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Class	10th	Chapter	04
Subject	Mathematics	Board	Punjab

CHAPTER 04: Functions and Graphs
EXERCISE 4.1

Question 1

Given $f(x) = 2x + 5$ and $g(x) = 3x - 2$:

(i) Find $f(x) + g(x)$

$$f(x) + g(x) = (2x + 5) + (3x - 2)$$
$$= 2x + 3x + 5 - 2 = \mathbf{5x + 3}$$

(ii) Find $f(x) - g(x)$

$$f(x) - g(x) = (2x + 5) - (3x - 2)$$
$$= 2x + 5 - 3x + 2 = \mathbf{-x + 7}$$

Question 2

Given $f(x) = x + 2$ and $g(x) = 2x + 4$:

(i) Find $f(x) \cdot g(x)$

$$f(x) \cdot g(x) = (x + 2)(2x + 4)$$
$$= 2x^2 + 4x + 4x + 8 = \mathbf{2x^2 + 8x + 8}$$

(ii) Find $g(x) \cdot f(x)$

$$g(x) \cdot f(x) = (2x + 4)(x + 2) = \mathbf{2x^2 + 8x + 8}$$

(iii) Find $\frac{f(x)}{g(x)}$

$$\frac{f(x)}{g(x)} = \frac{x + 2}{2x + 4} = \frac{x + 2}{2(x + 2)} = \mathbf{\frac{1}{2}} \quad (x \neq -2)$$

(iv) Find $\frac{g(x)}{f(x)}$

$$\frac{g(x)}{f(x)} = \frac{2x + 4}{x + 2} = \frac{2(x + 2)}{x + 2} = \mathbf{2} \quad (x \neq -2)$$

Question 3

Find (a) $(f \circ g)(x)$, (b) $(g \circ f)(x)$, (c) $(f \circ f)(x)$, (d) $(g \circ g)(x)$ for each part:

(i) $f(x) = 2x + 3, g(x) = x^3$

- **(a)** $(f \circ g)(x) = f\big(g(x)\big) = f(x^3) = \mathbf{2x^3 + 3}$

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- **(b)** $(g \circ f)(x) = g\big(f(x)\big) = g(2x+3) = \mathbf{(2x+3)^3}$
- **(c)** $(f \circ f)(x) = f\big(f(x)\big) = f(2x+3) = 2(2x+3)+3 = 4x+6+3 = \mathbf{4x+9}$
- **(d)** $(g \circ g)(x) = g\big(g(x)\big) = g(x^3) = (x^3)^3 = \mathbf{x^9}$

(ii) $\boldsymbol{f(x) = \frac{2}{x}\,(x \neq 0),\ g(x) = 2x^2 - 1}$

- **(a)** $(f \circ g)(x) = f\big(g(x)\big) = f(2x^2-1) = \mathbf{\frac{2}{2x^2-1}}$
- **(b)** $(g \circ f)(x) = g\big(f(x)\big) = g\left(\frac{2}{x}\right) = 2\left(\frac{2}{x}\right)^2 - 1 = 2\left(\frac{4}{x^2}\right) - 1 = \mathbf{\frac{8}{x^2} - 1}$
- **(c)** $(f \circ f)(x) = f\big(f(x)\big) = f\left(\frac{2}{x}\right) = \frac{2}{\frac{2}{x}} = 2 \cdot \frac{x}{2} = \mathbf{x}$
- **(d)** $(g \circ g)(x) = g\big(g(x)\big) = g(2x^2-1) = 2(2x^2-1)^2 - 1$

$$= 2(4x^4 - 4x^2 + 1) - 1 = 8x^4 - 8x^2 + 2 - 1 = \mathbf{8x^4 - 8x^2 + 1}$$

(iii) $\boldsymbol{f(x) = 2x - 1,\ g(x) = \frac{x+1}{2}}$

- **(a)** $(f \circ g)(x) = f\big(g(x)\big) = f\left(\frac{x+1}{2}\right) = 2\left(\frac{x+1}{2}\right) - 1 = x + 1 - 1 = \mathbf{x}$
- **(b)** $(g \circ f)(x) = g\big(f(x)\big) = g(2x-1) = \frac{(2x-1)+1}{2} = \frac{2x}{2} = \mathbf{x}$
- **(c)** $(f \circ f)(x) = f\big(f(x)\big) = f(2x-1) = 2(2x-1) - 1 = 4x - 2 - 1 = \mathbf{4x - 3}$
- **(d)** $(g \circ g)(x) = g\big(g(x)\big) = g\left(\frac{x+1}{2}\right) = \frac{\frac{x+1}{2}+1}{2} = \frac{\frac{x+3}{2}}{2} = \mathbf{\frac{x+3}{4}}$

Question 4

Find k such that $(f \circ g)(x) = (g \circ f)(x)$, where $f(x) = 3x + 2$ and $g(x) = 6x - k$.

