = 0 \Rightarrow (2x+3)(x-3) = 0$
  - **Solution Set:** $\left\{-\frac{3}{2}, 3\right\}$
- **(v)** $\frac{3x-8}{x-2} = \frac{5x-2}{x+5}$
  - Cross-multiply: $(3x-8)(x+5) = (5x-2)(x-2) \Rightarrow 3x^2 + 7x - 40 = 5x^2 - 12x + 4$
  - Simplifying: $2x^2 - 19x + 44 = 0 \Rightarrow (x-4)(2x-11) = 0$
  - **Solution Set:** $\left\{4, \frac{11}{2}\right\}$
- **(vi)** $\frac{1}{x-1} - \frac{1}{x+3} = \frac{1}{35}$
  - Combine LHS: $\frac{(x+3)-(x-1)}{x^2+2x-3} = \frac{1}{35} \Rightarrow \frac{4}{x^2+2x-3} = \frac{1}{35} \Rightarrow x^2 + 2x - 143 = 0$
  - Factorization: $(x+13)(x-11) = 0$
  - **Solution Set:** $\{-13, 11\}$

Q3. Solve the following quadratic equations by completing square method:

- **(i)** $2x^2 + 5x + 2 = 0$
  - Divide by 2: $x^2 + \frac{5}{2}x + 1 = 0 \Rightarrow x^2 + \frac{5}{2}x = -1$
  - Adding $\left(\frac{5}{4}\right)^2$ to both sides: $\left(x + \frac{5}{4}\right)^2 = -1 + \frac{25}{16} = \frac{9}{16} \Rightarrow x + \frac{5}{4} = \pm\frac{3}{4}$
  - **Solution Set:** $\left\{-2, -\frac{1}{2}\right\}$
- **(ii)** $x^2 + x = 42$
  - Completing square: $\left(x + \frac{1}{2}\right)^2 = 42 + \frac{1}{4} = \frac{169}{4} \Rightarrow x + \frac{1}{2} = \pm\frac{13}{2}$
  - **Solution Set:** $\{-7, 6\}$
- **(iii)** $12x^2 + 7x = 12$
  - Divide by 12: $x^2 + \frac{7}{12}x = 1 \Rightarrow \left(x + \frac{7}{24}\right)^2 = 1 + \frac{49}{576} = \frac{625}{576} \Rightarrow x + \frac{7}{24} = \pm\frac{25}{24}$
  - **Solution Set:** $\left\{-\frac{4}{3}, \frac{3}{4}\right\}$
- **(iv)** $\frac{x+3}{2x-7} = \frac{2x-1}{x-3}$

- Cross-multiply: $x^2 - 9 = 4x^2 - 16x + 7 \Rightarrow 3x^2 - 16x + 16 = 0 \Rightarrow (x-4)(3x-4) = 0$
- **Solution Set:** $\left\{\frac{4}{3}, 4\right\}$

- **(v)** $\frac{1}{1+x} - \frac{1}{3-x} = \frac{6}{35}$
  - Simplifying: $3x^2 - 41x + 26 = 0 \Rightarrow (3x-2)(x-13) = 0$
  - **Solution Set:** $\left\{\frac{2}{3}, 13\right\}$
- **(vi)** $\frac{3x-1}{4x+7} = 1 - \frac{6}{x+7}$
  - Simplifying: $x^2 - 9x + 14 = 0 \Rightarrow (x-2)(x-7) = 0$
  - **Solution Set:** $\{2,7\}$

Q4. Use quadratic formula to solve the following equations: ($x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$)

- **(i)** $2x^2 - 5x + 3 = 0 \rightarrow x = \frac{5 \pm \sqrt{25-24}}{4} \Rightarrow$ **Solution Set:** $\left\{1, \frac{3}{2}\right\}$
- **(ii)** $2x^2 - 7x - 15 = 0 \rightarrow x = \frac{7 \pm \sqrt{49+120}}{4} \Rightarrow$ **Solution Set:** $\left\{-\frac{3}{2}, 5\right\}$
- **(iii)** $2x^2 + 7x = 15 \rightarrow$ **Solution Set:** $\left\{-5, \frac{3}{2}\right\}$
- **(iv)** $x^2 + 11 = 7x \rightarrow$ **Solution Set:** $\left\{\frac{7 \pm \sqrt{5}}{2}\right\}$
- **(v)** $\frac{x+4}{x-4} + \frac{x-2}{x-3} = 6\frac{1}{3} \rightarrow 13x^2 - 118x + 240 = 0 \Rightarrow$ **Solution Set:** $\left\{\frac{40}{13}, 6\right\}$
- **(vi)** $\frac{3x-3}{x+1} = \frac{2x-1}{x-1} \rightarrow x^2 - 7x + 4 = 0 \Rightarrow$ **Solution Set:** $\left\{\frac{7 \pm \sqrt{33}}{2}\right\}$

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EXERCISE 2.2

Question 1: Find the points of intersection of the following linear equations with coordinate axes graphically

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Rule:

- To find the **x-intercept** (where the line crosses the x-axis), put $y = 0$.
- To find the **y-intercept** (where the line crosses the y-axis), put $x = 0$.

(i) $x + y = 8$

1. **For x-intercept:** Put $y = 0$:

$$x + 0 = 8 \Longrightarrow x = 8$$

Point on x-axis: $(8,0)$

2. **For y-intercept:** Put $x = 0$:

$$0 + y = 8 \Longrightarrow y = 8$$

Point on y-axis: $(0,8)$

- **Table of Values:**

x	**0**	**8**
y	8	0

- **Points of intersection:** $(8,0)$ and $(0,8)$

(ii) $x - y = 1$

1. **For x-intercept:** Put $y = 0$:

$$x - 0 = 1 \Longrightarrow x = 1$$

Point on x-axis: $(1,0)$

2. **For y-intercept:** Put $x = 0$:

$$0 - y = 1 \Longrightarrow y = -1$$

Point on y-axis: $(0, -1)$

- **Table of Values:**

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x	0	1
y	-1	0

- **Points of intersection:** $(1,0)$ and $(0,-1)$

(iii) $x - 2y = 1$

1. **For x-intercept:** Put $y = 0$:

$$x - 2(0) = 1 \Longrightarrow x = 1$$

Point on x-axis: $(1,0)$

2. **For y-intercept:** Put $x = 0$:

$$0 - 2y = 1 \Longrightarrow -2y = 1 \Longrightarrow y = -\frac{1}{2} = -0.5$$

Point on y-axis: $(0,-0.5)$

- **Table of Values:**

x	0	1
y	-0.5	0

- **Points of intersection:** $(1,0)$ and $(0,-0.5)$

(iv) $x - 2y + 2 = 0$

1. **For x-intercept:** Put $y = 0$:

$$x - 2(0) + 2 = 0 \Longrightarrow x + 2 = 0 \Longrightarrow x = -2$$

Point on x-axis: $(-2,0)$

2. **For y-intercept:** Put $x = 0$:

$$0 - 2y + 2 = 0 \Longrightarrow -2y = -2 \Longrightarrow y = 1$$

Point on y-axis: $(0,1)$

- **Table of Values:**

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x	0	-2
y	1	0

- **Points of intersection:** $(-2,0)$ and $(0,1)$

(v) $5x - 5y = 1$

1. **For x-intercept:** Put $y = 0$:

$$5x - 5(0) = 1 \Longrightarrow 5x = 1 \Longrightarrow x = \frac{1}{5} = 0.2$$

Point on x-axis: $(0.2,0)$

2. **For y-intercept:** Put $x = 0$:

$$5(0) - 5y = 1 \Longrightarrow -5y = 1 \Longrightarrow y = -\frac{1}{5} = -0.2$$

