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Class	10th	Chapter	03
Subject	Mathematics	Board	Punjab

Chapter 03: Matrices and Determinants

EXERCISE 3.1

Question 1: Write the number of rows and number of columns in each matrix.

A matrix of dimension $m \times n$ has m horizontal rows and n vertical columns.

- **Matrix** $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$:
  - Number of rows = **2**
  - Number of columns = **2**
- **Matrix** $B = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$:
  - Number of rows = **3**
  - Number of columns = **1**
- **Matrix** $C = [8 \quad -10 \quad 11]$:
  - Number of rows = **1**
  - Number of columns = **3**
- **Matrix** $D = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & 3 \\ 4 & 5 & 6 \end{bmatrix}$:
  - Number of rows = **3**
  - Number of columns = **3**
- **Matrix** $E = \begin{bmatrix} 5 & 9 & -2 \\ -3 & 4 & 5 \end{bmatrix}$:
  - Number of rows = **2**
  - Number of columns = **3**

Question 2: Write the order of each matrix

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The order of a matrix is given as Number of Rows × Number of Columns.

- $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow$ Order = 2×1

- $B = [3 \quad 4] \rightarrow$ Order = 1×2
- $C = \begin{bmatrix} 3 & 5 \\ 5 & 6 \end{bmatrix} \rightarrow$ Order = 2×2
- $D = \begin{bmatrix} 0 & 3 & 4 \\ 5 & 0 & 9 \end{bmatrix} \rightarrow$ Order = 2×3
- $E = \begin{bmatrix} 0 & 1 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \rightarrow$ Order = 3×2
- $F = [5] \rightarrow$ Order = 1×1

Question 3: Which of the following matrices are equal?

Two matrices are equal if they have the same order and their corresponding elements are equal.

Let's simplify the given matrices first:

- $A = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$
- $B = \begin{bmatrix} 9 \\ -7 \end{bmatrix}$
- $C = \begin{bmatrix} 2 \times 3 & 2 - 1 \\ 2 \times 2 & 4 - 2 \\ 4 + 4 & 3 + 0 \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 4 & 2 \\ 8 & 3 \end{bmatrix}$
- $D = \begin{bmatrix} 5 + 4 \\ -8 + 1 \end{bmatrix} = \begin{bmatrix} 9 \\ -7 \end{bmatrix}$
- $E = \begin{bmatrix} 6 & 1 \\ 4 & 2 \\ 8 & 3 \end{bmatrix}$
- $F = \begin{bmatrix} 3 - 3 & 3 \\ 3 + 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$

Comparison:

1. A and F have the exact same order (2×2) and identical corresponding elements $\begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$.
2. B and D have the exact same order (2×1) and identical corresponding elements $\begin{bmatrix} 9 \\ -7 \end{bmatrix}$.
3. C and E have the exact same order (3×2) and identical corresponding elements $\begin{bmatrix} 6 & 1 \\ 4 & 2 \\ 8 & 3 \end{bmatrix}$.

Answer:

- $A = F$
- $B = D$
- $C = E$

Question 4: If $\begin{bmatrix} a+2 & c-3 \\ b-1 & d+4 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 6 & 4 \end{bmatrix}$, find the values of a, b, c and d

.

By equating corresponding elements:

1. **Find a:**

$$a + 2 = 5 \Longrightarrow a = 5 - 2 \Longrightarrow \mathbf{a = 3}$$

2. **Find c:**

$$c - 3 = 8 \Longrightarrow c = 8 + 3 \Longrightarrow \mathbf{c = 11}$$

3. **Find b:**

$$b - 1 = 6 \Longrightarrow b = 6 + 1 \Longrightarrow \mathbf{b = 7}$$

4. **Find d:**

$$d + 4 = 4 \Longrightarrow d = 4 - 4 \Longrightarrow \mathbf{d = 0}$$

Answer: $a = 3, b = 7, c = 11, d = 0$

Question 5: If $\begin{bmatrix} 2x+1 & 4 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 7 & y \end{bmatrix}$, find the values of x and y

.

By equating corresponding elements:

1. **Find x:**

$$2x + 1 = 9$$
$$2x = 9 - 1$$
$$2x = 8 \Longrightarrow \mathbf{x = 4}$$

2. **Find y:**

$$5 = y \Longrightarrow \mathbf{y = 5}$$

Answer: $x = 4, y = 5$

Question 6: If $\begin{bmatrix} a+b & 2d-1 \\ 3b+2 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 11 & c \end{bmatrix}$, find the values of a, b, c and d

.

By equating corresponding elements:

1. **Find c:**

$$4 = c \Longrightarrow \mathbf{c = 4}$$

2. **Find d:**

$$2d - 1 = 5$$
$$2d = 6 \Longrightarrow \mathbf{d = 3}$$

3. **Find b:**

$$3b + 2 = 11$$
$$3b = 11 - 2$$
$$3b = 9 \Longrightarrow \mathbf{b = 3}$$

4. **Find a:**

$$a + b = 10$$

Substitute $b = 3$:

$$a + 3 = 10 \Longrightarrow a = 10 - 3 \Longrightarrow \mathbf{a = 7}$$

Answer: $a = 7, b = 3, c = 4, d = 3$

Question 7: If $\begin{bmatrix} p+q & 5 \\ 11 & p-2q \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 11 & 0 \end{bmatrix}$, find the values of p and q

.

Equating the corresponding elements gives a system of two equations:

1. $p + q = 6$ *(Equation 1)*
2. $p - 2q = 0$ *(Equation 2)*

Step 1: From Equation 2, express p in terms of q:

$$p = 2q$$

Step 2: Substitute $p = 2q$ into Equation 1:

$$2q + q = 6$$
$$2q + q = 6$$

Step 3: Substitute $q = 2$ back into $p = 2q$:

$$p = 2(2) \Longrightarrow \mathbf{p = 4}$$

Answer: $p = 4, q = 2$

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EXERCISE 3.2

Question 1: From the following matrices identify unit matrices, row matrices, column matrices, and null matrices.

- **Definitions:**
  - **Row matrix:** Has only 1 row.
  - **Column matrix:** Has only 1 column.
  - **Unit (Identity) matrix:** A diagonal matrix where all main diagonal entries are equal to 1 and all non-diagonal entries are 0.
  - **Null matrix:** All elements are 0.

- $A = [5 \quad 7 \quad 8]$: **Row matrix** (1 row, 3 columns)
- $B = [0]$: **Null matrix** (also a row and column matrix, but primarily a 1×1 null matrix)
- $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$: **Unit (Identity) matrix**
- $D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$: **Null matrix**
- $E = \begin{bmatrix} 6 \\ 0 \\ 8 \end{bmatrix}$: **Column matrix** (3 rows, 1 column)
- $F = \begin{bmatrix} 7 \\ 1 \\ 9 \end{bmatrix}$: **Column matrix** (3 rows, 1 column)

Question 2: Identify type of the given matrices as row, column, square, and rectangular matrices

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- **Definitions:**
  - **Square matrix:** Number of rows = Number of columns.
  - **Rectangular matrix:** Number of rows ≠ Number of columns.

- $A = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$ (Order 3×1): **Column matrix** and **Rectangular matrix**
- $B = \begin{bmatrix} 1 & 6 \\ 4 & 1 \end{bmatrix}$ (Order 2×2): **Square matrix**
- $C = \begin{bmatrix} 3 & 5 & 8 \\ 0 & 4 & -2 \end{bmatrix}$ (Order 2×3): **Rectangular matrix**
- $D = \begin{bmatrix} 5 & -5 \\ 2 & 7 \end{bmatrix}$ (Order 2×2): **Square matrix**
- $E = \begin{bmatrix} 3 & 2 \\ 4 & 1 \\ 5 & 0 \end{bmatrix}$ (Order 3×2): **Rectangular matrix**
- $F = [5 \quad -3 \quad 7]$ (Order 1×3): **Row matrix** and **Rectangular matrix**
- $G = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 3 & 4 \\ 5 & 2 & -3 \end{bmatrix}$ (Order 3×3): **Square matrix**

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- $H = \begin{bmatrix} 3 & 5 \\ 4 & 4 \\ 5 & 2 \end{bmatrix}$ (Order 3×2): **Rectangular matrix**

Question 3: Identify diagonal, scalar, and unit matrices

.

- **Definitions:**
  - **Diagonal matrix:** A square matrix where all non-diagonal elements are zero, and at least one diagonal element is non-zero.
  - **Scalar matrix:** A diagonal matrix where all diagonal elements are equal and non-zero.
  - **Unit matrix:** A scalar matrix where all diagonal elements are equal to 1.
- $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$: **Unit matrix** (also scalar and diagonal)
- $B = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$: **Scalar matrix** (also diagonal)
- $C = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$: **Diagonal matrix**
- $D = \begin{bmatrix} 7 & 0 \\ 0 & 6 \end{bmatrix}$: **Diagonal matrix**
- $E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$: **Scalar matrix** (also diagonal)

Question 4: Find transpose of each of the following matrices

.

The transpose of a matrix is obtained by swapping its rows and columns (M^t).

- $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$:

$$A^t = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

- $B = \begin{bmatrix} 3 \\ 7 \\ 6 \end{bmatrix}$:

$$B^t = \begin{bmatrix} 3 & 7 & 6 \end{bmatrix}$$

- $C = \begin{bmatrix} 5 & -2 & 4 \end{bmatrix}$:

$$C^t = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

- $D = \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$:

$$D^t = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

Question 5: Find negative of the following matrices

.

The negative of a matrix M is $-M$, obtained by changing the signs of all its entries.