1. **Calculate $(f \circ g)(x)$:**

$$(f \circ g)(x) = f(6x-k) = 3(6x-k) + 2 = 18x - 3k + 2$$

2. **Calculate $(g \circ f)(x)$:**

$$(g \circ f)(x) = g(3x+2) = 6(3x+2) - k = 18x + 12 - k$$

3. **Set them equal:**

$$18x - 3k + 2 = 18x + 12 - k$$
$$-3k + 2 = 12 - k$$
$$-2k = 10 \Longrightarrow \mathbf{k = -5}$$

Question 5

Given $f(x) = 3x + 2$ and $g(x) = 2x + 3$:

(i) Find $\boldsymbol{g\big(f(4)\big)}$

$$f(4) = 3(4) + 2 = 14$$
$$g\big(f(4)\big) = g(14) = 2(14) + 3 = 28 + 3 = \mathbf{31}$$

(ii) Find $\boldsymbol{f\big(f(3)\big)}$

$$f(3) = 3(3) + 2 = 11$$
$$f\big(f(3)\big) = f(11) = 3(11) + 2 = 33 + 2 = \mathbf{35}$$

(iii) Find $\boldsymbol{f\big(g(-2)\big)}$

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$$g(-2) = 2(-2) + 3 = -4 + 3 = -1$$
$$f\big(g(-2)\big) = f(-1) = 3(-1) + 2 = -3 + 2 = \mathbf{-1}$$

Question 6

Find $f^{-1}(x)$ for each:

(i) $f(x) = 2x - 3$

Let $y = 2x - 3 \Rightarrow 2x = y + 3 \Rightarrow x = \frac{y+3}{2}$

$$\mathbf{f^{-1}(x) = \frac{x+3}{2}}$$

(ii) $f(x) = 4x^3 - 1$

Let $y = 4x^3 - 1 \Rightarrow 4x^3 = y + 1 \Rightarrow x^3 = \frac{y+1}{4} \Rightarrow x = \sqrt[3]{\frac{y+1}{4}}$

$$\mathbf{f^{-1}(x) = \left(\frac{x+1}{4}\right)^{1/3}}$$

(iii) $f(x) = \sqrt{x-1}, \quad x \geq 1$

Let $y = \sqrt{x-1} \Rightarrow y^2 = x - 1 \Rightarrow x = y^2 + 1$

$$\mathbf{f^{-1}(x) = x^2 + 1} \quad (x \geq 0)$$

(iv) $f(x) = \frac{x+1}{3x-2}, \quad x \neq \frac{2}{3}$

Let $y = \frac{x+1}{3x-2}$

$$y(3x - 2) = x + 1 \Rightarrow 3xy - 2y = x + 1$$
$$3xy - x = 2y + 1 \Rightarrow x(3y - 1) = 2y + 1 \Rightarrow x = \frac{2y+1}{3y-1}$$
$$\mathbf{f^{-1}(x) = \frac{2x+1}{3x-1}} \quad \left(x \neq \frac{1}{3}\right)$$

Question 7

Given $f(x) = 4x + 2$ and $g(x) = 6x - 18$:

(i) Find $f^{-1}(x)$ and $g^{-1}(x)$

- For $f(x)$: $y = 4x + 2 \Rightarrow x = \frac{y-2}{4} \Rightarrow \mathbf{f^{-1}(x) = \frac{x-2}{4}}$
- For $g(x)$: $y = 6x - 18 \Rightarrow x = \frac{y+18}{6} \Rightarrow \mathbf{g^{-1}(x) = \frac{x+18}{6}}$

(ii) Find x if $f^{-1}(x) = g^{-1}(x)$

$$\frac{x-2}{4} = \frac{x+18}{6}$$

Multiply both sides by 12:

$$3(x - 2) = 2(x + 18)$$
$$3x - 6 = 2x + 36$$
$$3x - 2x = 36 + 6 \Rightarrow \mathbf{x = 42}$$

Question 8

Verify that $f\big(f^{-1}(x)\big) = f^{-1}\big(f(x)\big) = x$:

(i) $f(x) = x - 6$

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- Find $f^{-1}(x)$: $y = x - 6 \Longrightarrow x = y + 6 \Longrightarrow f^{-1}(x) = x + 6$
- $f\left(f^{-1}(x)\right) = f(x + 6) = (x + 6) - 6 = x$
- $f^{-1}\left(f(x)\right) = f^{-1}(x - 6) = (x - 6) + 6 = x$ **(Verified)**

(ii) $\boldsymbol{f(x) = 7x - 4}$

- Find $f^{-1}(x)$: $y = 7x - 4 \Longrightarrow x = \frac{y+4}{7} \Longrightarrow f^{-1}(x) = \frac{x+4}{7}$
- $f\left(f^{-1}(x)\right) = f\left(\frac{x+4}{7}\right) = 7\left(\frac{x+4}{7}\right) - 4 = x + 4 - 4 = x$
- $f^{-1}\left(f(x)\right) = f^{-1}(7x - 4) = \frac{(7x-4)+4}{7} = \frac{7x}{7} = x$ **(Verified)**

(iii) $\boldsymbol{f(x) = \frac{x-3}{4}}$

- Find $f^{-1}(x)$: $y = \frac{x-3}{4} \Longrightarrow 4y = x - 3 \Longrightarrow x = 4y + 3 \Longrightarrow f^{-1}(x) = 4x + 3$
- $f\left(f^{-1}(x)\right) = f(4x + 3) = \frac{(4x+3)-3}{4} = \frac{4x}{4} = x$
- $f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{x-3}{4}\right) = 4\left(\frac{x-3}{4}\right) + 3 = x - 3 + 3 = x$ **(Verified)**