Point on y-axis: $(0, -0.2)$

- **Table of Values:**

x	0	0.2
y	-0.2	0

- **Points of intersection:** $(0.2,0)$ and $(0, -0.2)$

Question 2: Solve the following system of linear equations graphically

:

(i) $x + y = 8$ and $3x - y = 4$

- **Equation (1):** $x + y = 8 \Longrightarrow y = 8 - x$

x	0	3	8
y	8	5	0

- **Equation (2):** $3x - y = 4 \Longrightarrow y = 3x - 4$

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x	0	2	3
y	-4	2	5

- **Graphical Intersection & Solution:** Both lines intersect at the point $(3,5)$.

Solution Set: $\{(3,5)\}$

(ii) $x - y = 1$ and $x + 2y = 7$

- **Equation (1):** $x - y = 1 \Longrightarrow y = x - 1$

x	0	1	3
y	-1	0	2

- **Equation (2):** $x + 2y = 7 \Longrightarrow y = \frac{7-x}{2}$

x	1	3	7
y	3	2	0

- **Graphical Intersection & Solution:** Both lines intersect at the point $(3,2)$.

Solution Set: $\{(3,2)\}$

(iii) $x - 2y = 1$ and $2x + y = 2$

- **Equation (1):** $x - 2y = 1 \Longrightarrow y = \frac{x-1}{2}$

x	-1	1	3
y	-1	0	1

- **Equation (2):** $2x + y = 2 \Longrightarrow y = 2 - 2x$

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x	0	1	2
y	2	0	-2

- **Graphical Intersection & Solution:** Both lines intersect at the point $(1,0)$.

Solution Set: $\{(1,0)\}$

(iv) $y = 2x + 2$ and $3x + 2y = 4$

- **Equation (1):** $y = 2x + 2$

x	-1	0	1
y	0	2	4

- **Equation (2):** $3x + 2y = 4 \Longrightarrow y = \frac{4-3x}{2}$

x	0	2	4
y	2	-1	-4

- **Graphical Intersection & Solution:** Both lines intersect at the point $(0,2)$.

Solution Set: $\{(0,2)\}$

(v) $3y = 2x + 8$ and $x + y = 1$

- **Equation (1):** $3y = 2x + 8 \Longrightarrow y = \frac{2x+8}{3}$

x	-4	-1	2
y	0	2	4

- **Equation (2):** $x + y = 1 \Longrightarrow y = 1 - x$

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x	-1	0	1
y	2	1	0

- **Graphical Intersection & Solution:** Both lines intersect at the point $(-1,2)$.

Solution Set: $\{(-1,2)\}$

Question 3: Solve the following equations graphically

:

(i) $y = 8x - 32$ and $y = x^2 - 6x + 8$

1. **Equate both expressions for y to find algebraic solution points:**

$$x^2 - 6x + 8 = 8x - 32$$

$$x^2 - 14x + 40 = 0$$

$$(x-4)(x-10) = 0 \Longrightarrow x = 4 \text{ or } x = 10$$

2. **Calculate corresponding y-values:**
   - When $x = 4$: $y = 8(4) - 32 = 0$
   - When $x = 10$: $y = 8(10) - 32 = 48$
3. **Table of Values:**
   - **Line $y = 8x - 32$:**

x	0	4	10
y	-32	0	48

- **Parabola $y = x^2 - 6x + 8$:**

x	2	3	4	6	10
y	0	-1	0	8	48

- **Graphical Intersection & Solution:** The straight line and the parabola intersect at $(4,0)$ and $(10,48)$.

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Solution Set: $\{(4,0), (10,48)\}$

(ii) $y + x = 2$ and $y = 2x^2 + x - 10$

1. **Express the first equation in terms of y:**

$$y = 2 - x$$

2. **Equate both equations for y:**

$$2x^2 + x - 10 = 2 - x$$

$$2x^2 + 2x - 12 = 0$$

Divide by 2:

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0 \Rightarrow x = -3 \text{ or } x = 2$$

3. **Calculate corresponding y-values:**
   - When $x = -3$: $y = 2 - (-3) = 5$
   - When $x = 2$: $y = 2 - 2 = 0$
4. **Table of Values:**
   - **Line $y = 2 - x$:**

x	**−3**	**0**	**2**
y	5	2	0

- **Parabola $y = 2x^2 + x - 10$:**

x	**−3**	**−0.25**	**0**	**2**
y	5	−10.125	−10	0

- **Graphical Intersection & Solution:** The straight line and the parabola intersect at $(-3,5)$ and $(2,0)$.

Solution Set: $\{(-3,5), (2,0)\}$

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EXERCISE 2.3

General Formula for Forming a Quadratic Equation:

If the roots of a quadratic equation are given, the required equation is:

$$x^2 - Sx + P = 0$$

where:

- $S =$ Sum of the roots
- $P =$ Product of the roots

Question 1: Form a quadratic equation whose roots are given below

:

(i) Roots: $-4, 9$

1. **Sum of roots (S):**

$$S = -4 + 9 = 5$$

2. **Product of roots (P):**

$$P = (-4) \times 9 = -36$$

3. **Required Equation:**

$$x^2 - Sx + P = 0 \Longrightarrow x^2 - 5x - 36 = 0$$

(ii) Roots: $5, -7$

1. **Sum of roots (S):**

$$S = 5 + (-7) = -2$$

2. **Product of roots (P):**

$$P = 5 \times (-7) = -35$$

3. **Required Equation:**

$$x^2 - Sx + P = 0 \Longrightarrow x^2 - (-2)x + (-35) = 0 \Longrightarrow x^2 + 2x - 35 = 0$$