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- $A = \begin{bmatrix} -3 & 0 \\ 5 & 6 \end{bmatrix}$:

$$-A = \begin{bmatrix} 3 & 0 \\ -5 & -6 \end{bmatrix}$$

- $B = \begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix}$:

$$-B = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}$$

- $C = \begin{bmatrix} -9 & 1 \\ 1 & -7 \end{bmatrix}$:

$$-C = \begin{bmatrix} 9 & -1 \\ -1 & 7 \end{bmatrix}$$

Question 6: If $\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} \mathbf{7} & \mathbf{6} \\ \mathbf{5} & \mathbf{8} \end{bmatrix}$, then verify that

:

(i) $(\boldsymbol{A^t})^t = \boldsymbol{A}$

1. **Find A^t:**

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \Longrightarrow A^t = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

2. **Find $(A^t)^t$:** Taking the transpose of A^t:

$$(A^t)^t = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

3. **Conclusion:**

$$(A^t)^t = A \quad \text{(Hence verified)}$$

(ii) $(\boldsymbol{B^t})^t = \boldsymbol{B}$

1. **Find B^t:**

$$B = \begin{bmatrix} 7 & 6 \\ 5 & 8 \end{bmatrix} \Longrightarrow B^t = \begin{bmatrix} 7 & 5 \\ 6 & 8 \end{bmatrix}$$

2. **Find $(B^t)^t$:** Taking the transpose of B^t:

$$(B^t)^t = \begin{bmatrix} 7 & 6 \\ 5 & 8 \end{bmatrix}$$

3. **Conclusion:**

$$(B^t)^t = B \quad \text{(Hence verified)}$$

Question 7: Show that $\boldsymbol{L} = \begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{3} & \mathbf{-2} & \mathbf{5} \\ \mathbf{4} & \mathbf{5} & \mathbf{0} \end{bmatrix}$ is a symmetric matrix

.

A matrix L is symmetric if $L^t = L$.

1. **Given Matrix:**

$$L = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 4 & 5 & 0 \end{bmatrix}$$

2. **Find the Transpose (L^t):** Interchanging rows into columns:

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- **Row 1** [2,3,4] becomes **Column 1**: $\begin{bmatrix}2\\3\\4\end{bmatrix}$
- **Row 2** [3, −2,5] becomes **Column 2**: $\begin{bmatrix}3\\-2\\5\end{bmatrix}$
- **Row 3** [4,5,0] becomes **Column 3**: $\begin{bmatrix}4\\5\\0\end{bmatrix}$

$$L^t = \begin{bmatrix}2 & 3 & 4\\3 & -2 & 5\\4 & 5 & 0\end{bmatrix}$$

3. **Conclusion:** Since $L^t = L$, the matrix L **is a symmetric matrix**.

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EXERCISE 3.3

Question 1: Which of the following matrices are conformable for addition and subtraction?

Rule: Matrices can be added or subtracted ***only if they have the same order (dimensions)****.*

Let's find the order of each matrix:

- $A = \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}$ (Order: 2×2)
- $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ (Order: 2×1)
- $C = [5 \quad 2]$ (Order: 1×2)
- $D = \begin{bmatrix} 1 & 7 \\ 2 & 5 \end{bmatrix}$ (Order: 2×2)
- $E = [2]$ (Order: 1×1)
- $F = [7 \quad 11]$ (Order: 1×2)
- $G = \begin{bmatrix} a \\ b \end{bmatrix}$ (Order: 2×1)
- $H = [3]$ (Order: 1×1)
- $M = \begin{bmatrix} l \\ m \end{bmatrix}$ (Order: 2×1)

Answer:

- A **and** D are conformable for addition and subtraction (both are 2×2).
- $B, G,$ **and** M are conformable for addition and subtraction (all are 2×1).
- C **and** F are conformable for addition and subtraction (both are 1×2).
- E **and** H are conformable for addition and subtraction (both are 1×1).

Question 2: Given matrices X, Y, Z, find the following:

Given:

$$X = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad Z = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$$

(i) $X + Y$

$$X + Y = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1+1 & -1+2 \\ -2+3 & 2+4 \end{bmatrix} = \begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{6} \end{bmatrix}$$

(ii) $Y + 7Z$

First, calculate $7Z$:

$$7Z = 7\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 21 & 0 \\ 0 & -14 \end{bmatrix}$$

Now, add Y and $7Z$:

$$Y + 7Z = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 21 & 0 \\ 0 & -14 \end{bmatrix} = \begin{bmatrix} 1+21 & 2+0 \\ 3+0 & 4+(-14) \end{bmatrix} = \begin{bmatrix} \mathbf{22} & \mathbf{2} \\ \mathbf{3} & \mathbf{-10} \end{bmatrix}$$

(iii) $4X - Z$

First, calculate $4X$:

$$4X = 4\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -8 & 8 \end{bmatrix}$$

Now, subtract Z:

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$$4X - Z = \begin{bmatrix} 4 & -4 \\ -8 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4-3 & -4-0 \\ -8-0 & 8-(-2) \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{-4} \\ \mathbf{-8} & \mathbf{10} \end{bmatrix}$$

(iv) $X + 2Y + 3Z$

Calculate scalar multiples:

$$2Y = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$3Z = 3\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & -6 \end{bmatrix}$$

Add them together:

$$X + 2Y + 3Z = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+9 & -1+4+0 \\ -2+6+0 & 2+8+(-6) \end{bmatrix} = \begin{bmatrix} \mathbf{12} & \mathbf{3} \\ \mathbf{4} & \mathbf{4} \end{bmatrix}$$

(v) $X - 4Y + Z$

Calculate $4Y$:

$$4Y = 4\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$

Combine the operations:

$$X - 4Y + Z = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4+3 & -1-8+0 \\ -2-12+0 & 2-16+(-2) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{-9} \\ \mathbf{-14} & \mathbf{-16} \end{bmatrix}$$

(vi) $Z - Z$

$$Z - Z = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3-3 & 0-0 \\ 0-0 & -2-(-2) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Question 3: Find the additive inverse of the following matrices:

The additive inverse of a matrix M is $-M$, obtained by changing the signs of all entries.

- **(i)** $P = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$

Additive Inverse $(-P) = \begin{bmatrix} \mathbf{-5} \\ \mathbf{7} \end{bmatrix}$

- **(ii)** $Q = [9 \quad -3]$

Additive Inverse $(-Q) = [\mathbf{-9} \quad \mathbf{3}]$

- **(iii)** $R = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

Additive Inverse $(-R) = \begin{bmatrix} \mathbf{-2} & \mathbf{1} \\ \mathbf{2} & \mathbf{-3} \end{bmatrix}$

- **(iv)** $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Additive Inverse $(-S) = \begin{bmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{bmatrix}$

Question 4: If $A = \begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-3} & \mathbf{2} \end{bmatrix}$, $B = \begin{bmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{5} & \mathbf{6} \end{bmatrix}$, and $C = \begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{0} & \mathbf{5} \end{bmatrix}$, then verify that

:

(i) $A + B = B + A$

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- **L.H.S.:**

$$A + B = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 2+3 & 3+4 \\ -3+5 & 2+6 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix}$$

- **R.H.S.:**

$$B + A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 3+2 & 4+3 \\ 5+(-3) & 6+2 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

(ii) $(A + B) + C = A + (B + C)$

- **L.H.S.:**

$$(A + B) + C = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5+1 & 7-2 \\ 2+0 & 8+5 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 13 \end{bmatrix}$$

- **R.H.S.:**

$$B + C = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix} = \begin{bmatrix} 2+4 & 3+2 \\ -3+5 & 2+11 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 13 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

(iii) $(2A + B) + C = 2A + (B + C)$

First, calculate $2A$:

$$2A = 2\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix}$$

- **L.H.S.:**

$$(2A + B) = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ -1 & 10 \end{bmatrix}$$

$$(2A + B) + C = \begin{bmatrix} 7 & 10 \\ -1 & 10 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix}$$

- **R.H.S.:**

$$(B + C) = \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix}$$

$$2A + (B + C) = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

(iv) $3(A + B) = 3A + 3B$

- **L.H.S.:**

$$A + B = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix}$$

$$3(A + B) = 3\begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix}$$

- **R.H.S.:**

$$3A = 3\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ -9 & 6 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 12 \\ 15 & 18 \end{bmatrix}$$

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$$3A + 3B = \begin{bmatrix} 6 & 9 \\ -9 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 15 & 18 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

Question 5: If $\boldsymbol{A} = \begin{bmatrix} \mathbf{5} & \mathbf{6} \\ \mathbf{7} & \mathbf{-2} \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} \mathbf{-5} & \mathbf{-6} \\ \mathbf{-7} & \mathbf{2} \end{bmatrix}$, then show that $\boldsymbol{B}$ is additive inverse of $\boldsymbol{A}$ and $\boldsymbol{A}$ is additive inverse of $\boldsymbol{B}$.

Two matrices A and B are additive inverses of each other if $A + B = O$ (Zero matrix) and $B + A = O$.

1. **Check $A + B$:**

$$A + B = \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix} + \begin{bmatrix} -5 & -6 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} 5 + (-5) & 6 + (-6) \\ 7 + (-7) & -2 + 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

2. **Check $B + A$:**

$$B + A = \begin{bmatrix} -5 & -6 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix} = \begin{bmatrix} -5 + 5 & -6 + 6 \\ -7 + 7 & 2 + (-2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Conclusion: Since $A + B = O = B + A$, **B is the additive inverse of A and A is the additive inverse of B.**

Question 6: If $\boldsymbol{A} = \begin{bmatrix} \mathbf{6} & \mathbf{-2} \\ \mathbf{0} & \mathbf{3} \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} \mathbf{8} & \mathbf{-1} \\ \mathbf{3} & \mathbf{0} \end{bmatrix}$, then verify that:

(i) $(\boldsymbol{A} + \boldsymbol{B})^{\boldsymbol{t}} = \boldsymbol{A}^{\boldsymbol{t}} + \boldsymbol{B}^{\boldsymbol{t}}$

- **L.H.S.:**

$$A + B = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 14 & -3 \\ 3 & 3 \end{bmatrix}$$

$$(A + B)^t = \begin{bmatrix} 14 & 3 \\ -3 & 3 \end{bmatrix}$$

- **R.H.S.:**

$$A^t = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix}, \quad B^t = \begin{bmatrix} 8 & 3 \\ -1 & 0 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ -3 & 3 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

(ii) $(\boldsymbol{A} - \boldsymbol{B})^{\boldsymbol{t}} = \boldsymbol{A}^{\boldsymbol{t}} - \boldsymbol{B}^{\boldsymbol{t}}$

- **L.H.S.:**

$$A - B = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 6 - 8 & -2 - (-1) \\ 0 - 3 & 3 - 0 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -3 & 3 \end{bmatrix}$$

$$(A - B)^t = \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix}$$

- **R.H.S.:**

$$A^t = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix}, \quad B^t = \begin{bmatrix} 8 & 3 \\ -1 & 0 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 6 - 8 & 0 - 3 \\ -2 - (-1) & 3 - 0 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

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EXERCISE 3.4

Question 1: Find AB and BA, if possible.