(iv) $\boldsymbol{f(x) = \frac{x-4}{x+2}, \quad x \neq -2}$

- Find $f^{-1}(x)$: $y = \frac{x-4}{x+2} \Longrightarrow y(x + 2) = x - 4 \Longrightarrow xy + 2y = x - 4$

$$xy - x = -2y - 4 \Longrightarrow x(y - 1) = -(2y + 4) \Longrightarrow x = \frac{2y + 4}{1 - y}$$

$$f^{-1}(x) = \frac{2x + 4}{1 - x}$$

- $f\left(f^{-1}(x)\right) = f\left(\frac{2x+4}{1-x}\right) = \frac{\frac{2x+4}{1-x} - 4}{\frac{2x+4}{1-x} + 2} = \frac{(2x+4)-4(1-x)}{(2x+4)+2(1-x)} = \frac{6x}{6} = x$
- $f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{x-4}{x+2}\right) = \frac{2\left(\frac{x-4}{x+2}\right)+4}{1-\left(\frac{x-4}{x+2}\right)} = \frac{2(x-4)+4(x+2)}{(x+2)-(x-4)} = \frac{6x}{6} = x$ **(Verified)**

Question 9

Without finding $f^{-1}(x)$, find the domain and range of $f^{-1}(x)$. *(Rule:* $\text{Domain}(f^{-1}) = \text{Range}(f)$ *and* $\text{Range}(f^{-1}) = \text{Domain}(f)$*)*

(i) $\boldsymbol{f(x) = 12x - 3}$

- Domain of $f = \mathbb{R}$, Range of $f = \mathbb{R}$
- Domain of $f^{-1} = \mathbb{R}$
- Range of $f^{-1} = \mathbb{R}$

(ii) $\boldsymbol{f(x) = \frac{1}{2}x + 8}$

- Domain of $f = \mathbb{R}$, Range of $f = \mathbb{R}$
- Domain of $f^{-1} = \mathbb{R}$
- Range of $f^{-1} = \mathbb{R}$

(iii) $\boldsymbol{f(x) = \frac{x}{1+x}, \quad x \neq -1}$

- Domain of $f = \mathbb{R} \setminus \{-1\}$

- Range of $f = \mathbb{R} \setminus \{1\}$
- Domain of $f^{-1} = \mathbb{R} \setminus \{1\}$
- Range of $f^{-1} = \mathbb{R} \setminus \{-1\}$

(iv) $f(x) = \sqrt{x-2}, \quad x \geq 2$

- Domain of $f = [2, \infty)$
- Range of $f = [0, \infty)$
- Domain of $f^{-1} = [0, \infty)$
- Range of $f^{-1} = [2, \infty)$

Question 10

Given $f(x) = x^2 + 9$ and $g(x) = x + 21$. Find the values of a such that $f(a) = g(a)$:

$$f(a) = a^2 + 9$$
$$g(a) = a + 21$$

Set them equal:

$$a^2 + 9 = a + 21$$
$$a^2 - a - 12 = 0$$

Factorize:

$$(a-4)(a+3) = 0$$
$$\mathbf{a = 4} \quad \text{or} \quad \mathbf{a = -3}$$

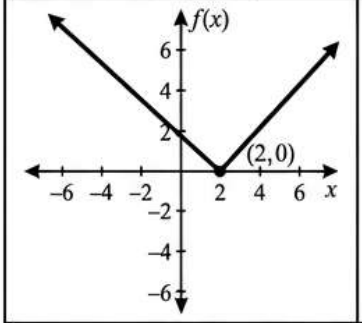
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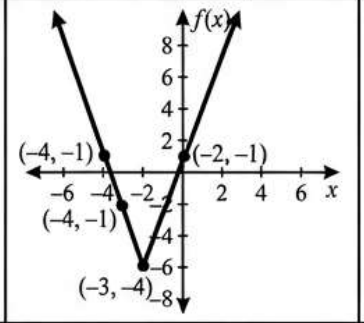
EXERCISE 4.2