(iii) Roots: $\frac{-7}{5}, \frac{-6}{5}$

1. **Sum of roots (S):**

$$S = -\frac{7}{5} + \left(-\frac{6}{5}\right) = -\frac{13}{5}$$

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2. **Product of roots (P):**

$$P = \left(-\frac{7}{5}\right) \times \left(-\frac{6}{5}\right) = \frac{42}{25}$$

3. **Required Equation:**

$$x^2 - \left(-\frac{13}{5}\right)x + \frac{42}{25} = 0 \Longrightarrow x^2 + \frac{13}{5}x + \frac{42}{25} = 0$$

Multiply the entire equation by 25:

$$25x^2 + 65x + 42 = 0$$

(iv) Roots: $\frac{-3}{2}, \frac{7}{2}$

1. **Sum of roots (S):**

$$S = -\frac{3}{2} + \frac{7}{2} = \frac{4}{2} = 2$$

2. **Product of roots (P):**

$$P = \left(-\frac{3}{2}\right) \times \left(\frac{7}{2}\right) = -\frac{21}{4}$$

3. **Required Equation:**

$$x^2 - 2x - \frac{21}{4} = 0$$

Multiply the entire equation by 4:

$$4x^2 - 8x - 21 = 0$$

(v) Roots: $3 + \sqrt{5}, 3 - \sqrt{5}$

1. **Sum of roots (S):**

$$S = \left(3 + \sqrt{5}\right) + \left(3 - \sqrt{5}\right) = 6$$

2. **Product of roots (P):**

$$P = \left(3 + \sqrt{5}\right)\left(3 - \sqrt{5}\right) = (3)^2 - \left(\sqrt{5}\right)^2 = 9 - 5 = 4$$

3. **Required Equation:**

$$x^2 - Sx + P = 0 \Longrightarrow x^2 - 6x + 4 = 0$$

(vi) Roots: $-2 + \sqrt{3}, -2 - \sqrt{3}$

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1. **Sum of roots (S):**

$$S = (-2 + \sqrt{3}) + (-2 - \sqrt{3}) = -4$$

2. **Product of roots (P):**

$$P = (-2 + \sqrt{3})(-2 - \sqrt{3}) = (-2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$$

3. **Required Equation:**

$$x^2 - Sx + P = 0 \Longrightarrow x^2 - (-4)x + 1 = 0 \Longrightarrow x^2 + 4x + 1 = 0$$

Question 2: Find the quadratic equation with roots exceeding by 2 than those of roots of $x^2 + 9x + 20 = 0$

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1. **Find roots of given equation:**

$$x^2 + 9x + 20 = 0$$

$$x^2 + 5x + 4x + 20 = 0$$

$$(x + 5)(x + 4) = 0 \Longrightarrow x = -5 \text{ or } x = -4$$

2. **New roots (exceeding given roots by 2):**

$$\text{First new root} = -5 + 2 = -3$$

$$\text{Second new root} = -4 + 2 = -2$$

3. **Form the new equation:**
   - **Sum (S):** $S = -3 + (-2) = -5$
   - **Product (P):** $P = (-3) \times (-2) = 6$
4. **Required Equation:**

$$x^2 - (-5)x + 6 = 0 \Longrightarrow x^2 + 5x + 6 = 0$$

Question 3: Find the equation whose roots are double the roots of $x^2 - px + q = 0$

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1. Let α, β be roots of $x^2 - px + q = 0$:
   - $\alpha + \beta = -\frac{-p}{1} = p$
   - $\alpha\beta = \frac{q}{1} = q$

2. **New roots are $2\alpha, 2\beta$:**
    - **Sum of new roots (S):**

$$S = 2\alpha + 2\beta = 2(\alpha + \beta) = 2p$$

    - **Product of new roots (P):**

$$P = (2\alpha)(2\beta) = 4\alpha\beta = 4q$$

3. **Required Equation:**

$$x^2 - Sx + P = 0 \Longrightarrow x^2 - 2px + 4q = 0$$

Question 4: If $\boldsymbol{\alpha, \beta}$ are the roots of $\boldsymbol{x^2 + 2x + 4 = 0}$, find the equation whose roots are :

Given Equation: $x^2 + 2x + 4 = 0$

- $\alpha + \beta = -2$
- $\alpha\beta = 4$

(i) Roots: $\frac{1}{\alpha}, \frac{1}{\beta}$

1. **Sum (S):**

$$S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-2}{4} = -\frac{1}{2}$$

2. **Product (P):**

$$P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{4}$$

3. **Required Equation:**

$$x^2 - \left(-\frac{1}{2}\right)x + \frac{1}{4} = 0 \Longrightarrow x^2 + \frac{1}{2}x + \frac{1}{4} = 0$$

Multiply by 4:

$$4x^2 + 2x + 1 = 0$$

(ii) Roots: $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

1. **Sum (S):**

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$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(-2)^2 - 2(4)}{4} = \frac{4-8}{4} = \frac{-4}{4} = -1$$

2. **Product (P):**

$$P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

3. **Required Equation:**

$$x^2 - (-1)x + 1 = 0 \Rightarrow x^2 + x + 1 = 0$$

(iii) Roots: $2\alpha - \frac{1}{\beta}, 2\beta - \frac{1}{\alpha}$

1. **Sum (S):**

$$S = \left(2\alpha - \frac{1}{\beta}\right) + \left(2\beta - \frac{1}{\alpha}\right) = 2(\alpha+\beta) - \left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$S = 2(-2) - \left(\frac{-2}{4}\right) = -4 + \frac{1}{2} = -\frac{7}{2}$$

2. **Product (P):**

$$P = \left(2\alpha - \frac{1}{\beta}\right)\left(2\beta - \frac{1}{\alpha}\right) = 4\alpha\beta - 2 - 2 + \frac{1}{\alpha\beta} = 4\alpha\beta - 4 + \frac{1}{\alpha\beta}$$

$$P = 4(4) - 4 + \frac{1}{4} = 16 - 4 + \frac{1}{4} = 12 + \frac{1}{4} = \frac{49}{4}$$

3. **Required Equation:**

$$x^2 - \left(-\frac{7}{2}\right)x + \frac{49}{4} = 0 \Rightarrow x^2 + \frac{7}{2}x + \frac{49}{4} = 0$$

Multiply by 4:

$$4x^2 + 14x + 49 = 0$$

(iv) Roots: α^2, β^2

1. **Sum (S):**

$$S = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-2)^2 - 2(4) = 4 - 8 = -4$$

2. **Product (P):**

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$$P = \alpha^2 \cdot \beta^2 = (\alpha\beta)^2 = 4^2 = 16$$