Rule for Matrix Multiplication: Matrix multiplication $X \times Y$ is possible **only if** the number of columns in X equals the number of rows in Y.

(i) $\boldsymbol{A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}}$ **(Order: 2 × 2),** $\boldsymbol{B = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}}$ **(Order: 2 × 2)**

- **Find AB:**

$$AB = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} (1)(3)+(2)(1) & (1)(2)+(2)(-1) \\ (-1)(3)+(0)(1) & (-1)(2)+(0)(-1) \end{bmatrix}$$
$$= \begin{bmatrix} 3+2 & 2-2 \\ -3+0 & -2+0 \end{bmatrix} = \begin{bmatrix} \mathbf{5} & \mathbf{0} \\ \mathbf{-3} & \mathbf{-2} \end{bmatrix}$$

- **Find BA:**

$$BA = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} (3)(1)+(2)(-1) & (3)(2)+(2)(0) \\ (1)(1)+(-1)(-1) & (1)(2)+(-1)(0) \end{bmatrix}$$
$$= \begin{bmatrix} 3-2 & 6+0 \\ 1+1 & 2+0 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{6} \\ \mathbf{2} & \mathbf{2} \end{bmatrix}$$

(ii) $\boldsymbol{A = [1 \quad -2]}$ **(Order: 1 × 2),** $\boldsymbol{B = \begin{bmatrix} 3 \\ -4 \end{bmatrix}}$ **(Order: 2 × 1)**

- **Find AB:**

$$AB = [1 \quad -2]\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$
$$= [(1)(3)+(-2)(-4)] = [3+8] = [\mathbf{11}]$$

- **Find BA:**

$$BA = \begin{bmatrix} 3 \\ -4 \end{bmatrix}[1 \quad -2]$$
$$= \begin{bmatrix} (3)(1) & (3)(-2) \\ (-4)(1) & (-4)(-2) \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{-6} \\ \mathbf{-4} & \mathbf{8} \end{bmatrix}$$

(iii) $\boldsymbol{A = \begin{bmatrix} 4 \\ 4 \end{bmatrix}}$ **(Order: 2 × 1),** $\boldsymbol{B = [2 \quad 5]}$ **(Order: 1 × 2)**

- **Find AB:**

$$AB = \begin{bmatrix} 4 \\ 4 \end{bmatrix}[2 \quad 5]$$
$$= \begin{bmatrix} (4)(2) & (4)(5) \\ (4)(2) & (4)(5) \end{bmatrix} = \begin{bmatrix} \mathbf{8} & \mathbf{20} \\ \mathbf{8} & \mathbf{20} \end{bmatrix}$$

- **Find BA:**

$$BA = [2 \quad 5]\begin{bmatrix} 4 \\ 4 \end{bmatrix}$$
$$= [(2)(4)+(5)(4)] = [8+20] = [\mathbf{28}]$$

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(iv) $\boldsymbol{A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}}$ **(Order: 3 × 2),** $\boldsymbol{B = \begin{bmatrix} -1 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}}$ **(Order: 2 × 3)**

- **Find** AB**:**

$$AB = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(-1)+(2)(2) & (1)(4)+(2)(3) & (1)(1)+(2)(1) \\ (-1)(-1)+(1)(2) & (-1)(4)+(1)(3) & (-1)(1)+(1)(1) \\ (3)(-1)+(0)(2) & (3)(4)+(0)(3) & (3)(1)+(0)(1) \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 & 4+6 & 1+2 \\ 1+2 & -4+3 & -1+1 \\ -3+0 & 12+0 & 3+0 \end{bmatrix} = \boldsymbol{\begin{bmatrix} 3 & 10 & 3 \\ 3 & -1 & 0 \\ -3 & 12 & 3 \end{bmatrix}}$$

- **Find** BA**:**

$$BA = \begin{bmatrix} -1 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(1)+(4)(-1)+(1)(3) & (-1)(2)+(4)(1)+(1)(0) \\ (2)(1)+(3)(-1)+(1)(3) & (2)(2)+(3)(1)+(1)(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1-4+3 & -2+4+0 \\ 2-3+3 & 4+3+0 \end{bmatrix} = \boldsymbol{\begin{bmatrix} -2 & 2 \\ 2 & 7 \end{bmatrix}}$$

(v) $\boldsymbol{A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \end{bmatrix}}$ **(Order: 2 × 3),** $\boldsymbol{B = \begin{bmatrix} 1 & 5 \\ 2 & 4 \\ 1 & 6 \end{bmatrix}}$ **(Order: 3 × 2)**

- **Find** AB**:**

$$AB = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 2 & 4 \\ 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(2)(2)+(2)(1) & (1)(5)+(2)(4)+(2)(6) \\ (3)(1)+(1)(2)+(1)(1) & (3)(5)+(1)(4)+(1)(6) \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+2 & 5+8+12 \\ 3+2+1 & 15+4+6 \end{bmatrix} = \boldsymbol{\begin{bmatrix} 7 & 25 \\ 6 & 25 \end{bmatrix}}$$

- **Find** BA**:**

$$BA = \begin{bmatrix} 1 & 5 \\ 2 & 4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(5)(3) & (1)(2)+(5)(1) & (1)(2)+(5)(1) \\ (2)(1)+(4)(3) & (2)(2)+(4)(1) & (2)(2)+(4)(1) \\ (1)(1)+(6)(3) & (1)(2)+(6)(1) & (1)(2)+(6)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+15 & 2+5 & 2+5 \\ 2+12 & 4+4 & 4+4 \\ 1+18 & 2+6 & 2+6 \end{bmatrix} = \boldsymbol{\begin{bmatrix} 16 & 7 & 7 \\ 14 & 8 & 8 \\ 19 & 8 & 8 \end{bmatrix}}$$

Question 2: Verify each statement using $\boldsymbol{A = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}}$**,** $\boldsymbol{B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}}$**, and** $\boldsymbol{C = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}}$

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(i) $AB \neq BA$

- **Find AB:**

$$AB = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 10+0 & -5+3 \\ -2+0 & 1+12 \end{bmatrix} = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

- **Find BA:**

$$BA = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 10+1 & 2-4 \\ 0-3 & 0+12 \end{bmatrix} = \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix}$$

Since $\begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix} \neq \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix}$, **AB ≠ BA** is verified.

(ii) $A(B-C) = AB - AC$

- **L.H.S. $A(B-C)$:**

$$B - C = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2-5 & -1-1 \\ 0-1 & 3-4 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix}$$

$$A(B-C) = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -15-1 & -10-1 \\ 3-4 & 2-4 \end{bmatrix} = \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix}$$

- **R.H.S. $AB - AC$:**

$$AB = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

$$AC = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 25+1 & 5+4 \\ -5+4 & -1+16 \end{bmatrix} = \begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix} - \begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix} = \begin{bmatrix} 10-26 & -2-9 \\ -2-(-1) & 13-15 \end{bmatrix} = \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

(iii) $A(BC) = (AB)C$

- **L.H.S. $A(BC)$:**

$$BC = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 10-1 & 2-4 \\ 0+3 & 0+12 \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix} = \begin{bmatrix} 45+3 & -10+12 \\ -9+12 & 2+48 \end{bmatrix} = \begin{bmatrix} 48 & 2 \\ 3 & 50 \end{bmatrix}$$

- **R.H.S. $(AB)C$:**

$$AB = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 50-2 & 10-8 \\ -10+13 & -2+52 \end{bmatrix} = \begin{bmatrix} 48 & 2 \\ 3 & 50 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

(iv) $(BC)^t = C^t B^t$

- **L.H.S. $(BC)^t$:**

$$BC = \begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix} \Rightarrow (BC)^t = \begin{bmatrix} 9 & 3 \\ -2 & 12 \end{bmatrix}$$

- **R.H.S. $C^t B^t$:**

$$C^t = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}, \quad B^t = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$$

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$$C^tB^t = \begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}\begin{bmatrix}2 & 0\\ -1 & 3\end{bmatrix} = \begin{bmatrix}10-1 & 0+3\\ 2-4 & 0+12\end{bmatrix} = \begin{bmatrix}9 & 3\\ -2 & 12\end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

(v) $(\boldsymbol{B+C})\boldsymbol{A} = \boldsymbol{BA} + \boldsymbol{CA}$

- **L.H.S.** $(B+C)A$**:**

$$B + C = \begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix} + \begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix} = \begin{bmatrix}7 & 0\\ 1 & 7\end{bmatrix}$$

$$(B+C)A = \begin{bmatrix}7 & 0\\ 1 & 7\end{bmatrix}\begin{bmatrix}5 & 1\\ -1 & 4\end{bmatrix} = \begin{bmatrix}35+0 & 7+0\\ 5-7 & 1+28\end{bmatrix} = \begin{bmatrix}35 & 7\\ -2 & 29\end{bmatrix}$$

- **R.H.S.** $BA + CA$**:**

$$BA = \begin{bmatrix}11 & -2\\ -3 & 12\end{bmatrix}$$

$$CA = \begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}\begin{bmatrix}5 & 1\\ -1 & 4\end{bmatrix} = \begin{bmatrix}25-1 & 5+4\\ 5-4 & 1+16\end{bmatrix} = \begin{bmatrix}24 & 9\\ 1 & 17\end{bmatrix}$$

$$BA + CA = \begin{bmatrix}11 & -2\\ -3 & 12\end{bmatrix} + \begin{bmatrix}24 & 9\\ 1 & 17\end{bmatrix} = \begin{bmatrix}11+24 & -2+9\\ -3+1 & 12+17\end{bmatrix} = \begin{bmatrix}35 & 7\\ -2 & 29\end{bmatrix}$$

L.H.S. = R.H.S. (Verified)

Question 3: If $\begin{bmatrix} 4 & a \\ b & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$, then find the values of a and b.