EXERCISE 4.2 Absolute Value Graphs & Number Lines

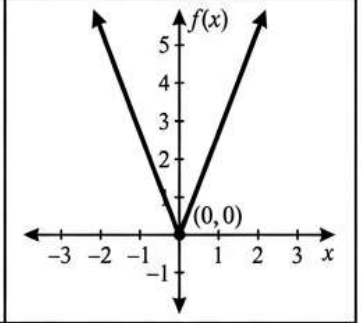
(i) $f(x) = |x - 2|$



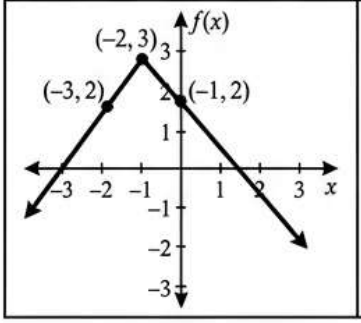
(ii) $f(x) = 3|x + 3| - 4$



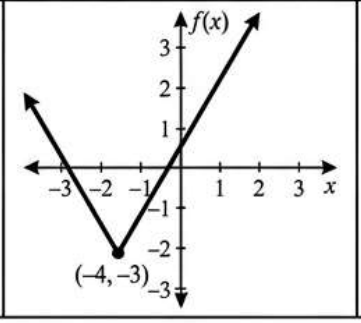
(iii) $f(x) = 5|x|$



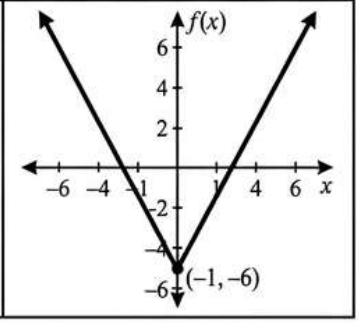
(iv) $f(x) = -|x + 2| + 3$



(v) $f(x) = 2|x + 4| - 3$



(vi) $f(x) = 2|x + 1| - 6$



Number Line Solutions

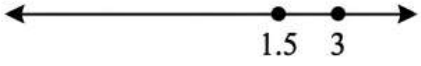
(i) $|x - 2| = 6$



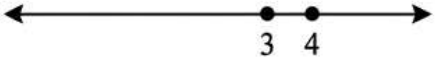
(ii) $|2x + 1| = 7$



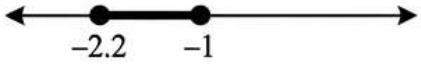
(iii) $|4x - 9| = 3$



(iv) $|7 - 2x| = 1$



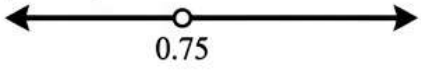
(v) $|5x + 8| \leq 3$



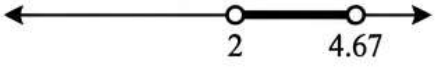
(vi) $|4x - 12| \leq 0$



(vii) $|3 - 4x| > 0$



(viii) $1 - 2|1.5x - 5| > -3$



(ix) $|3x - 2| \leq 12$



(x) $|1 - 2x| > 5$



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Question 2: Solve Equations and Express on Number Line

(i) $|x-2|=6$

- **Step 1:** Set up two equations:

$$x-2=6 \quad \text{or} \quad x-2=-6$$

- **Step 2:** Solve for x:

$$x=6+2=8 \quad \text{or} \quad x=-6+2=-4$$

- **Solution Set:** $\{-4,8\}$
- **Number Line Representation:** Closed/solid dots placed specifically at -4 and 8.

(ii) $|2x+1|=7$

- **Step 1:** Set up two equations:

$$2x+1=7 \quad \text{or} \quad 2x+1=-7$$

- **Step 2:** Solve for x:

$$2x=6 \Longrightarrow x=3$$
$$2x=-8 \Longrightarrow x=-4$$

- **Solution Set:** $\{-4,3\}$
- **Number Line Representation:** Solid dots at -4 and 3.

(iii) $|4x-9|=3$

- **Step 1:** Set up two equations:

$$4x-9=3 \quad \text{or} \quad 4x-9=-3$$

- **Step 2:** Solve for x:

$$4x=12 \Longrightarrow x=3$$
$$4x=6 \Longrightarrow x=\frac{6}{4}=\frac{3}{2}=1.5$$

- **Solution Set:** $\{1.5,3\}$
- **Number Line Representation:** Solid dots at 1.5 and 3.

(iv) $|7-2x|=1$

- **Step 1:** Set up two equations:

$$7-2x=1 \quad \text{or} \quad 7-2x=-1$$

- **Step 2:** Solve for x:

$$-2x=-6 \Longrightarrow x=3$$
$$-2x=-8 \Longrightarrow x=4$$

- **Solution Set:** $\{3,4\}$
- **Number Line Representation:** Solid dots at 3 and 4.

Question 3: Solve Inequalities and Express on Number Line

(i) $|5x+8|\leq 3$

- **Step 1:** Rewrite without absolute value:

$$-3\leq 5x+8\leq 3$$

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- **Step 2:** Subtract 8 from all parts:

$$-3-8 \leq 5x \leq 3-8$$
$$-11 \leq 5x \leq -5$$

- **Step 3:** Divide by 5:

$$-\frac{11}{5} \leq x \leq -1 \quad \Longrightarrow \quad -2.2 \leq x \leq -1$$

- **Solution Interval:** $[-2.2, -1]$
- **Number Line Representation:** Solid segment spanning between -2.2 and -1 with solid dots at both endpoints.

(ii) $|4x-12| \leq 0$

- **Step 1:** Since absolute value is always non-negative ($|a| \geq 0$), $|4x-12|$ can only equal 0:

$$4x-12=0 \Longrightarrow 4x=12 \Longrightarrow x=3$$

- **Solution Set:** $\{3\}$
- **Number Line Representation:** A single solid dot at 3.

(iii) $|3-4x| > 0$

- **Step 1:** Absolute value is strictly positive for all real numbers except when the expression equals zero:

$$3-4x \neq 0 \Longrightarrow 4x \neq 3 \Longrightarrow x \neq \frac{3}{4}=0.75$$

- **Solution Set:** $(-\infty, 0.75) \cup (0.75, \infty)$ or $\mathbb{R} \setminus \{0.75\}$
- **Number Line Representation:** The entire number line shaded with an **open circle** at 0.75.

(iv) $1-2\left|\frac{3}{2}x-5\right| > -3$

- **Step 1:** Isolate the absolute value expression:

$$-2\left|\frac{3}{2}x-5\right| > -3-1$$
$$-2\left|\frac{3}{2}x-5\right| > -4$$

- **Step 2:** Divide by -2 (flip the inequality sign):

$$\left|\frac{3}{2}x-5\right| < 2$$

- **Step 3:** Rewrite as a compound inequality:

$$-2 < \frac{3}{2}x-5 < 2$$

- **Step 4:** Add 5 to all parts:

$$3 < \frac{3}{2}x < 7$$

- **Step 5:** Multiply by $\frac{2}{3}$:

$$2 < x < \frac{14}{3} \quad \Longrightarrow \quad 2 < x < 4.67$$

- **Solution Interval:** $\left(2, \frac{14}{3}\right)$
- **Number Line Representation:** Open region shaded between 2 and $\frac{14}{3}$ (approx. 4.67) with **open circles** at both endpoints.