3. **Required Equation:**

$$x^2 - (-4)x + 16 = 0 \Longrightarrow x^2 + 4x + 16 = 0$$

(v) Roots: $2\alpha - 1, 2\beta - 1$

1. **Sum (S):**

$$S = (2\alpha - 1) + (2\beta - 1) = 2(\alpha + \beta) - 2 = 2(-2) - 2 = -6$$

2. **Product (P):**

$$P = (2\alpha - 1)(2\beta - 1) = 4\alpha\beta - 2(\alpha + \beta) + 1 = 4(4) - 2(-2) + 1 = 16 + 4 + 1 = 21$$

3. **Required Equation:**

$$x^2 - (-6)x + 21 = 0 \Longrightarrow x^2 + 6x + 21 = 0$$

Question 5: Find the condition that roots of $ax^2 + bx + c = 0$ should be reciprocals of each other

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1. Let the roots be α and $\frac{1}{\alpha}$:
2. **Product of roots:**

$$\text{Product} = \frac{c}{a}$$

$$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$$

$$1 = \frac{c}{a} \Longrightarrow c = a$$

Condition: $c = a$ (or $a = c$)

Question 6: Find the value of k, given that one root of $x^2 - (2k + 4)x + (7k + 1) = 0$ is 3

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1. Since $x = 3$ is a root, substitute $x = 3$ into the equation:

$$(3)^2 - (2k + 4)(3) + (7k + 1) = 0$$

2. Simplify the terms:

$$9 - 3(2k + 4) + 7k + 1 = 0$$

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$$9 - 6k - 12 + 7k + 1 = 0$$

$$k - 2 = 0 \Longrightarrow k = 2$$

Answer: $k = 2$

Question 7: Find the value of m in $2x^2 + 3x + m = 0$ when sum of its roots is equal to double the product of its roots

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1. **Given Equation:** $2x^2 + 3x + m = 0$ Here $a = 2, b = 3, c = m$
2. **Sum of roots (S):**

$$S = -\frac{b}{a} = -\frac{3}{2}$$

3. **Product of roots (P):**

$$P = \frac{c}{a} = \frac{m}{2}$$

4. **Given Condition:** Sum $= 2 \times$ Product

$$-\frac{3}{2} = 2\left(\frac{m}{2}\right)$$

$$-\frac{3}{2} = m \Longrightarrow m = -\frac{3}{2}$$

Answer: $m = -\frac{3}{2}$

Question 8: If α, β are the roots of $x^2 + ax + b = 0$ and α^2, β^2 are the roots of $x^2 + Ax + B = 0$, prove that $A = 2b - a^2$ and $B = b^2$

.

1. For $x^2 + ax + b = 0$ with roots α, β:
   - $\alpha + \beta = -a$
   - $\alpha\beta = b$
2. For $x^2 + Ax + B = 0$ with roots α^2, β^2:
   - **Sum of roots:**

$$\alpha^2 + \beta^2 = -A$$

$$(\alpha + \beta)^2 - 2\alpha\beta = -A$$

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Substitute $\alpha + \beta = -a$ and $\alpha\beta = b$:

$$(-a)^2 - 2(b) = -A$$

$$a^2 - 2b = -A \Longrightarrow A = 2b - a^2 \quad \textbf{(Proved)}$$

- **Product of roots:**

$$\alpha^2 \cdot \beta^2 = B$$

$$(\alpha\beta)^2 = B$$

Substitute $\alpha\beta = b$:

$$(b)^2 = B \Longrightarrow B = b^2 \quad \textbf{(Proved)}$$

Question 9: If α, β are the roots of $x^2 + px + q = 0$, find the condition that

:

Given: $\alpha + \beta = -p$ and $\alpha\beta = q$

(i) Condition when $\alpha = \beta$

1. Since $\alpha = \beta$, roots are equal.
2. Equal roots mean the discriminant of $x^2 + px + q = 0$ must be zero:

$$b^2 - 4ac = 0$$

$$p^2 - 4(1)(q) = 0 \Longrightarrow p^2 - 4q = 0$$

Condition: $p^2 = 4q$ (or $p^2 - 4q = 0$)

(ii) Condition when $\alpha = \frac{1}{\beta}$

1. Cross-multiplying gives:

$$\alpha\beta = 1$$

2. Since product of roots $\alpha\beta = q$:

$$q = 1$$

Condition: $q = 1$

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EXERCISE 2.4

General Rule for the Nature of Roots

For a quadratic equation in standard form $ax^2 + bx + c = 0$, the **discriminant** is given by:

$$D = b^2 - 4ac$$

- If $D > 0$, the roots are **real and unequal (distinct)**.
  - If D is a perfect square, the roots are **rational and unequal**.
  - If D is not a perfect square, the roots are **irrational and unequal**.
- If $D = 0$, the roots are **real and equal (repeated)**.
- If $D < 0$, the roots are **imaginary (complex conjugate/not real)**.

Question 1: Examine the nature of roots of the following quadratic equations

(i) $3x^2 - 9x - 2 = 0$

- **Identify coefficients:** $a = 3, b = -9, c = -2$
- **Calculate Discriminant (D):**

$$D = b^2 - 4ac$$
$$D = (-9)^2 - 4(3)(-2)$$
$$D = 81 + 24 = 105$$

- **Conclusion:** Since $D = 105 > 0$ (and not a perfect square), the roots are **real, irrational, and unequal**.

(ii) $x^2 + 6x + 9 = 0$

- **Identify coefficients:** $a = 1, b = 6, c = 9$
- **Calculate Discriminant (D):**

$$D = b^2 - 4ac$$
$$D = (6)^2 - 4(1)(9)$$
$$D = 36 - 36 = 0$$

- **Conclusion:** Since $D = 0$, the roots are **real and equal**.

(iii) $2x^2 + 4x + 5 = 0$

- **Identify coefficients:** $a = 2, b = 4, c = 5$
- **Calculate Discriminant (D):**

$$D = b^2 - 4ac$$
$$D = (4)^2 - 4(2)(5)$$
$$D = 16 - 40 = -24$$

- **Conclusion:** Since $D = -24 < 0$, the roots are **imaginary (not real)**.

(iv) $7x^2 - 6x - 1 = 0$

- **Identify coefficients:** $a = 7, b = -6, c = -1$
- **Calculate Discriminant (D):**

$$D = b^2 - 4ac$$
$$D = (-6)^2 - 4(7)(-1)$$
$$D = 36 + 28 = 64$$

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- **Conclusion:** Since $D = 64 > 0$ and is a perfect square (8^2), the roots are **real, rational, and unequal**.

(v) $5x^2 - 2x + 10 = 0$

- **Identify coefficients:** $a = 5, b = -2, c = 10$
- **Calculate Discriminant (D):**

$$D = b^2 - 4ac$$
$$D = (-2)^2 - 4(5)(10)$$
$$D = 4 - 200 = -196$$

- **Conclusion:** Since $D = -196 < 0$, the roots are **imaginary (not real)**.