Step 1: Multiply the matrices on the Left Hand Side:

Step 2: Equate corresponding elements:

1. **Find** a**:**

$$24 + 6a = 6$$
$$6a = 6 - 24$$
$$6a = -18 \Rightarrow \mathbf{a = -3}$$

2. **Find** b**:**

$$6b + 18 = 3$$
$$6b = 3 - 18$$
$$6b = -15 \Rightarrow b = -\frac{15}{6} \Rightarrow \mathbf{b = -\frac{5}{2}}$$

Answer: $a = -3$, $b = -\frac{5}{2}$ (or -2.5)

Question 4: If $\begin{bmatrix} x & 1 \\ y & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$, then find the values of x and y. **Step 1:** Multiply the matrices on the Left Hand Side:

Step 2: Equate corresponding elements:

1. **Find** x**:**

$$x + 3 = 7 \Rightarrow x = 7 - 3 \Rightarrow \mathbf{x = 4}$$

2. **Find** y**:**

$$y + 6 = 4 \Rightarrow y = 4 - 6 \Rightarrow \mathbf{y = -2}$$

Answer: $x = 4$, $y = -2$

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EXERCISE 3.5

1. Find the values of each of the determinant.

The determinant of a 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is calculated as $(a \times d) - (b \times c)$.

(i) $\begin{vmatrix} 10 & 5 \\ 4 & 6 \end{vmatrix}$

$$= (10 \times 6) - (5 \times 4)$$
$$= 60 - 20$$
$$= \mathbf{40}$$

(ii) $\begin{vmatrix} -5 & 8 \\ -3 & -7 \end{vmatrix}$

$$= (-5 \times -7) - (8 \times -3)$$
$$= 35 - (-24)$$
$$= 35 + 24$$
$$= \mathbf{59}$$

(iii) $\begin{vmatrix} 3 & 8 \\ 0 & 2 \end{vmatrix}$

$$= (3 \times 2) - (8 \times 0)$$
$$= 6 - 0$$
$$= \mathbf{6}$$

2. Find whether the following matrices are singular or non-singular.

- **Singular Matrix:** Determinant equals 0 ($|M| = 0$).
- **Non-singular Matrix:** Determinant is non-zero ($|M| \neq 0$).

Matrix A: $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

$$|A| = (5 \times 2) - (3 \times 3) = 10 - 9 = 1$$

Since $|A| \neq 0$, A is a **non-singular matrix**.

Matrix B: $B = \begin{bmatrix} 7 & 21 \\ 2 & 6 \end{bmatrix}$

$$|B| = (7 \times 6) - (21 \times 2) = 42 - 42 = 0$$

Since $|B| = 0$, B is a **singular matrix**.

Matrix C: $C = \begin{bmatrix} 13 & 5 \\ 7 & 3 \end{bmatrix}$

$$|C| = (13 \times 3) - (5 \times 7) = 39 - 35 = 4$$

Since $|C| \neq 0$, C is a **non-singular matrix**.

Matrix D: $D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

$$|D| = (2 \times 0) - (0 \times 0) = 0 - 0 = 0$$

Since $|D| = 0$, D is a **singular matrix**.

3. Find the value of x when $A = \begin{bmatrix} x & 6 \\ 5 & 15 \end{bmatrix}$ is a singular matrix.

Since A is singular, $|A| = 0$:

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$$|A| = \begin{vmatrix} x & 6 \\ 5 & 15 \end{vmatrix} = 0$$
$$(x \times 15) - (6 \times 5) = 0$$
$$15x - 30 = 0$$
$$15x = 30$$
$$x = \frac{30}{15}$$
$$\mathbf{x = 2}$$

4. Find the adjoint of the following matrices:

For a 2×2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the adjoint is $\text{adj}(M) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (swap main diagonal entries, change signs of off-diagonal entries).

Matrix A: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\text{adj}(A) = \begin{bmatrix} \mathbf{d} & \mathbf{-b} \\ \mathbf{-c} & \mathbf{a} \end{bmatrix}$$

Matrix B: $B = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$

$$\text{adj}(B) = \begin{bmatrix} \mathbf{7} & \mathbf{2} \\ \mathbf{-3} & \mathbf{5} \end{bmatrix}$$

Matrix C: $C = \begin{bmatrix} -3 & 5 \\ -3 & 2 \end{bmatrix}$

$$\text{adj}(C) = \begin{bmatrix} \mathbf{2} & \mathbf{-5} \\ \mathbf{3} & \mathbf{-3} \end{bmatrix}$$

5. Find multiplicative inverse of the following matrices:

The formula for multiplicative inverse is $M^{-1} = \frac{1}{|M|}\text{adj}(M)$, provided $|M| \neq 0$.

(i) $M = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

- $|M| = (5 \times 5) - (0 \times 0) = 25$
- $\text{adj}(M) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
- $M^{-1} = \frac{1}{25}\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} \mathbf{1/5} & \mathbf{0} \\ \mathbf{0} & \mathbf{1/5} \end{bmatrix}$

(ii) $M = \begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$

- $|M| = (-4 \times 2) - (8 \times 7) = -8 - 56 = -64$
- $\text{adj}(M) = \begin{bmatrix} 2 & -8 \\ -7 & -4 \end{bmatrix}$
- $M^{-1} = \frac{1}{-64}\begin{bmatrix} 2 & -8 \\ -7 & -4 \end{bmatrix} = \begin{bmatrix} \mathbf{-1/32} & \mathbf{1/8} \\ \mathbf{7/64} & \mathbf{1/16} \end{bmatrix}$

(iii) $M = \begin{bmatrix} 40 & 8 \\ 5 & 2 \end{bmatrix}$

- $|M| = (40 \times 2) - (8 \times 5) = 80 - 40 = 40$
- $\text{adj}(M) = \begin{bmatrix} 2 & -8 \\ -5 & 40 \end{bmatrix}$

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- $M^{-1} = \frac{1}{40}\begin{bmatrix} 2 & -8 \\ -5 & 40 \end{bmatrix} = \begin{bmatrix} \mathbf{1/20} & \mathbf{-1/5} \\ \mathbf{-1/8} & \mathbf{1} \end{bmatrix}$

(iv) $M = \begin{bmatrix} 3 & 5 \\ 5 & -3 \end{bmatrix}$

- $|M| = (3 \times -3) - (5 \times 5) = -9 - 25 = -34$
- $\text{adj}(M) = \begin{bmatrix} -3 & -5 \\ -5 & 3 \end{bmatrix}$
- $M^{-1} = \frac{1}{-34}\begin{bmatrix} -3 & -5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \mathbf{3/34} & \mathbf{5/34} \\ \mathbf{5/34} & \mathbf{-3/34} \end{bmatrix}$

(v) $M = \begin{bmatrix} 10 & 8 \\ 3 & 3 \end{bmatrix}$

- $|M| = (10 \times 3) - (8 \times 3) = 30 - 24 = 6$
- $\text{adj}(M) = \begin{bmatrix} 3 & -8 \\ -3 & 10 \end{bmatrix}$
- $M^{-1} = \frac{1}{6}\begin{bmatrix} 3 & -8 \\ -3 & 10 \end{bmatrix} = \begin{bmatrix} \mathbf{1/2} & \mathbf{-4/3} \\ \mathbf{-1/2} & \mathbf{5/3} \end{bmatrix}$

(vi) $M = \begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$

- $|M| = (-2 \times 5) - (-3 \times 4) = -10 + 12 = 2$
- $\text{adj}(M) = \begin{bmatrix} 5 & 3 \\ -4 & -2 \end{bmatrix}$
- $M^{-1} = \frac{1}{2}\begin{bmatrix} 5 & 3 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} \mathbf{5/2} & \mathbf{3/2} \\ \mathbf{-2} & \mathbf{-1} \end{bmatrix}$

6. If $\boldsymbol{A = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}}$, then find $\boldsymbol{A^{-1}}$ and prove that $\boldsymbol{AA^{-1} = A^{-1}A = I}$.

Step 1: Find A^{-1}

- $|A| = (5 \times -1) - (-3 \times 2) = -5 + 6 = 1$
- $\text{adj}(A) = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$
- $A^{-1} = \frac{1}{1}\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$

Step 2: Calculate AA^{-1}

$$AA^{-1} = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} (5)(-1) + (-3)(-2) & (5)(3) + (-3)(5) \\ (2)(-1) + (-1)(-2) & (2)(3) + (-1)(5) \end{bmatrix}$$
$$= \begin{bmatrix} -5 + 6 & 15 - 15 \\ -2 + 2 & 6 - 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Step 3: Calculate $A^{-1}A$

$$A^{-1}A = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} (-1)(5) + (3)(2) & (-1)(-3) + (3)(-1) \\ (-2)(5) + (5)(2) & (-2)(-3) + (5)(-1) \end{bmatrix}$$
$$= \begin{bmatrix} -5 + 6 & 3 - 3 \\ -10 + 10 & 6 - 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

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Hence, $AA^{-1} = A^{-1}A = I$ **is proved.**

7. Show that the following matrices are multiplicative inverse of each other.

(To prove this, show that their product equals identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$).

(i) $X = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}, Y = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$$XY = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} (2)(2) + (-1)(3) & (2)(1) + (-1)(2) \\ (-3)(2) + (2)(3) & (-3)(1) + (2)(2) \end{bmatrix}$$
$$= \begin{bmatrix} 4 - 3 & 2 - 2 \\ -6 + 6 & -3 + 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence proved.

(ii) $X = \begin{bmatrix} 4 & 15 \\ 2 & 8 \end{bmatrix}, Y = \begin{bmatrix} 4 & -\frac{15}{2} \\ -1 & 2 \end{bmatrix}$

$$XY = \begin{bmatrix} 4 & 15 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 4 & -\frac{15}{2} \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} (4)(4) + (15)(-1) & (4)\left(-\frac{15}{2}\right) + (15)(2) \\ (2)(4) + (8)(-1) & (2)\left(-\frac{15}{2}\right) + (8)(2) \end{bmatrix}$$
$$= \begin{bmatrix} 16 - 15 & -30 + 30 \\ 8 - 8 & -15 + 16 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence proved.