(v) $|3x - 2| \leq 12$

- **Step 1:** Rewrite without absolute value:

$$-12 \leq 3x - 2 \leq 12$$

- **Step 2:** Add 2 to all parts:

$$-10 \leq 3x \leq 14$$

- **Step 3:** Divide by 3:

$$-\frac{10}{3} \leq x \leq \frac{14}{3} \quad \Longrightarrow \quad -3.33 \leq x \leq 4.67$$

- **Solution Interval:** $\left[-\frac{10}{3}, \frac{14}{3}\right]$
- **Number Line Representation:** A solid segment shaded between -3.33 and 4.67 with **solid dots** at both endpoints.

(vi) $|1 - 2x| > 5$

- **Step 1:** Set up two separate inequalities:

$$1 - 2x > 5 \quad \text{or} \quad 1 - 2x < -5$$

- **Step 2:** Solve each inequality:
  - $1 - 2x > 5 \Longrightarrow -2x > 4 \Longrightarrow x < -2$
  - $1 - 2x < -5 \Longrightarrow -2x < -6 \Longrightarrow x > 3$
- **Solution Interval:** $(-\infty, -2) \cup (3, \infty)$
- **Number Line Representation:** Two rays: one going left from -2 (with an **open circle** at -2) and one going right from 3 (with an **open circle** at 3).

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Exercise 4.3

Question 1

Problem: A function $B(t) = 5000 + 200t$ represents the total balance (in rupees) after t months. What will be the balance after 6 months?

Step-by-step Solution:

1. **Given function:**

$$B(t) = 5000 + 200t$$

2. **Given time:** $t = 6$ months
3. **Substitute $t = 6$ into the function:**

$$B(6) = 5000 + 200(6)$$
$$B(6) = 5000 + 1200$$
$$B(6) = 6200$$

Answer: The balance after 6 months will be **Rs. 6,200**.

Question 2

Problem: A function $f(k) = 150 + 20k$ represents the total fare (in rupees) for k kilometres. How much will the fare be for a 12 kilometres ride?

Step-by-step Solution:

1. **Given function:**

$$f(k) = 150 + 20k$$

2. **Given distance:** $k = 12$ km
3. **Substitute $k = 12$ into the function:**

$$f(12) = 150 + 20(12)$$
$$f(12) = 150 + 240$$
$$f(12) = 390$$

Answer: The fare for a 12 km ride will be **Rs. 390**.

Question 3

Problem: The cost of manufacturing fancy sofa set would be fixed charges Rs. 5500 which is modeled as $f(n) = 5500n$, where n is the number of sofa sets. Find the cost of 50 sofa sets.

Step-by-step Solution:

1. **Given function:**

$$f(n) = 5500n$$

2. **Given number of sofa sets:** $n = 50$
3. **Substitute $n = 50$ into the function:**

$$f(50) = 5500(50)$$
$$f(50) = 275000$$

Answer: The cost of 50 sofa sets will be **Rs. 275,000**.

Question 4

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Problem: A function $T(d) = \frac{d}{60}$ represents the time T in hours to travel a distance d kilometres. How long will it take to travel 180 km?

Step-by-step Solution:

1. **Given function:**

$$T(d) = \frac{d}{60}$$

2. **Given distance:** $d = 180$ km
3. **Substitute $d = 180$ into the function:**

$$T(180) = \frac{180}{60}$$
$$T(180) = 3$$

Answer: It will take **3 hours** to travel 180 km.

Question 5

Problem: A company charges Rs. 100 for an encoding work. In addition, the company charges Rs. 5 per page of printed output. The model of function $f(x) = 100 + 5x$, where x represents the number of pages printed out. How much will company charge for 55 page encoding and printing work?

Step-by-step Solution:

1. **Given function:**

$$f(x) = 100 + 5x$$

2. **Given number of pages:** $x = 55$
3. **Substitute $x = 55$ into the function:**

$$f(55) = 100 + 5(55)$$
$$f(55) = 100 + 275$$
$$f(55) = 375$$

Answer: The company will charge **Rs. 375**.

Question 6

Problem: A chemical reaction is stable at 37°C. The process must be stopped if the temperature deviates by more than 2.5°C. The condition is modeled as: $|T - 37°| > 2.5°$, T be the temperature. For what temperature values must the process be stopped?

Step-by-step Solution:

1. **Given inequality:**

$$|T - 37| > 2.5$$

2. **Apply absolute value property:** $|u| > a \Rightarrow u > a$ or $u < -a$
   - **Case 1:**

$$T - 37 > 2.5$$
$$T > 37 + 2.5$$
$$T > 39.5°\text{C}$$

   - **Case 2:**

$$T - 37 < -2.5$$
$$T < 37 - 2.5$$
$$T < 34.5°\text{C}$$

Answer: The process must be stopped if the temperature is **below** 34.5°C or **above** 39.5°C ($T < 34.5°\text{C}$ or $T > 39.5°\text{C}$).

Question 7

Problem: A factory produces metal rods that must be 2.5 metres long, with a tolerance of ± 0.04 metres. An absolute value inequality models this: $|x - 2.5| \leq 0.04$. What is the range of acceptable lengths?

Step-by-step Solution:

1. **Given inequality:**

$$|x - 2.5| \leq 0.04$$

2. **Apply absolute value property:** $|u| \leq a \Rightarrow -a \leq u \leq a$

$$-0.04 \leq x - 2.5 \leq 0.04$$

3. **Add 2.5 to all parts:**

$$-0.04 + 2.5 \leq x \leq 0.04 + 2.5$$
$$2.46 \leq x \leq 2.54$$

Answer: The range of acceptable lengths is from 2.46 metres **to** 2.54 metres ($2.46 \text{ m} \leq x \leq 2.54 \text{ m}$).