(vi) $x^2 - 8x + 16 = 0$

- **Identify coefficients:** $a = 1, b = -8, c = 16$
- **Calculate Discriminant (D):**

$$D = b^2 - 4ac$$
$$D = (-8)^2 - 4(1)(16)$$
$$D = 64 - 64 = 0$$

- **Conclusion:** Since $D = 0$, the roots are **real and equal**.

Question 2

For what values of t, the roots of $3x^2 + x + 9t = 0$ are real and unequal?

1. **Identify coefficients:** $a = 3, b = 1, c = 9t$
2. **Condition for real and unequal roots:** $D > 0$
3. **Set up the inequality:**

$$b^2 - 4ac > 0$$
$$(1)^2 - 4(3)(9t) > 0$$
$$1 - 108t > 0$$
$$1 > 108t$$
$$t < \frac{1}{108}$$

- **Answer:** The roots are real and unequal for $t < \frac{1}{108}$.

Question 3

If the quadratic equation $16x^2 + 7px + 49 = 0$ has equal roots, then find the values of p.

1. **Identify coefficients:** $a = 16, b = 7p, c = 49$
2. **Condition for equal roots:** $D = 0$
3. **Set up the equation:**

$$b^2 - 4ac = 0$$
$$(7p)^2 - 4(16)(49) = 0$$
$$49p^2 - 3136 = 0$$
$$49p^2 = 3136$$

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$$p^2 = \frac{3136}{49}$$
$$p^2 = 64$$
$$p = \pm\sqrt{64} = \pm 8$$

- **Answer:** $p = \pm 8$ (or $p = 8, -8$).

Question 4

If the quadratic equation $4u^2 + 8u + q = 0$ has unequal and real roots, find the possible values for q.

1. **Identify coefficients:** $a = 4, b = 8, c = q$
2. **Condition for real and unequal roots:** $D > 0$
3. **Set up the inequality:**

$$b^2 - 4ac > 0$$
$$(8)^2 - 4(4)(q) > 0$$
$$64 - 16q > 0$$
$$64 > 16q$$
$$q < \frac{64}{16}$$
$$q < 4$$

- **Answer:** The possible values are $q < 4$.

Question 5

Find the value for m, if the quadratic equation $mx^2 - 8x + 1 = 0$ has real and equal roots.

1. **Identify coefficients:** $a = m, b = -8, c = 1$
2. **Condition for real and equal roots:** $D = 0$ (with $a \neq 0$, so $m \neq 0$)
3. **Set up the equation:**

$$b^2 - 4ac = 0$$
$$(-8)^2 - 4(m)(1) = 0$$
$$64 - 4m = 0$$
$$4m = 64$$
$$m = \frac{64}{4}$$
$$m = 16$$

- **Answer:** $m = 16$.

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EXERCISE 2.5

General Method for Solving Quadratic Inequalities

1. **Find Critical Values: Set the quadratic expression equal to 0 and solve for x (find the roots).**
2. **Factorize: Write the quadratic in factored form $(x - r_1)(x - r_2) = 0$.**
3. **Analyze Intervals: Use the critical points to test intervals or apply standard rules:**
   - **For $(x - a)(x - b) > 0$ with $a < b$: Solution is $x < a$ or $x > b$.**
   - **For $(x - a)(x - b) < 0$ with $a < b$: Solution is $a < x < b$.**

(i) $x^2 + 3x - 4 > 0$

1. **Find Critical Values:**

$$x^2 + 3x - 4 = 0$$

$$x^2 + 4x - x - 4 = 0$$

$$x(x + 4) - 1(x + 4) = 0$$

$$(x + 4)(x - 1) = 0$$

$$x = -4 \quad \text{or} \quad x = 1$$

2. **Solve the Inequality: Since we want $(x + 4)(x - 1) > 0$, the expression is positive outside the roots.**

- **Solution: $x < -4$ or $x > 1$ (or in interval notation: $(-\infty, -4) \cup (1, \infty)$)**

(ii) $2x^2 - 8x + 6 > 0$

1. **Simplify by dividing by 2:**

$$x^2 - 4x + 3 > 0$$

2. **Find Critical Values:**

$$x^2 - 4x + 3 = 0$$

$$x^2 - 3x - x + 3 = 0$$

$$x(x - 3) - 1(x - 3) = 0$$

$$(x - 3)(x - 1) = 0$$

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$$x = 1 \quad \text{or} \quad x = 3$$

3. **Solve the Inequality: Since we want $(x-1)(x-3) > 0$, the expression is positive outside the roots.**

- **Solution: $x < 1$ or $x > 3$ (or in interval notation: $(-\infty, 1) \cup (3, \infty)$)**

(iii) $x^2 + x - 6 < 0$

1. **Find Critical Values:**

$$x^2 + x - 6 = 0$$

$$x^2 + 3x - 2x - 6 = 0$$

$$x(x+3) - 2(x+3) = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3 \quad \text{or} \quad x = 2$$

2. **Solve the Inequality: Since we want $(x+3)(x-2) < 0$, the expression is negative between the roots.**

- **Solution: $-3 < x < 2$ (or in interval notation: $(-3, 2)$)**

(iv) $x^2 - 6x + 9 < 0$

1. **Factorize the expression:**

$$x^2 - 6x + 9 = (x-3)^2$$

2. **Analyze the Inequality:**

$$(x-3)^2 < 0$$

Since any real number squared is non-negative ($(x-3)^2 \geq 0$ for all real x), it can never be strictly less than 0.

- **Solution: No solution (or $\emptyset$)**

(v) $4x^2 - 16x + 15 \leq 0$

1. **Find Critical Values:**

$$4x^2 - 16x + 15 = 0$$

$$4x^2 - 10x - 6x + 15 = 0$$

$$2x(2x-5) - 3(2x-5) = 0$$

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$$(2x-3)(2x-5)=0$$

$$x=\frac{3}{2} \quad \text{or} \quad x=\frac{5}{2}$$

2. **Solve the Inequality: Since we want $(2x-3)(2x-5)\leq 0$, the values lie between (and include) the roots.**

- **Solution: $\frac{3}{2}\leq x\leq \frac{5}{2}$ (or in interval notation: $\left[\frac{3}{2},\frac{5}{2}\right]$)**

(vi) $-x^2+3x-2\geq 0$

1. **Multiply by -1 (remember to reverse the inequality sign):**

$$x^2-3x+2\leq 0$$

2. **Find Critical Values:**

$$x^2-3x+2=0$$

$$(x-1)(x-2)=0$$

$$x=1 \quad \text{or} \quad x=2$$

3. **Solve the Inequality: Since we want $(x-1)(x-2)\leq 0$, the values lie between (and include) the roots.**

- **Solution: $1\leq x\leq 2$ (or in interval notation: $[1,2]$)**

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EXERCISE 2.6

Question 1

Make F the subject of the formula: $C^\circ = \frac{5}{9}(F^\circ - 32)$

1. Given equation:

$$C^\circ = \frac{5}{9}(F^\circ - 32)$$

2. Multiply both sides by 9:

$$9C^\circ = 5(F^\circ - 32)$$

3. Divide both sides by 5:

$$\frac{9}{5}C^\circ = F^\circ - 32$$

4. Add 32 to both sides:

$$F^\circ = \frac{9}{5}C^\circ + 32$$

Question 2

The formula for finding simple interest is $I = PRT$.