8. Prove that $(\boldsymbol{AB})^{-1} = \boldsymbol{B^{-1}A^{-1}}$

(i) Given: $A = \begin{bmatrix} -3 & -2 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

LHS: $(AB)^{-1}$

1. Product AB:

$$AB = \begin{bmatrix} -3 & -2 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -6 + 6 & 3 - 4 \\ 10 - 18 & -5 + 12 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -8 & 7 \end{bmatrix}$$

2. Inverse $(AB)^{-1}$:
   - $|AB| = (0 \times 7) - (-1 \times -8) = 0 - 8 = -8$
   - $\text{adj}(AB) = \begin{bmatrix} 7 & 1 \\ 8 & 0 \end{bmatrix}$
   - $(AB)^{-1} = -\frac{1}{8}\begin{bmatrix} 7 & 1 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} -7/8 & -1/8 \\ -1 & 0 \end{bmatrix}$

RHS: $B^{-1}A^{-1}$

1. B^{-1}:
   - $|B| = (2 \times 2) - (-1 \times -3) = 4 - 3 = 1$
   - $B^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

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2. A^{-1}:
   - $|A| = (-3 \times 6) - (-2 \times 5) = -18 + 10 = -8$
   - $A^{-1} = -\frac{1}{8}\begin{bmatrix} 6 & 2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} -6/8 & -2/8 \\ 5/8 & 3/8 \end{bmatrix}$
3. Multiply $B^{-1}A^{-1}$:

$$B^{-1}A^{-1} = -\frac{1}{8}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 6 & 2 \\ -5 & -3 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 12-5 & 4-3 \\ 18-10 & 6-6 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 7 & 1 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} -7/8 & -1/8 \\ -1 & 0 \end{bmatrix}$$

LHS = RHS, hence proved.

(ii) Given: $A = \begin{bmatrix} 1 & 2 \\ 8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 5 & 2 \end{bmatrix}$

LHS: $(AB)^{-1}$

1. Product AB:

$$AB = \begin{bmatrix} 1 & 2 \\ 8 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 0+10 & -1+4 \\ 0+0 & -8+0 \end{bmatrix} = \begin{bmatrix} 10 & 3 \\ 0 & -8 \end{bmatrix}$$

2. Inverse $(AB)^{-1}$:
   - $|AB| = (10 \times -8) - (3 \times 0) = -80$
   - $\text{adj}(AB) = \begin{bmatrix} -8 & -3 \\ 0 & 10 \end{bmatrix}$
   - $(AB)^{-1} = -\frac{1}{80}\begin{bmatrix} -8 & -3 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1/10 & 3/80 \\ 0 & -1/8 \end{bmatrix}$

RHS: $B^{-1}A^{-1}$

1. B^{-1}:
   - $|B| = (0 \times 2) - (-1 \times 5) = 5$
   - $B^{-1} = \frac{1}{5}\begin{bmatrix} 2 & 1 \\ -5 & 0 \end{bmatrix}$
2. A^{-1}:
   - $|A| = (1 \times 0) - (2 \times 8) = -16$
   - $A^{-1} = -\frac{1}{16}\begin{bmatrix} 0 & -2 \\ -8 & 1 \end{bmatrix}$
3. Multiply $B^{-1}A^{-1}$:

$$B^{-1}A^{-1} = \left(\frac{1}{5}\right)\left(-\frac{1}{16}\right)\begin{bmatrix} 2 & 1 \\ -5 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ -8 & 1 \end{bmatrix}$$

$$= -\frac{1}{80}\begin{bmatrix} 0-8 & -4+1 \\ 0+0 & 10+0 \end{bmatrix} = -\frac{1}{80}\begin{bmatrix} -8 & -3 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1/10 & 3/80 \\ 0 & -1/8 \end{bmatrix}$$

LHS = RHS, hence proved.

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Exercise 3.6.

1. Solve by Matrix Inversion Method, if possible:

(i)

$$2x + 5y = 19$$
$$4x - 3y = -1$$

$$2x + 5y = 19$$
$$4x - 3y = -1$$

Step 1: Write in matrix form $AX = B$

$$\begin{bmatrix} 2 & 5 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ -1 \end{bmatrix}$$

Step 2: Find the determinant $|A|$

$$|A| = \begin{vmatrix} 2 & 5 \\ 4 & -3 \end{vmatrix} = (2)(-3) - (5)(4) = -6 - 20 = -26$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 4 & -3 \end{vmatrix} = (2)(-3) - (5)(4) = -6 - 20 = -26$$

Since $|A| \neq 0$, the solution exists.

Step 3: Find the adjoint $\text{adj}(A)$ **and inverse** A^{-1}

$$\text{adj}(A) = \begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{-26}\begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{-26}\begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}$$

Step 4: Solve for $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-26}\begin{bmatrix} -3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 19 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-26}\begin{bmatrix} (-3)(19) + (-5)(-1) \\ (-4)(19) + (2)(-1) \end{bmatrix} = \frac{1}{-26}\begin{bmatrix} -57 + 5 \\ -76 - 2 \end{bmatrix} = \frac{1}{-26}\begin{bmatrix} -52 \\ -78 \end{bmatrix}$$

$$x = \frac{-52}{-26} = 2,\ y = \frac{-78}{-26} = 3$$

$$x = \frac{-52}{-26} = 2, \quad y = \frac{-78}{-26} = 3$$

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Answer: $x = 2, y = 3$

(ii)

$$3x + 2y = 7$$
$$5x - y = 16$$

$$3x + 2y = 7$$
$$5x - y = 16$$

Step 1: Matrix Form

$$\begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 16 \end{bmatrix}$$

Step 2: Determinant

$$| A | = (3)(-1) - (2)(5) = -3 - 10 = -13$$

$$|A| = (3)(-1) - (2)(5) = -3 - 10 = -13$$

Step 3: Matrix Inversion

$$A^{-1} = \frac{1}{-13}\begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-13}\begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-13}\begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-13}\begin{bmatrix} (-1)(7) + (-2)(16) \\ (-5)(7) + (3)(16) \end{bmatrix} = \frac{1}{-13}\begin{bmatrix} -7 - 32 \\ -35 + 48 \end{bmatrix} = \frac{1}{-13}\begin{bmatrix} -39 \\ 13 \end{bmatrix}$$

$$x = \frac{-39}{-13} = 3,\ y = \frac{13}{-13} = -1$$

$$x = \frac{-39}{-13} = 3, \quad y = \frac{13}{-13} = -1$$

Answer: $x = 3, y = -1$

(iii)

$$x - 2y = 9$$
$$2x + 7y = -4$$

$$x - 2y = 9$$
$$2x + 7y = -4$$

Step 1: Matrix Form

$$\begin{bmatrix} 1 & -2 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

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Step 2: Determinant

$$| A |= (1)(7) - (-2)(2) = 7 + 4 = 11$$

$$|A| = (1)(7) - (-2)(2) = 7 + 4 = 11$$

Step 3: Solution

$$A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 9 \\ -4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 63 - 8 \\ -18 - 4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 55 \\ -22 \end{bmatrix}$$

$$x = \frac{55}{11} = 5,\ y = \frac{-22}{11} = -2$$

$$x = \frac{55}{11} = 5, \quad y = \frac{-22}{11} = -2$$

Answer: $x = 5, y = -2$

(iv)

$$3x + 2y = 2$$
$$x - 2y = -2$$

$$3x + 2y = 2$$
$$x - 2y = -2$$

Step 1: Matrix Form

$$\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

Step 2: Determinant

$$| A |= (3)(-2) - (2)(1) = -6 - 2 = -8$$

$$|A| = (3)(-2) - (2)(1) = -6 - 2 = -8$$

Step 3: Solution

$$A^{-1} = \frac{1}{-8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -4 + 4 \\ -2 - 6 \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} 0 \\ -8 \end{bmatrix}$$

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$$x = \frac{0}{-8} = 0,\ y = \frac{-8}{-8} = 1$$

$$x = \frac{0}{-8} = 0, \quad y = \frac{-8}{-8} = 1$$

Answer: $x = 0, y = 1$

2. Use Cramer's Rule to solve the following pair of linear equations, if possible:

(i)

$$x + 4y = 4$$
$$2x - y = 5$$

$$x + 4y = 4$$
$$2x - y = 5$$

$$| A | = \begin{vmatrix} 1 & 4 \\ 2 & -1 \end{vmatrix} = (1)(-1) - (4)(2) = -1 - 8 = -9$$

$$|A| = \begin{vmatrix} 1 & 4 \\ 2 & -1 \end{vmatrix} = (1)(-1) - (4)(2) = -1 - 8 = -9$$

$$| A_x | = \begin{vmatrix} 4 & 4 \\ 5 & -1 \end{vmatrix} = (4)(-1) - (4)(5) = -4 - 20 = -24$$

$$|A_x| = \begin{vmatrix} 4 & 4 \\ 5 & -1 \end{vmatrix} = (4)(-1) - (4)(5) = -4 - 20 = -24$$

$$| A_y | = \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = (1)(5) - (4)(2) = 5 - 8 = -3$$

$$|A_y| = \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = (1)(5) - (4)(2) = 5 - 8 = -3$$

$$x = \frac{| A_x |}{| A |} = \frac{-24}{-9} = \frac{8}{3}$$

$$x = \frac{|A_x|}{|A|} = \frac{-24}{-9} = \frac{8}{3}$$

$$y = \frac{| A_y |}{| A |} = \frac{-3}{-9} = \frac{1}{3}$$

$$y = \frac{|A_y|}{|A|} = \frac{-3}{-9} = \frac{1}{3}$$

Answer: $x = \frac{8}{3}, y = \frac{1}{3}$

(ii)