Question 8

Problem: A machine part must be aligned so that its centre is exactly at 0. If it shifts more than 0.1 mm, the part is rejected. The model is given by $|x| > 0.1$. What positions of the centre cause rejection?

Step-by-step Solution:

1. **Given inequality:**

$$|x| > 0.1$$

2. **Apply absolute value property:** $|u| > a \Rightarrow u > a$ or $u < -a$

$$x > 0.1 \quad \text{or} \quad x < -0.1$$

Answer: The part will be rejected if its centre shifts to positions **less than** -0.1 mm or **greater than** 0.1 mm ($x < -0.1$ mm or $x > 0.1$ mm).

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REVIEW EXERCISE 4

Question 1: Multiple Choice Questions

(i) If $f(x) = \frac{5x-6}{3}$, then what is the value of $f(3)$?

- **Solution Steps:** Substitute $x = 3$into the function:

$$f(3) = \frac{5(3) - 6}{3} = \frac{15 - 6}{3} = \frac{9}{3} = 3$$

$$f(3) = \frac{5(3) - 6}{3} = \frac{15 - 6}{3} = \frac{9}{3} = 3$$

- **Options:** (a) -1\quad (b) 3\quad (c) 9\quad (d) 15
- **Correct Answer: (b)** 3

(ii) A function f from X to Y is represented by:

- **Solution Steps:** By definition, a function mapping from set $X$to set $Y$is denoted as $f: X \to Y$.
- **Options:** (a) $f: XY$\quad (b) $f: Y \to X$\quad (c) $f: X \to Y$\quad (d) $f: \frac{X}{Y}$
- **Correct Answer: (c)** $f: X \to Y$

(iii) $(fog)(x) =$

- **Solution Steps:** By definition of composite functions, $(fog)(x) = f\big(g(x)\big)$.
- **Options:** (a) $(f + g)(x)$\quad (b) $(f - g)(x)$\quad (c) $f\big(g(x)\big)$\quad (d) $f(x) \div g(x)$
- **Correct Answer: (c)** $f\big(g(x)\big)$

(iv) If $f(x) = 2x + 3$, $g(x) = x + 1$, then $f(x) + g(x) =$

- **Solution Steps:** Add the two functions:

$$f(x) + g(x) = (2x + 3) + (x + 1) = 2x + x + 3 + 1 = 3x + 4$$

$$f(x) + g(x) = (2x + 3) + (x + 1) = 2x + x + 3 + 1 = 3x + 4$$

- **Options:** (a) $3x$\quad (b) $3x + 4$\quad (c) 4\quad (d) $2x^2 + 3$
- **Correct Answer: (b)** $3x + 4$

(v) If $f(x) = 5x + 2$, $h(x) = 2x - 2$, then $f(x) - h(x) =$

- **Solution Steps:** Subtract $h(x)$from $f(x)$:

$$f(x) - h(x) = (5x + 2) - (2x - 2) = 5x + 2 - 2x + 2 = 3x + 4$$

$$f(x) - h(x) = (5x + 2) - (2x - 2) = 5x + 2 - 2x + 2 = 3x + 4$$

- **Options:** (a) $3x$\quad (b) $5x^2 - 4$\quad (c) $3x + 4$\quad (d) $-3x - 4$
- **Correct Answer: (c)** $3x + 4$

(vi) If $f(x) = 3x + 1$, $g(x) = 2x$, then $g(x) \times f(x) =$

- **Solution Steps:** Multiply the functions:

$$g(x) \times f(x) = 2x(3x + 1) = 6x^2 + 2x$$

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$$g(x) \times f(x) = 2x(3x+1) = 6x^2 + 2x$$

- **Options:** (a) $6x + 2x$\quad (b) $5x^2 + 1$\quad (c) $x + 1$\quad (d) $6x^2 + 2x$
- **Correct Answer: (d)** $6x^2 + 2x$

(vii) If $f(x) = x^2 - 4$, $g(x) = x + 2$, $x \neq -2$, then $\frac{f(x)}{g(x)} =$

- **Solution Steps:** Factorize the numerator using the difference of squares:

$$\frac{f(x)}{g(x)} = \frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{x+2} = x - 2 \text{ (for } x \neq -2)$$

$$\frac{f(x)}{g(x)} = \frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{x+2} = x - 2 \quad (\text{for } x \neq -2)$$

- **Options:** (a) $\frac{1}{x-2}$\quad (b) $\frac{1}{x+2}$\quad (c) $x + 2$\quad (d) $x - 2$
- **Correct Answer: (d)** $x - 2$

(viii) What is the shape of the graph of an absolute value function?