(a) Make *P* the subject of the formula:

1. Given equation:

$$I = PRT$$

2. Divide both sides by RT:

$$P = \frac{I}{RT}$$

(b) Make *T* the subject of the formula:

1. Given equation:

$$I = PRT$$

2. Divide both sides by PR:

$$T = \frac{I}{PR}$$

Question 3

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Make 'a' the subject of the formula: $S = 2a + (n-1)d$

1. Given equation:

$$S = 2a + (n-1)d$$

2. Subtract $(n-1)d$ from both sides:

$$S - (n-1)d = 2a$$

3. Divide both sides by 2:

$$a = \frac{S - (n-1)d}{2}$$

Question 4

The volume of a cylinder is given by the formula $V = \pi r^2 h$. Make 'h' the subject of the formula.

1. Given equation:

$$V = \pi r^2 h$$

2. Divide both sides by πr^2:

$$h = \frac{V}{\pi r^2}$$

Question 5

The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, make h the subject of the formula.

1. Given equation:

$$A = \frac{1}{2}h(b_1 + b_2)$$

2. Multiply both sides by 2:

$$2A = h(b_1 + b_2)$$

3. Divide both sides by $(b_1 + b_2)$:

$$h = \frac{2A}{b_1 + b_2}$$

Question 6

If $y = mx + c$, then make 'x' the subject of this equation.

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1. Given equation:

$$y = mx + c$$

2. Subtract c from both sides:

$$y - c = mx$$

3. Divide both sides by m:

$$x = \frac{y - c}{m}$$

Question 7

Perimeter (P) of a rectangle is $P = 2(\ell + w)$, make ℓ as the subject of this formula.

1. Given equation:

$$P = 2(\ell + w)$$

2. Divide both sides by 2:

$$\frac{P}{2} = \ell + w$$

3. Subtract w from both sides:

$$\ell = \frac{P}{2} - w \quad \text{or} \quad \ell = \frac{P - 2w}{2}$$

Question 8

The equation of a parabola is $y^2 = 4ax$, make 'x' as a subject of this equation.

1. Given equation:

$$y^2 = 4ax$$

2. Divide both sides by $4a$:

$$x = \frac{y^2}{4a}$$

Question 9

If $P = S - C$, where S is selling price and C is cost price. Make S as subject of the equation.

1. Given equation:

$$P = S - C$$

2. Add C to both sides:

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$$S = P + C$$

Question 10

Volume of the cone is $V = \frac{1}{3}\pi r^2 h$, make 'h' as subject of this formula.

1. Given equation:

$$V = \frac{1}{3}\pi r^2 h$$

2. Multiply both sides by 3:

$$3V = \pi r^2 h$$

3. Divide both sides by πr^2:

$$h = \frac{3V}{\pi r^2}$$

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EXERCISE 2.7

Question 1

A town's population is modeled by $P(t) = -2t^2 + 40t + 800$, where t is years since 2020. Find the years when the population will be at least 1000.

Solution:

1. **Set up the inequality:** We want $P(t) \geq 1000$:

$$-2t^2 + 40t + 800 \geq 1000$$

2. **Rearrange into standard quadratic form:**

$$-2t^2 + 40t + 800 - 1000 \geq 0$$

$$-2t^2 + 40t - 200 \geq 0$$

3. **Divide by** -2 (remembering to reverse the inequality sign):

$$t^2 - 20t + 100 \leq 0$$

4. **Factorize the quadratic expression:**

$$(t - 10)^2 \leq 0$$

5. **Solve for** t**:** Since a square of any real number cannot be negative (< 0), the inequality only holds when $(t - 10)^2 = 0$.

$$t - 10 = 0 \Longrightarrow t = 10$$

6. **Find the actual calendar year:**

$$\text{Year} = 2020 + t = 2020 + 10 = 2030$$

Answer: The population will be at least 1000 in the year **2030**.

Question 2

A company models its profit P in thousands of rupees by the equation: $P(x) = -5x^2 + 150x - 1000$, where x is the price per item in rupees. Find the price that gives maximum profit.

Solution:

1. **Identify the vertex formula for maximum value:** For a quadratic equation $P(x) = ax^2 + bx + c$, the maximum value occurs at the vertex $x = -\frac{b}{2a}$ because $a = -5 < 0$.

2. **Extract coefficients:**

$$a = -5, \quad b = 150, \quad c = -1000$$

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3. **Calculate x:**

$$x = -\frac{150}{2(-5)}$$

$$x = -\frac{150}{-10}$$

$$x = 15$$

Answer: The price per item that gives maximum profit is **15 rupees**.

Question 3

A toy car rolls down an incline and covers a distance given by the equation $d = t^2 - 0.5t$ metres, where t is the time in seconds. Find the time when the car has travelled a distance 12.5 metres.

Solution:

1. **Set up the equation:** Given $d = 12.5$:

$$t^2 - 0.5t = 12.5$$

2. **Rearrange into standard form:**

$$t^2 - 0.5t - 12.5 = 0$$

3. **Multiply by 2 to clear decimals:**

$$2t^2 - t - 25 = 0$$

4. **Apply the quadratic formula:** $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Here $a = 2$, $b = -1$, $c = -25$:

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-25)}}{2(2)}$$

$$t = \frac{1 \pm \sqrt{1 + 200}}{4}$$

$$t = \frac{1 \pm \sqrt{201}}{4}$$

5. **Calculate numerical value:**

$$\sqrt{201} \approx 14.177$$

Since time t must be positive ($t > 0$):

$$t = \frac{1 + 14.177}{4} = \frac{15.177}{4} \approx 3.79 \text{ seconds}$$

Answer: The car travels 12.5 metres at $t \approx 3.8$ **seconds** (or $t = \frac{1+\sqrt{201}}{4}$ s).

Question 4

A ball's height (in metres) after t seconds is $h(t) = -4t^2 + 24t$. For what time interval is the ball at least 20m above the ground?