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$$x + 2y = 7$$
$$3x - 2y = -3$$

$$x + 2y = 7$$
$$3x - 2y = -3$$
$$| A | = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = (1)(-2) - (2)(3) = -2 - 6 = -8$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = (1)(-2) - (2)(3) = -2 - 6 = -8$$
$$| A_x | = \begin{vmatrix} 7 & 2 \\ -3 & -2 \end{vmatrix} = (7)(-2) - (2)(-3) = -14 + 6 = -8$$

$$|A_x| = \begin{vmatrix} 7 & 2 \\ -3 & -2 \end{vmatrix} = (7)(-2) - (2)(-3) = -14 + 6 = -8$$
$$| A_y | = \begin{vmatrix} 1 & 7 \\ 3 & -3 \end{vmatrix} = (1)(-3) - (7)(3) = -3 - 21 = -24$$

$$|A_y| = \begin{vmatrix} 1 & 7 \\ 3 & -3 \end{vmatrix} = (1)(-3) - (7)(3) = -3 - 21 = -24$$
$$x = \frac{-8}{-8} = 1,\ y = \frac{-24}{-8} = 3$$

$$x = \frac{-8}{-8} = 1, \quad y = \frac{-24}{-8} = 3$$

Answer: $x = 1, y = 3$

(iii)

$$2x - 5y = -6$$
$$4x - 3y = -12$$

$$2x - 5y = -6$$
$$4x - 3y = -12$$
$$| A | = \begin{vmatrix} 2 & -5 \\ 4 & -3 \end{vmatrix} = (2)(-3) - (-5)(4) = -6 + 20 = 14$$

$$|A| = \begin{vmatrix} 2 & -5 \\ 4 & -3 \end{vmatrix} = (2)(-3) - (-5)(4) = -6 + 20 = 14$$
$$| A_x | = \begin{vmatrix} -6 & -5 \\ -12 & -3 \end{vmatrix} = (-6)(-3) - (-5)(-12) = 18 - 60 = -42$$

$$|A_x| = \begin{vmatrix} -6 & -5 \\ -12 & -3 \end{vmatrix} = (-6)(-3) - (-5)(-12) = 18 - 60 = -42$$

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$$| A_y |= \begin{vmatrix} 2 & -6 \\ 4 & -12 \end{vmatrix} = (2)(-12) - (-6)(4) = -24 + 24 = 0$$

$$|A_y| = \begin{vmatrix} 2 & -6 \\ 4 & -12 \end{vmatrix} = (2)(-12) - (-6)(4) = -24 + 24 = 0$$

$$x = \frac{-42}{14} = -3,\ y = \frac{0}{14} = 0$$

$$x = \frac{-42}{14} = -3, \quad y = \frac{0}{14} = 0$$

Answer: $x = -3, y = 0$

(iv)

$$3x + 2y = -1$$
$$5x + 6y = 5$$

$$3x + 2y = -1$$
$$5x + 6y = 5$$

$$| A |= \begin{vmatrix} 3 & 2 \\ 5 & 6 \end{vmatrix} = (3)(6) - (2)(5) = 18 - 10 = 8$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & 6 \end{vmatrix} = (3)(6) - (2)(5) = 18 - 10 = 8$$

$$| A_x |= \begin{vmatrix} -1 & 2 \\ 5 & 6 \end{vmatrix} = (-1)(6) - (2)(5) = -6 - 10 = -16$$

$$|A_x| = \begin{vmatrix} -1 & 2 \\ 5 & 6 \end{vmatrix} = (-1)(6) - (2)(5) = -6 - 10 = -16$$

$$| A_y |= \begin{vmatrix} 3 & -1 \\ 5 & 5 \end{vmatrix} = (3)(5) - (-1)(5) = 15 + 5 = 20$$

$$|A_y| = \begin{vmatrix} 3 & -1 \\ 5 & 5 \end{vmatrix} = (3)(5) - (-1)(5) = 15 + 5 = 20$$

$$x = \frac{-16}{8} = -2,\ y = \frac{20}{8} = \frac{5}{2}$$

$$x = \frac{-16}{8} = -2, \quad y = \frac{20}{8} = \frac{5}{2}$$

Answer: $x = -2, y = \frac{5}{2}$

3.

System of equations:

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$$x + y = 7$$
$$2x - y = 2$$

$$x + y = 7$$
$$2x - y = 2$$

Matrix Form:

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$| A |= (1)(-1) - (1)(2) = -1 - 2 = -3$$

$$|A| = (1)(-1) - (1)(2) = -1 - 2 = -3$$
$$A^{-1} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -7 - 2 \\ -14 + 2 \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -9 \\ -12 \end{bmatrix}$$

$$x = \frac{-9}{-3} = 3,\ y = \frac{-12}{-3} = 4$$

$$x = \frac{-9}{-3} = 3, \quad y = \frac{-12}{-3} = 4$$

Answer: $x = 3$amperes, $y = 4$amperes.

4.

Given condition: $F_1 + F_2 + F_3 = 0$

$$\begin{bmatrix} 8 \\ x \end{bmatrix} + \begin{bmatrix} -2 \\ -7 \end{bmatrix} + \begin{bmatrix} y \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 8 - 2 + y \\ x - 7 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 + y \\ x - 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Equating components:

$$6 + y = 0 \Longrightarrow y = -6$$

$$6 + y = 0 \Longrightarrow y = -6$$
$$x - 8 = 0 \Longrightarrow x = 8$$

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$$x - 8 = 0 \Longrightarrow x = 8$$

Answer: $x = 8, y = -6$

5.

Let the load on beam Abe $x$and on beam Bbe y.

1. Combined load: $x + y = 100$
2. Twice load Aplus three times load B: $2x + 3y = 240$

Matrix Form:

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 100 \\ 240 \end{bmatrix}$$

$$| A | = (1)(3) - (1)(2) = 3 - 2 = 1$$

$$|A| = (1)(3) - (1)(2) = 3 - 2 = 1$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 240 \end{bmatrix} = \begin{bmatrix} 300 - 240 \\ -200 + 240 \end{bmatrix} = \begin{bmatrix} 60 \\ 40 \end{bmatrix}$$

Answer: Load on beam $A = 60$ kN, Load on beam $B = 40$ kN.

6.

Let horizontal position be $x$and vertical position be y.

1. "Total of twice horizontal position and vertical position is 5":

$$2x + y = 5$$

$$2x + y = 5$$

2. "Horizontal position is 1 more than vertical position":

$$x = y + 1 \Longrightarrow x - y = 1$$

$$x = y + 1 \Longrightarrow x - y = 1$$

Matrix Form:

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$| A | = (2)(-1) - (1)(1) = -2 - 1 = -3$$

$$|A| = (2)(-1) - (1)(1) = -2 - 1 = -3$$

$$A^{-1} = \frac{1}{-3} \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$

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$$A^{-1} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -5-1 \\ -5+2 \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -6 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Answer: Point of intersection is $(2,1)$.

7.

Let the present age of the man be $x$and the son's present age be y.

1. **Two years ago:**

$$x - 2 = 5(y - 2) \Longrightarrow x - 2 = 5y - 10 \Longrightarrow x - 5y = -8$$

$$x - 2 = 5(y - 2) \Longrightarrow x - 2 = 5y - 10 \Longrightarrow x - 5y = -8$$

2. **After 6 years:**

$$x + 6 = 3(y + 6) \Longrightarrow x + 6 = 3y + 18 \Longrightarrow x - 3y = 12$$

$$x + 6 = 3(y + 6) \Longrightarrow x + 6 = 3y + 18 \Longrightarrow x - 3y = 12$$

Matrix Form:

$$\begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

$$| A | = (1)(-3) - (-5)(1) = -3 + 5 = 2$$

$$|A| = (1)(-3) - (-5)(1) = -3 + 5 = 2$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -8 \\ 12 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 24+60 \\ 8+12 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 84 \\ 20 \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix}$$

Answer: Man's age = 42years, Son's age = 10years.

8.

Let speed of faster cyclist $= x$ km/hand slower cyclist $= y$ km/h. Total distance $= 44$ km.

1. **Moving towards each other (opposite direction):** Relative speed $= x + y$.

$$\text{Distance} = \text{Speed} \times \text{Time} \Longrightarrow 44 = (x + y) \times 2 \Longrightarrow x + y = 22$$

$$\text{Distance} = \text{Speed} \times \text{Time} \Longrightarrow 44 = (x + y) \times 2 \Longrightarrow x + y = 22$$

2. **Moving in the same direction:** Relative speed $= x - y$. Time $= 7\frac{1}{2} = \frac{15}{2}$ hours.

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$$44 = (x - y) \times \frac{15}{2} \Rightarrow x - y = \frac{88}{15}$$

$$44 = (x - y) \times \frac{15}{2} \Rightarrow x - y = \frac{88}{15}$$

Matrix Form:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ \frac{88}{15} \end{bmatrix}$$

$$| A |= (1)(-1) - (1)(1) = -2$$

$$|A| = (1)(-1) - (1)(1) = -2$$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 22 \\ \frac{88}{15} \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -22 - \frac{88}{15} \\ -22 + \frac{88}{15} \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -\frac{418}{15} \\ -\frac{242}{15} \end{bmatrix} = \begin{bmatrix} \frac{209}{15} \\ \frac{121}{15} \end{bmatrix}$$

$$x = \frac{209}{15} = 13.93 \text{ km/h}, \; y = \frac{121}{15} = 8.07 \text{ km/h}$$

$$x = \frac{209}{15} = 13.93 \text{ km/h}, \quad y = \frac{121}{15} = 8.07 \text{ km/h}$$

Answer: Speeds are $\frac{209}{15}$ km/h($13\frac{14}{15}$ km/h) and $\frac{121}{15}$ km/h($8\frac{1}{15}$ km/h).

9.

Let numerator be $x$and denominator be y. Original fraction is $\frac{x}{y}$.

1. "Numerator is 7 less than denominator":

$$x = y - 7 \Rightarrow x - y = -7$$

$$x = y - 7 \Rightarrow x - y = -7$$

2. "If numerator is increased by 3, the fraction becomes $\frac{3}{4}$":

$$\frac{x + 3}{y} = \frac{3}{4} \Rightarrow 4(x + 3) = 3y \Rightarrow 4x + 12 = 3y \Rightarrow 4x - 3y = -12$$

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$$\frac{x+3}{y}=\frac{3}{4}\Longrightarrow 4(x+3)=3y\Longrightarrow 4x+12=3y\Longrightarrow 4x-3y=-12$$

Matrix Form:

$$\begin{bmatrix}1 & -1\\ 4 & -3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-7\\ -12\end{bmatrix}$$

$$|\ A\ |=(1)(-3)-(-1)(4)=-3+4=1$$

$$|A|=(1)(-3)-(-1)(4)=-3+4=1$$

$$A^{-1}=\begin{bmatrix}-3 & 1\\ -4 & 1\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}-3 & 1\\ -4 & 1\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-3 & 1\\ -4 & 1\end{bmatrix}\begin{bmatrix}-7\\ -12\end{bmatrix}=\begin{bmatrix}21-12\\ 28-12\end{bmatrix}=\begin{bmatrix}9\\ 16\end{bmatrix}$$

Answer: Original fraction $=\frac{9}{16}$

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بسم اللہ الرحمٰن الرحیم

Review Exercise 3.