- **Solution Steps:** The graph of an absolute value function $f(x) = |x|$forms a V-shape.
- **Options:** (a) U-shaped \quad (b) V-shaped \quad (c) L-shaped \quad (d) M-shaped
- **Correct Answer: (b) V-shaped**

(ix) If a graph represents a function, then every vertical line must intersect it at:

- **Solution Steps:** By the Vertical Line Test, a graph represents a function if any vertical line intersects it at most once (i.e., exactly 1point).
- **Options:** (a) 4points \quad (b) 3points \quad (c) 2points \quad (d) 1point
- **Correct Answer: (d) 1point**

(x) If $f(x) = x^3$, then $f(-2) =$

- **Solution Steps:** Substitute $x = -2$:

$$f(-2) = (-2)^3 = -8$$

$$f(-2) = (-2)^3 = -8$$

- **Options:** (a) -8\quad (b) 8\quad (c) 4\quad (d) -6
- **Correct Answer: (a)** -8

Question 2

Given: $f(x) = 25 - x^2$and $g(x) = 5 + x$

(i) Find $(f + g)(x)$

- **Step 1:** $(f + g)(x) = f(x) + g(x) = (25 - x^2) + (5 + x)$
- **Step 2:** Combine like terms:

$$(f + g)(x) = -x^2 + x + 30$$

$$(f + g)(x) = -x^2 + x + 30$$

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(ii) Find $(f-g)(x)$

- **Step 1:** $(f-g)(x) = f(x) - g(x) = (25 - x^2) - (5 + x) = 25 - x^2 - 5 - x$
- **Step 2:** Simplify:

$$(f-g)(x) = -x^2 - x + 20$$

$$(f-g)(x) = -x^2 - x + 20$$

(iii) Find $(f \cdot g)(x)$

- **Step 1:** $(f \cdot g)(x) = f(x) \cdot g(x) = (25 - x^2)(5 + x)$
- **Step 2:** Expand the product:

$$(f \cdot g)(x) = 25(5) + 25x - 5x^2 - x^3 = 125 + 25x - 5x^2 - x^3$$

$$(f \cdot g)(x) = 25(5) + 25x - 5x^2 - x^3 = 125 + 25x - 5x^2 - x^3$$

- **Step 3:** Rearrange in standard descending order:

$$(f \cdot g)(x) = -x^3 - 5x^2 + 25x + 125$$

$$(f \cdot g)(x) = -x^3 - 5x^2 + 25x + 125$$

(iv) Find $\frac{f(x)}{g(x)}$

- **Step 1:** $\frac{f(x)}{g(x)} = \frac{25-x^2}{5+x}$
- **Step 2:** Factorize the numerator:

$$\frac{(5-x)(5+x)}{5+x} = 5 - x \text{ (for } x \neq -5)$$

$$\frac{(5-x)(5+x)}{5+x} = 5 - x \quad \text{(for } x \neq -5)$$

(v) Find $f(7)$

- **Step 1:** $f(7) = 25 - (7)^2 = 25 - 49 = -24$

(vi) Find $g(-8)$

- **Step 1:** $g(-8) = 5 + (-8) = 5 - 8 = -3$

Question 3

Given: $f(x) = x^3$ and $g(x) = 14 + 2x$

(i) Find $(fog)(x)$

- **Step 1:** $(fog)(x) = f\big(g(x)\big) = f(14 + 2x) = (14 + 2x)^3$

(ii) Find $(gof)(x)$

- **Step 1:** $(gof)(x) = g\big(f(x)\big) = g(x^3) = 14 + 2x^3$

(iii) Find $(fof)(x)$

- **Step 1:** $(fof)(x) = f\big(f(x)\big) = f(x^3) = (x^3)^3 = x^9$

(iv) Find $(gog)(x)$

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- **Step 1:** $(gog)(x) = g\big(g(x)\big) = g(14 + 2x) = 14 + 2(14 + 2x)$
- **Step 2:** Simplify:

$$14 + 28 + 4x = 42 + 4x$$

$$14 + 28 + 4x = 42 + 4x$$

Question 4: Find $f^{-1}(x)$, if

(i) $f(x) = 9x - 1$

- **Step 1:** Let $y = 9x - 1$. Interchange $x$and y: $x = 9y - 1$.
- **Step 2:** Solve for y: $9y = x + 1 \Longrightarrow y = \frac{x+1}{9}$.
- **Answer:** $f^{-1}(x) = \frac{x+1}{9}$

(ii) $f(x) = \frac{5}{x-1}, x \neq 1$

- **Step 1:** Let $y = \frac{5}{x-1}$. Interchange $x$and y: $x = \frac{5}{y-1}$.
- **Step 2:** Solve for y: $y - 1 = \frac{5}{x} \Longrightarrow y = \frac{5}{x} + 1 = \frac{5+x}{x}$.
- **Answer:** $f^{-1}(x) = \frac{x+5}{x}$

(iii) $f(x) = \sqrt{x-5}, x \geq 5$

- **Step 1:** Let $y = \sqrt{x-5}$. Interchange $x$and y: $x = \sqrt{y-5}$.
- **Step 2:** Square both sides: $x^2 = y - 5 \Longrightarrow y = x^2 + 5$.
- **Answer:** $f^{-1}(x) = x^2 + 5 \quad (x \geq 0)$

(iv) $f(x) = \frac{3-x}{2}$

- **Step 1:** Let $y = \frac{3-x}{2}$. Interchange $x$and y: $x = \frac{3-y}{2}$.
- **Step 2:** Solve for y: $2x = 3 - y \Longrightarrow y = 3 - 2x$.
- **Answer:** $f^{-1}(x) = 3 - 2x$

Question 5: Plot graph for the following absolute valued functions:

(i) $f(x) = 7|x|$

- **Steps:**
  1. Determine the vertex at $(0,0)$.
  2. Calculate coordinates for points such as $(-1,7)$, $(1,7)$, $(-2,14)$, and $(2,14)$.
  3. Draw two rays starting from $(0,0)$passing through these points to form a steep V-shaped graph.

(ii) $f(x) = |x + 6| - 2$

- **Steps:**
  1. Find the vertex by setting $x + 6 = 0 \Longrightarrow x = -6$, yielding $f(-6) = -2$. The vertex is $(-6, -2)$.
  2. Calculate points around the vertex: $(-8,0)$, $(-7, -1)$, $(-5, -1)$, and $(-4,0)$.
  3. Plot the vertex and points, then sketch the upward-opening V-shape.