Solution:

1. **Set up the inequality:**

$$h(t) \geq 20$$

$$-4t^2 + 24t \geq 20$$

2. **Rearrange into standard form:**

$$-4t^2 + 24t - 20 \geq 0$$

3. **Divide by** -4 (reversing the inequality sign):

$$t^2 - 6t + 5 \leq 0$$

4. **Factorize:**

$$(t - 1)(t - 5) \leq 0$$

5. **Solve the inequality:** The product is negative or zero between its roots $t = 1$ and $t = 5$:

$$1 \leq t \leq 5$$

Answer: The ball is at least 20m above the ground during the time interval $1 \leq t \leq 5$ **seconds** (from 1 second to 5 seconds).

Question 5

A ball is thrown upward with an initial velocity of 40 ms^{-1}. Calculate the maximum height it reaches above ground level.

Solution:

- **Method 1: Physics Formula**
    1. Using third equation of motion:

$$v^2 = u^2 - 2gh$$

    2. At maximum height, final velocity $v = 0$:

$$0 = (40)^2 - 2gh$$

$$2gh = 1600$$

3. Taking $g = 9.8$ ms^{-2}:

$$h = \frac{1600}{2 \times 9.8} = \frac{1600}{19.6} \approx 81.63 \text{ metres}$$

(Note: If taking $g = 10$ ms^{-2}, $h = \frac{1600}{20} = 80$ metres)

- **Method 2: Quadratic Model ($h(t) = ut - \frac{1}{2}gt^2$)**

1. Height equation with $u = 40$ and $g = 9.8$:

$$h(t) = -4.9t^2 + 40t$$

2. Time to reach maximum height:

$$t = -\frac{b}{2a} = -\frac{40}{2(-4.9)} \approx 4.08 \text{ seconds}$$

3. Maximum height:

$$h(4.08) = -4.9(4.08)^2 + 40(4.08) \approx 81.63 \text{ metres}$$

Answer: The maximum height reached is 81.63 m (or 80 m using $g = 10$ ms^{-2}).

Question 6

A freelancer's earnings follow the model $E(h) = -2h^2 + 40h$, where E is earning in rupees and h is hours worked per week. What is the maximum number of hours he should work to maximize earnings?

Solution:

1. **Identify the quadratic function:**

$$E(h) = -2h^2 + 40h$$

This is a parabola opening downward ($a = -2 < 0$), so its peak is at the vertex.

2. **Extract coefficients:**

$$a = -2, \quad b = 40, \quad c = 0$$

3. **Calculate the vertex h:**

$$h = -\frac{b}{2a}$$

$$h = -\frac{40}{2(-2)}$$

$$h = -\frac{40}{-4}$$

$$h = 10$$

Answer: He should work **10 hours per week** to maximize his earnings.

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Review Exercise 2.

Question 1: Multiple Choice Questions (MCQs)

(i) The type of the equation $2x^2 - x + 1 = 0$ is:

- **Options:** (a) quadratic (b) linear (c) third degree (d) Pure quadratic
- **Steps:**
  - An equation of the form $ax^2 + bx + c = 0$ with degree 2 and $b \neq 0$ is a **quadratic** equation.
- **Correct Answer: (a) quadratic**

(ii) What is the discriminant of $x^2 + 5x - 5 = 0$?

- **Options:** (a) -20 (b) 20 (c) 25 (d) 45
- **Steps:**
  - $a = 1, b = 5, c = -5$
  - Discriminant $D = b^2 - 4ac$
  - $D = (5)^2 - 4(1)(-5) = 25 + 20 = 45$
- **Correct Answer: (d) 45**

(iii) The solution set of $3x^2 - 9 = 0$ is:

- **Options:** (a) $\{3\}$ (b) $\{\pm 3\}$ (c) $\{\pm\sqrt{3}\}$ (d) $\{\sqrt{3}\}$
- **Steps:**
  - $3x^2 = 9 \Longrightarrow x^2 = 3 \Longrightarrow x = \pm\sqrt{3}$
  - Solution set = $\{\pm\sqrt{3}\}$
- **Correct Answer: (c) $\{\pm\sqrt{3}\}$**

(iv) Sum of the roots of $3x^2 + 5x - 12 = 0$ is:

- **Options:** (a) $\frac{5}{3}$ (b) $-\frac{5}{3}$ (c) $\frac{12}{3}$ (d) $\frac{3}{5}$
- **Steps:**
  - $a = 3, b = 5, c = -12$
  - Sum of roots $S = -\frac{b}{a} = -\frac{5}{3}$

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- **Correct Answer: (b)** $-\frac{5}{3}$

(v) Product of the roots of $3x^2 + 5x - 12 = 0$ is:

- **Options:** (a) -4 (b) 3 (c) 4 (d) 5
- **Steps:**
  - $a = 3, b = 5, c = -12$
  - Product of roots $P = \frac{c}{a} = \frac{-12}{3} = -4$
- **Correct Answer: (a)** -4

(vi) What are the roots of $(x - 3)(x + 3) = 0$?

- **Options:** (a) $3, -3$ (b) 3,3 (c) $-3, -3$ (d) 9,0
- **Steps:**
  - $x - 3 = 0 \Longrightarrow x = 3$
  - $x + 3 = 0 \Longrightarrow x = -3$
- **Correct Answer: (a)** $3, -3$

(vii) 3 and 2 are the roots of:

- **Options:** (a) $x^2 + 5x + 6 = 0$ (b) $x^2 + 6x + 5 = 0$ (c) $x^2 - 5x + 6 = 0$ (d) $x^2 + 6x - 5 = 0$
- **Steps:**
  - Sum of roots $S = 3 + 2 = 5$
  - Product of roots $P = 3 \times 2 = 6$
  - Quadratic equation formula: $x^2 - Sx + P = 0 \Longrightarrow x^2 - 5x + 6 = 0$
- **Correct Answer: (c)** $x^2 - 5x + 6 = 0$

(viii) If $b^2 - 4ac > 0$ and is a perfect square, then the roots of $ax^2 + bx + c = 0$ are:

- **Options:** (a) equal (b) unequal (c) imaginary (d) irrational
- **Steps:**
  - When $b^2 - 4ac > 0$ and is a perfect square, the roots are **rational and unequal**. Since "unequal" is listed, it is the correct choice among the given options.
- **Correct Answer: (b) unequal**

(ix) If $b^2 - 4ac = 0$, then the roots of $ax^2 + bx + c = 0$ are:

- **Options:** (a) unequal (b) irrational (c) imaginary (d) equal
- **Steps:**
  - When the discriminant equals zero, the roots are real and **equal**.
- **Correct Answer: (d) equal**

(x) Subject "c" of $x - 2c = b$ is:

- **Options:** (a) $x + b$ (b) $b - x$ (c) $\frac{x-b}{2}$ (d) $\frac{b-x}{2}$
- **Steps:**
  - $x - 2c = b$
  - $x - b = 2c$
  - $c = \frac{x-b}{2}$
- **Correct Answer: (c)** $\frac{x-b}{2}$