1. Multiple Choice Questions (MCQs)

(i) If $\begin{bmatrix} a + 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, then $a =$

- Options: (a) 3 (b) 5 (c) 6 (d) 7
- **Steps:** Comparing corresponding elements:

$$a + 2 = 7 \Rightarrow a = 7 - 2 = 5$$

- **Correct Answer: (b) 5**

(ii) $A = [3 \quad 5 \quad 0]$ is a _____ matrix.

- Options: (a) row (b) square (c) column (d) null
- **Steps:** The matrix A consists of only one row, so it is a row matrix.
- **Correct Answer: (a) row**

(iii) $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a _____ matrix.

- Options: (a) identity (b) square (c) row (d) column
- **Steps:** The matrix B consists of only one column, so it is a column matrix.
- **Correct Answer: (d) column**

(iv) $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a/an _____ matrix.

- Options: (a) rectangular (b) identity (c) column (d) row
- **Steps:** A square matrix with diagonal elements equal to 1 and all other elements equal to 0 is called an identity (or unit) matrix.
- **Correct Answer: (b) identity**

(v) If $A^t = -A$, then A is _____ matrix.

- Options: (a) symmetric (b) row (c) rectangular (d) skew-symmetric
- **Steps:** By definition, if the transpose of a square matrix equals its negative, $A^t = -A$, it is a skew-symmetric matrix.
- **Correct Answer: (d) skew-symmetric**

(vi) If $A = [3 \quad 4]$, $B = [7 \quad 8]$, then $A + B =$

- Options: (a) $[21 \quad 32]$ (b) $[24 \quad 28]$ (c) $[10 \quad 12]$ (d) $[11 \quad 11]$
- **Steps:**

$$A + B = [3 + 7 \quad 4 + 8] = [10 \quad 12]$$

- **Correct Answer: (c)** $[10 \quad 12]$

(vii) If $A = [3 \quad 4]$, $B = [7 \quad 8]$, then $B - A =$

- Options: (a) $[10 \quad 12]$ (b) $[11 \quad 11]$ (c) $[4 \quad 4]$ (d) $[-4 \quad -4]$
- **Steps:**

$$B - A = [7 - 3 \quad 8 - 4] = [4 \quad 4]$$

- **Correct Answer: (c)** $[4 \quad 4]$

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(viii) What is the additive inverse of $\begin{bmatrix}3\\4\end{bmatrix}$?

- Options: (a) $\begin{bmatrix}-3\\4\end{bmatrix}$ (b) $\begin{bmatrix}-3\\-4\end{bmatrix}$ (c) $\begin{bmatrix}3\\4\end{bmatrix}$ (d) $\begin{bmatrix}-1\\1\end{bmatrix}$
- **Steps:** The additive inverse of a matrix X is $-X$, obtained by changing the signs of all elements:

$$-\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}-3\\-4\end{bmatrix}$$

- **Correct Answer: (b)** $\begin{bmatrix}-3\\-4\end{bmatrix}$

(ix) If $\boldsymbol{A}=\begin{bmatrix}3&1&3\\2&0&2\end{bmatrix}$, then order of $\boldsymbol{A}$ is:

- Options: (a) 3-by-2 (b) 2-by-3 (c) 3-by-3 (d) 2-by-2
- **Steps:** Matrix A has 2 rows and 3 columns, so its order is 2-by-3.
- **Correct Answer: (b) 2-by-3**

(x) $\begin{vmatrix} 3 & 1 \\ 0 & 4 \end{vmatrix} =$

- Options: (a) 11 (b) 12 (c) 13 (d) -11
- **Steps:**
- **Correct Answer: (b) 12**

2. If $\boldsymbol{A}=\begin{bmatrix}4&2\\7&6\end{bmatrix}, \boldsymbol{B}=\begin{bmatrix}5&5\\8&8\end{bmatrix}$, then find:

(i) $(\boldsymbol{A}-\boldsymbol{B})^{\boldsymbol{t}}$

1. Find $A-B$:

$$A-B=\begin{bmatrix}4&2\\7&6\end{bmatrix}-\begin{bmatrix}5&5\\8&8\end{bmatrix}=\begin{bmatrix}4-5&2-5\\7-8&6-8\end{bmatrix}=\begin{bmatrix}-1&-3\\-1&-2\end{bmatrix}$$

2. Take transpose:

$$(A-B)^t=\begin{bmatrix}-1&-1\\-3&-2\end{bmatrix}$$

(ii) $\boldsymbol{B^t-A^t}$

1. Find A^t and B^t:

$$A^t=\begin{bmatrix}4&7\\2&6\end{bmatrix},\quad B^t=\begin{bmatrix}5&8\\5&8\end{bmatrix}$$

2. Subtract:

$$B^t-A^t=\begin{bmatrix}5&8\\5&8\end{bmatrix}-\begin{bmatrix}4&7\\2&6\end{bmatrix}=\begin{bmatrix}5-4&8-7\\5-2&8-6\end{bmatrix}=\begin{bmatrix}1&1\\3&2\end{bmatrix}$$

(iii) $\boldsymbol{2A+3B}$

$$2A=2\begin{bmatrix}4&2\\7&6\end{bmatrix}=\begin{bmatrix}8&4\\14&12\end{bmatrix}$$

$$3B=3\begin{bmatrix}5&5\\8&8\end{bmatrix}=\begin{bmatrix}15&15\\24&24\end{bmatrix}$$

$$2A+3B=\begin{bmatrix}8+15&4+15\\14+24&12+24\end{bmatrix}=\begin{bmatrix}23&19\\38&36\end{bmatrix}$$

3. If $\boldsymbol{A}=\begin{bmatrix}2&3\\4&5\end{bmatrix}, \boldsymbol{B}=\begin{bmatrix}5&6\\2&1\end{bmatrix}, \boldsymbol{C}=\begin{bmatrix}3&2\\1&7\end{bmatrix}$, then verify that:

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(i) $2(A+B)=2A+2B$

- **LHS:**

$$A+B=\begin{bmatrix}2+5 & 3+6\\ 4+2 & 5+1\end{bmatrix}=\begin{bmatrix}7 & 9\\ 6 & 6\end{bmatrix}$$

$$2(A+B)=2\begin{bmatrix}7 & 9\\ 6 & 6\end{bmatrix}=\begin{bmatrix}14 & 18\\ 12 & 12\end{bmatrix}$$

- **RHS:**

$$2A=\begin{bmatrix}4 & 6\\ 8 & 10\end{bmatrix},\quad 2B=\begin{bmatrix}10 & 12\\ 4 & 2\end{bmatrix}$$

$$2A+2B=\begin{bmatrix}4+10 & 6+12\\ 8+4 & 10+2\end{bmatrix}=\begin{bmatrix}14 & 18\\ 12 & 12\end{bmatrix}$$

LHS = RHS (Verified)

(ii) $(A+B)+C=A+(B+C)$

- **LHS:**

$$(A+B)+C=\begin{bmatrix}7 & 9\\ 6 & 6\end{bmatrix}+\begin{bmatrix}3 & 2\\ 1 & 7\end{bmatrix}=\begin{bmatrix}10 & 11\\ 7 & 13\end{bmatrix}$$

- **RHS:**

$$B+C=\begin{bmatrix}5+3 & 6+2\\ 2+1 & 1+7\end{bmatrix}=\begin{bmatrix}8 & 8\\ 3 & 8\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}+\begin{bmatrix}8 & 8\\ 3 & 8\end{bmatrix}=\begin{bmatrix}10 & 11\\ 7 & 13\end{bmatrix}$$

LHS = RHS (Verified)

(iii) $(A+B)C=AC+BC$

- **LHS:**

$$(A+B)C=\begin{bmatrix}7 & 9\\ 6 & 6\end{bmatrix}\begin{bmatrix}3 & 2\\ 1 & 7\end{bmatrix}=\begin{bmatrix}7(3)+9(1) & 7(2)+9(7)\\ 6(3)+6(1) & 6(2)+6(7)\end{bmatrix}=\begin{bmatrix}30 & 77\\ 24 & 54\end{bmatrix}$$

- **RHS:**

$$AC=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}3 & 2\\ 1 & 7\end{bmatrix}=\begin{bmatrix}2(3)+3(1) & 2(2)+3(7)\\ 4(3)+5(1) & 4(2)+5(7)\end{bmatrix}=\begin{bmatrix}9 & 25\\ 17 & 43\end{bmatrix}$$

$$BC=\begin{bmatrix}5 & 6\\ 2 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ 1 & 7\end{bmatrix}=\begin{bmatrix}5(3)+6(1) & 5(2)+6(7)\\ 2(3)+1(1) & 2(2)+1(7)\end{bmatrix}=\begin{bmatrix}21 & 52\\ 7 & 11\end{bmatrix}$$

$$AC+BC=\begin{bmatrix}9+21 & 25+52\\ 17+7 & 43+11\end{bmatrix}=\begin{bmatrix}30 & 77\\ 24 & 54\end{bmatrix}$$

LHS = RHS (Verified)

(iv) $C(A-B)=CA-CB$

- **LHS:**

$$A-B=\begin{bmatrix}2-5 & 3-6\\ 4-2 & 5-1\end{bmatrix}=\begin{bmatrix}-3 & -3\\ 2 & 4\end{bmatrix}$$

$$C(A-B)=\begin{bmatrix}3 & 2\\ 1 & 7\end{bmatrix}\begin{bmatrix}-3 & -3\\ 2 & 4\end{bmatrix}=\begin{bmatrix}3(-3)+2(2) & 3(-3)+2(4)\\ 1(-3)+7(2) & 1(-3)+7(4)\end{bmatrix}=\begin{bmatrix}-5 & -1\\ 11 & 25\end{bmatrix}$$