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Question 6: Solve and express the solution on number line:

(i) $|3x - 2| = 1$

- **Step 1:** Split into two equations:

$$3x - 2 = 1 \text{ or } 3x - 2 = -1$$

$$3x - 2 = 1 \quad \text{or} \quad 3x - 2 = -1$$

- **Step 2:** Solve:

$$3x = 3 \Longrightarrow x = 1$$

$$3x = 3 \Longrightarrow x = 1$$
$$3x = 1 \Longrightarrow x = \frac{1}{3}$$

$$3x = 1 \Longrightarrow x = \frac{1}{3}$$

- **Solution Set:** $\left\{\frac{1}{3}, 1\right\}$ *(Represented with solid dots at* $\frac{1}{3}$ *and* 1 *on the number line).*

(ii) $|6x + 1| = 9$

- **Step 1:** Split into two equations:

$$6x + 1 = 9 \text{ or } 6x + 1 = -9$$

$$6x + 1 = 9 \quad \text{or} \quad 6x + 1 = -9$$

- **Step 2:** Solve:

$$6x = 8 \Longrightarrow x = \frac{4}{3}$$

$$6x = 8 \Longrightarrow x = \frac{4}{3}$$
$$6x = -10 \Longrightarrow x = -\frac{5}{3}$$

$$6x = -10 \Longrightarrow x = -\frac{5}{3}$$

- **Solution Set:** $\left\{-\frac{5}{3}, \frac{4}{3}\right\}$ *(Represented with solid dots at* $-\frac{5}{3}$ *and* $\frac{4}{3}$ *on the number line).*

Question 7: Solve and express the solution on number line:

(i) $|7 - 2x| \leq 1$

- **Step 1:** Rewrite as a compound inequality:

$$-1 \leq 7 - 2x \leq 1$$

$$-1 \leq 7 - 2x \leq 1$$

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- **Step 2:** Subtract 7from all parts:

$$-8 \leq -2x \leq -6$$

$$-8 \leq -2x \leq -6$$

- **Step 3:** Divide by $-2$and reverse the inequality signs:

$$4 \geq x \geq 3 \Longrightarrow 3 \leq x \leq 4$$

$$4 \geq x \geq 3 \Longrightarrow 3 \leq x \leq 4$$

- **Solution Interval:** $[3,4]$*(Shaded segment between* 3*and* 4*with solid endpoints).*

(ii) $|6x + 18| \leq 24$

- **Step 1:** Rewrite as:

$$-24 \leq 6x + 18 \leq 24$$

$$-24 \leq 6x + 18 \leq 24$$

- **Step 2:** Subtract 18:

$$-42 \leq 6x \leq 6$$

$$-42 \leq 6x \leq 6$$

- **Step 3:** Divide by 6:

$$-7 \leq x \leq 1$$

$$-7 \leq x \leq 1$$

- **Solution Interval:** $[-7,1]$*(Shaded segment between* -7*and* 1*with solid endpoints).*

Question 8: Given that $f(x) = 3x + 7$, find:

(i) $f^{-1}(x)$

- **Step 1:** Let $y = 3x + 7 \Longrightarrow x = 3y + 7$.
- **Step 2:** Solve for y: $3y = x - 7 \Longrightarrow y = \frac{x-7}{3}$.
- **Answer:** $f^{-1}(x) = \frac{x-7}{3}$

(ii) the value of xfor which $f(x) = f^{-1}(x)$

- **Step 1:** Set them equal:

$$3x + 7 = \frac{x - 7}{3}$$

$$3x + 7 = \frac{x - 7}{3}$$

- **Step 2:** Multiply by 3:

$$9x + 21 = x - 7$$

$$9x + 21 = x - 7$$

- **Step 3:** Solve for x:

$$8x = -28 \Longrightarrow x = -\frac{28}{8} = -3.5$$

$$8x = -28 \Longrightarrow x = -\frac{28}{8} = -3.5$$

Question 9

- **Step 1:** Set profit $P(x) = 0$to find the break-even point:

$$100x - 10{,}000 = 0$$

$$100x - 10{,}000 = 0$$

- **Step 2:** Solve for x:

$$100x = 10{,}000 \Longrightarrow x = 100$$

$$100x = 10{,}000 \Longrightarrow x = 100$$

- **Answer: 100 items** must be sold.

Question 10

- **Step 1:** Use the given discount function $D(p) = 0.85p$with original price $p = 2{,}000$.
- **Step 2:** Calculate:

$$D(2000) = 0.85(2000) = 1700$$

$$D(2000) = 0.85(2000) = 1700$$

- **Answer: Rs. 1,700**

Question 11

- **Step 1:** Substitute actual position $x = 100$into the accuracy range inequality:

$$|\ 100 - r\ | \leq 6$$

$$|100 - r| \leq 6$$

- **Step 2:** Rewrite as a compound inequality:

$$-6 \leq 100 - r \leq 6$$

$$-6 \leq 100 - r \leq 6$$

- **Step 3:** Subtract 100:

$$-106 \leq -r \leq -94$$

$$-106 \leq -r \leq -94$$

- **Step 4:** Multiply by $-1$and reverse inequality signs:

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$$94 \leq r \leq 106$$

$$94 \leq r \leq 106$$

- **Answer:** The range of acceptable reported locations is $94 \leq r \leq 106$**metres**.

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