Question 2: Solve the quadratic equations by 3 methods

(i) $8x^2 = x + 7 \Rightarrow 8x^2 - x - 7 = 0$

Method A: Factorization

1. Split middle term: $-x = -8x + 7x$

$$8x^2 - 8x + 7x - 7 = 0$$

$$8x(x - 1) + 7(x - 1) = 0$$

$$(8x + 7)(x - 1) = 0$$

2. Solve:

$$8x + 7 = 0 \Rightarrow x = -\frac{7}{8}$$

$$x - 1 = 0 \Rightarrow x = 1$$

- **Solution Set:** $\left\{1, -\frac{7}{8}\right\}$

Method B: Completing Square

1. Divide by 8:

$$x^2 - \frac{1}{8}x - \frac{7}{8} = 0 \Rightarrow x^2 - \frac{1}{8}x = \frac{7}{8}$$

2. Add $\left(\frac{1}{2}\cdot\frac{1}{8}\right)^2 = \left(\frac{1}{16}\right)^2 = \frac{1}{256}$ to both sides:

$$\left(x - \frac{1}{16}\right)^2 = \frac{7}{8} + \frac{1}{256}$$

$$\left(x - \frac{1}{16}\right)^2 = \frac{224+1}{256} = \frac{225}{256}$$

3. Take square root:

$$x - \frac{1}{16} = \pm\frac{15}{16}$$

$$x = \frac{1}{16} \pm \frac{15}{16}$$

$$x = \frac{1+15}{16} = \frac{16}{16} = 1 \quad \text{or} \quad x = \frac{1-15}{16} = -\frac{14}{16} = -\frac{7}{8}$$

- **Solution Set:** $\left\{1, -\frac{7}{8}\right\}$

Method C: Quadratic Formula

1. Coefficients: $a = 8, b = -1, c = -7$
2. Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-7)}}{2(8)}$$

$$x = \frac{1 \pm \sqrt{1+224}}{16} = \frac{1 \pm \sqrt{225}}{16} = \frac{1 \pm 15}{16}$$

$$x = \frac{1+15}{16} = 1 \quad \text{or} \quad x = \frac{1-15}{16} = -\frac{7}{8}$$

- **Solution Set:** $\left\{1, -\frac{7}{8}\right\}$

(ii) $2x^2 - x - 10 = 0$

Method A: Factorization

1. Split middle term: $-x = -5x + 4x$

$$2x^2 + 4x - 5x - 10 = 0$$

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$$2x(x+2) - 5(x+2) = 0$$

$$(2x-5)(x+2) = 0$$

2. Solve:

$$2x - 5 = 0 \Longrightarrow x = \frac{5}{2}$$

$$x + 2 = 0 \Longrightarrow x = -2$$

- **Solution Set:** $\left\{-2, \frac{5}{2}\right\}$

Method B: Completing Square

1. Divide by 2:

$$x^2 - \frac{1}{2}x - 5 = 0 \Longrightarrow x^2 - \frac{1}{2}x = 5$$

2. Add $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$ to both sides:

$$\left(x - \frac{1}{4}\right)^2 = 5 + \frac{1}{16} = \frac{81}{16}$$

3. Take square root:

$$x - \frac{1}{4} = \pm\frac{9}{4}$$

$$x = \frac{1}{4} \pm \frac{9}{4}$$

$$x = \frac{1+9}{4} = \frac{10}{4} = \frac{5}{2} \quad \text{or} \quad x = \frac{1-9}{4} = -\frac{8}{4} = -2$$

- **Solution Set:** $\left\{-2, \frac{5}{2}\right\}$

Method C: Quadratic Formula

1. Coefficients: $a = 2, b = -1, c = -10$
2. Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-10)}}{2(2)}$$

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$$x = \frac{1 \pm \sqrt{1+80}}{4} = \frac{1 \pm \sqrt{81}}{4} = \frac{1 \pm 9}{4}$$

$$x = \frac{1+9}{4} = \frac{5}{2} \quad \text{or} \quad x = \frac{1-9}{4} = -2$$

- **Solution Set:** $\left\{-2, \frac{5}{2}\right\}$

Question 3: Form a quadratic equation whose roots are $6, \frac{3}{2}$

1. **Calculate Sum of roots (S):**

$$S = 6 + \frac{3}{2} = \frac{12+3}{2} = \frac{15}{2}$$

2. **Calculate Product of roots (P):**

$$P = 6 \times \frac{3}{2} = 9$$

3. **Form the Quadratic Equation:**

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{15}{2}x + 9 = 0$$

4. **Multiply by 2 to clear the fraction:**

$$2x^2 - 15x + 18 = 0$$

- **Answer:** $2x^2 - 15x + 18 = 0$

Question 4: Examine the nature of the roots of the following equations

(i) $15x^2 + 11x + 2 = 0$

- **Coefficients:** $a = 15, b = 11, c = 2$
- **Discriminant (D):**

$$D = b^2 - 4ac = (11)^2 - 4(15)(2) = 121 - 120 = 1$$

- **Conclusion:** Since $D = 1 > 0$ and is a perfect square (1^2), the roots are **real, rational, and unequal**.

(ii) $x^2 - x - 1 = 0$

- **Coefficients:** $a = 1, b = -1, c = -1$
- **Discriminant (D):**

$$D = b^2 - 4ac = (-1)^2 - 4(1)(-1) = 1 + 4 = 5$$

- **Conclusion:** Since $D = 5 > 0$ and is not a perfect square, the roots are **real, irrational, and unequal**.

Question 5: Rearrange the formula $h = \frac{v^2}{2g}$ to make v the subject

1. Given formula:

$$h = \frac{v^2}{2g}$$

2. Multiply both sides by $2g$:

$$v^2 = 2gh$$

3. Take the square root of both sides:

$$v = \sqrt{2gh}$$

- **Answer:** $v = \sqrt{2gh}$

Question 6: If $x^2 + 2(1 + k)x + k^2 = 0$ has equal roots, find k

1. **Identify Coefficients:**

$$a = 1, \quad b = 2(1 + k) = 2 + 2k, \quad c = k^2$$

2. **Condition for Equal Roots:** Discriminant $D = 0$

$$b^2 - 4ac = 0$$

$$[2(1 + k)]^2 - 4(1)(k^2) = 0$$

$$4(1 + 2k + k^2) - 4k^2 = 0$$

$$4 + 8k + 4k^2 - 4k^2 = 0$$

$$4 + 8k = 0$$

$$8k = -4$$

$$k = -\frac{4}{8} = -\frac{1}{2}$$

- **Answer:** $k = -\frac{1}{2}$

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