- **RHS:**

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$$CA = \begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix} = \begin{bmatrix}3(2)+2(4) & 3(3)+2(5)\\1(2)+7(4) & 1(3)+7(5)\end{bmatrix} = \begin{bmatrix}14 & 19\\30 & 38\end{bmatrix}$$

$$CB = \begin{bmatrix}3 & 2\\1 & 7\end{bmatrix}\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix} = \begin{bmatrix}3(5)+2(2) & 3(6)+2(1)\\1(5)+7(2) & 1(6)+7(1)\end{bmatrix} = \begin{bmatrix}19 & 20\\19 & 13\end{bmatrix}$$

$$CA - CB = \begin{bmatrix}14-19 & 19-20\\30-19 & 38-13\end{bmatrix} = \begin{bmatrix}-5 & -1\\11 & 25\end{bmatrix}$$

LHS = RHS (Verified)

(v) $(AB)^{-1} = B^{-1}A^{-1}$

1. **Calculate AB:**

$$AB = \begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}\begin{bmatrix}5 & 6\\2 & 1\end{bmatrix} = \begin{bmatrix}2(5)+3(2) & 2(6)+3(1)\\4(5)+5(2) & 4(6)+5(1)\end{bmatrix} = \begin{bmatrix}16 & 15\\30 & 29\end{bmatrix}$$

$$|AB| = (16)(29) - (15)(30) = 464 - 450 = 14$$

$$(AB)^{-1} = \frac{1}{14}\begin{bmatrix}29 & -15\\-30 & 16\end{bmatrix}$$

2. **Calculate A^{-1} and B^{-1}:**

$$|A| = (2)(5) - (3)(4) = 10 - 12 = -2 \Longrightarrow A^{-1} = \frac{1}{-2}\begin{bmatrix}5 & -3\\-4 & 2\end{bmatrix}$$

$$|B| = (5)(1) - (6)(2) = 5 - 12 = -7 \Longrightarrow B^{-1} = \frac{1}{-7}\begin{bmatrix}1 & -6\\-2 & 5\end{bmatrix}$$

3. **Calculate $B^{-1}A^{-1}$:**

$$B^{-1}A^{-1} = \frac{1}{14}\begin{bmatrix}1 & -6\\-2 & 5\end{bmatrix}\begin{bmatrix}5 & -3\\-4 & 2\end{bmatrix} = \frac{1}{14}\begin{bmatrix}1(5)+(-6)(-4) & 1(-3)+(-6)(2)\\-2(5)+5(-4) & -2(-3)+5(2)\end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix}29 & -15\\-30 & 16\end{bmatrix}$$

LHS = RHS (Verified)

(vi) $AA^{-1} = A^{-1}A = I$

$$A = \begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}, \quad A^{-1} = -\frac{1}{2}\begin{bmatrix}5 & -3\\-4 & 2\end{bmatrix}$$

- AA^{-1}:

$$AA^{-1} = -\frac{1}{2}\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}\begin{bmatrix}5 & -3\\-4 & 2\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}10-12 & -6+6\\20-20 & -12+10\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$$

- $A^{-1}A$:

$$A^{-1}A = -\frac{1}{2}\begin{bmatrix}5 & -3\\-4 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}10-12 & 15-15\\-8+8 & -12+10\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$$

$AA^{-1} = A^{-1}A = I$ (Verified)

(vii) $(AB)^t = B^tA^t$

- **LHS:**

$$AB = \begin{bmatrix}16 & 15\\30 & 29\end{bmatrix} \Longrightarrow (AB)^t = \begin{bmatrix}16 & 30\\15 & 29\end{bmatrix}$$

- **RHS:**

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$$A^t = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}, \quad B^t = \begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix}$$

$$B^tA^t = \begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 5(2)+2(3) & 5(4)+2(5) \\ 6(2)+1(3) & 6(4)+1(5) \end{bmatrix} = \begin{bmatrix} 16 & 30 \\ 15 & 29 \end{bmatrix}$$

LHS = RHS (Verified)

(viii) $(AB)C = A(BC)$

- **LHS:**

$$(AB)C = \begin{bmatrix} 16 & 15 \\ 30 & 29 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 16(3)+15(1) & 16(2)+15(7) \\ 30(3)+29(1) & 30(2)+29(7) \end{bmatrix} = \begin{bmatrix} 63 & 137 \\ 119 & 263 \end{bmatrix}$$

- **RHS:**

$$BC = \begin{bmatrix} 21 & 52 \\ 7 & 11 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 21 & 52 \\ 7 & 11 \end{bmatrix} = \begin{bmatrix} 2(21)+3(7) & 2(52)+3(11) \\ 4(21)+5(7) & 4(52)+5(11) \end{bmatrix} = \begin{bmatrix} 63 & 137 \\ 119 & 263 \end{bmatrix}$$

LHS = RHS (Verified)

4. If $A = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$, then find:

(i) $|B|$

$$|B| = \begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} = (5)(4) - (3)(2) = 20 - 6 = 14$$

(ii) Adj$B$

$$\text{Adj}B = \begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$$

(iii) A^{-1}

$$|A| = \begin{vmatrix} 7 & 3 \\ 2 & 1 \end{vmatrix} = (7)(1) - (3)(2) = 7 - 6 = 1$$

$$\text{Adj}A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

(iv) $A^{-1}A$

$$A^{-1}A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1(7)+(-3)(2) & 1(3)+(-3)(1) \\ -2(7)+7(2) & -2(3)+7(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(v) $(AB)^t$

$$AB = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 7(5)+3(2) & 7(3)+3(4) \\ 2(5)+1(2) & 2(3)+1(4) \end{bmatrix} = \begin{bmatrix} 41 & 33 \\ 12 & 10 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} 41 & 12 \\ 33 & 10 \end{bmatrix}$$

(vi) $(B^t)^t$

$$B^t = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$$

$$(B^t)^t = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} = B$$

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5. Solve the following pair of linear equations, if possible, using Matrix Inversion Method and Cramer's Rule:

(i)

$$3x + 4y = 7$$
$$5x - y = 2$$

Matrix Inversion Method:

$$|A| = (3)(-1) - (4)(5) = -3 - 20 = -23$$
$$A^{-1} = \frac{1}{-23}\begin{bmatrix} -1 & -4 \\ -5 & 3 \end{bmatrix}$$

Cramer's Rule:

$$|A| = -23$$
$$|A_x| = \begin{vmatrix} 7 & 4 \\ 2 & -1 \end{vmatrix} = -7 - 8 = -15 \Rightarrow x = \frac{-15}{-23} = \frac{15}{23}$$
$$|A_y| = \begin{vmatrix} 3 & 7 \\ 5 & 2 \end{vmatrix} = 6 - 35 = -29 \Rightarrow y = \frac{-29}{-23} = \frac{29}{23}$$

Answer: $x = \frac{15}{23}, y = \frac{29}{23}$

(ii)

$$x - 6y = -15$$
$$2x + 6y = -3$$

Matrix Inversion Method:

$$|A| = (1)(6) - (-6)(2) = 6 + 12 = 18$$
$$A^{-1} = \frac{1}{18}\begin{bmatrix} 6 & 6 \\ -2 & 1 \end{bmatrix}$$

Cramer's Rule:

$$|A| = 18$$
$$|A_x| = \begin{vmatrix} -15 & -6 \\ -3 & 6 \end{vmatrix} = -90 - 18 = -108 \Rightarrow x = \frac{-108}{18} = -6$$
$$|A_y| = \begin{vmatrix} 1 & -15 \\ 2 & -3 \end{vmatrix} = -3 - (-30) = 27 \Rightarrow y = \frac{27}{18} = \frac{3}{2}$$

Answer: $x = -6, y = \frac{3}{2}$

(iii)

$$2x + y = 5$$
$$x + 3y = 3$$

Matrix Inversion Method:

$$|A| = (2)(3) - (1)(1) = 6 - 1 = 5$$
$$A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

Cramer's Rule:

$$|A| = 5$$
$$|A_x| = \begin{vmatrix} 5 & 1 \\ 3 & 3 \end{vmatrix} = 15 - 3 = 12 \Rightarrow x = \frac{12}{5}$$

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$$|A_y| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1 \Longrightarrow y = \frac{1}{5}$$

Answer: $x = \frac{12}{5}, y = \frac{1}{5}$

6. Find two numbers by using matrices such that twice the first added to the second makes 21 and twice the second added to the first makes 27.

Let the first number be x and the second number be y.

1. $2x + y = 21$
2. $x + 2y = 27$

Matrix Form:

$$|A| = (2)(2) - (1)(1) = 4 - 1 = 3$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Answer: The two numbers are 5 and 11.

7. 4 knives and 6 forks cost Rs. 136, whereas 6 knives and 5 forks cost Rs. 164. Find the cost of a knife and a fork by using matrices.

Let the cost of a knife be x and the cost of a fork be y.

1. $4x + 6y = 136$
2. $6x + 5y = 164$

Matrix Form:

$$|A| = (4)(5) - (6)(6) = 20 - 36 = -16$$

$$A^{-1} = \frac{1}{-16}\begin{bmatrix} 5 & -6 \\ -6 & 4 \end{bmatrix}$$

Answer:

- Cost of a knife = **Rs. 19**
- Cost of a fork = **Rs. 10**

8. A shop employs 5 men and 3 women, pays total daily wages Rs. 3500. If the number of men is reduced to 2 and 3 extra women are taken on, the daily wages amount to Rs. 5000. Find daily wages of a man and a woman by using matrices.

Let daily wage of a man $= x$ and daily wage of a woman $= y$.

1. $5x + 3y = 3500$
2. Men reduced to 2, women become $3 + 3 = 6$:

$$2x + 6y = 5000$$

Matrix Form:

$$|A| = (5)(6) - (3)(2) = 30 - 6 = 24$$

$$A^{-1} = \frac{1}{24}\begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}$$

Answer:

- Daily wage of a man = **Rs. 250**
- Daily wage of a woman = **Rs. 750**

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