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Class	10th	Chapter	06
Subject	Mathematics	Board	Punjab

CHAPTER 06: Vectors in Plane

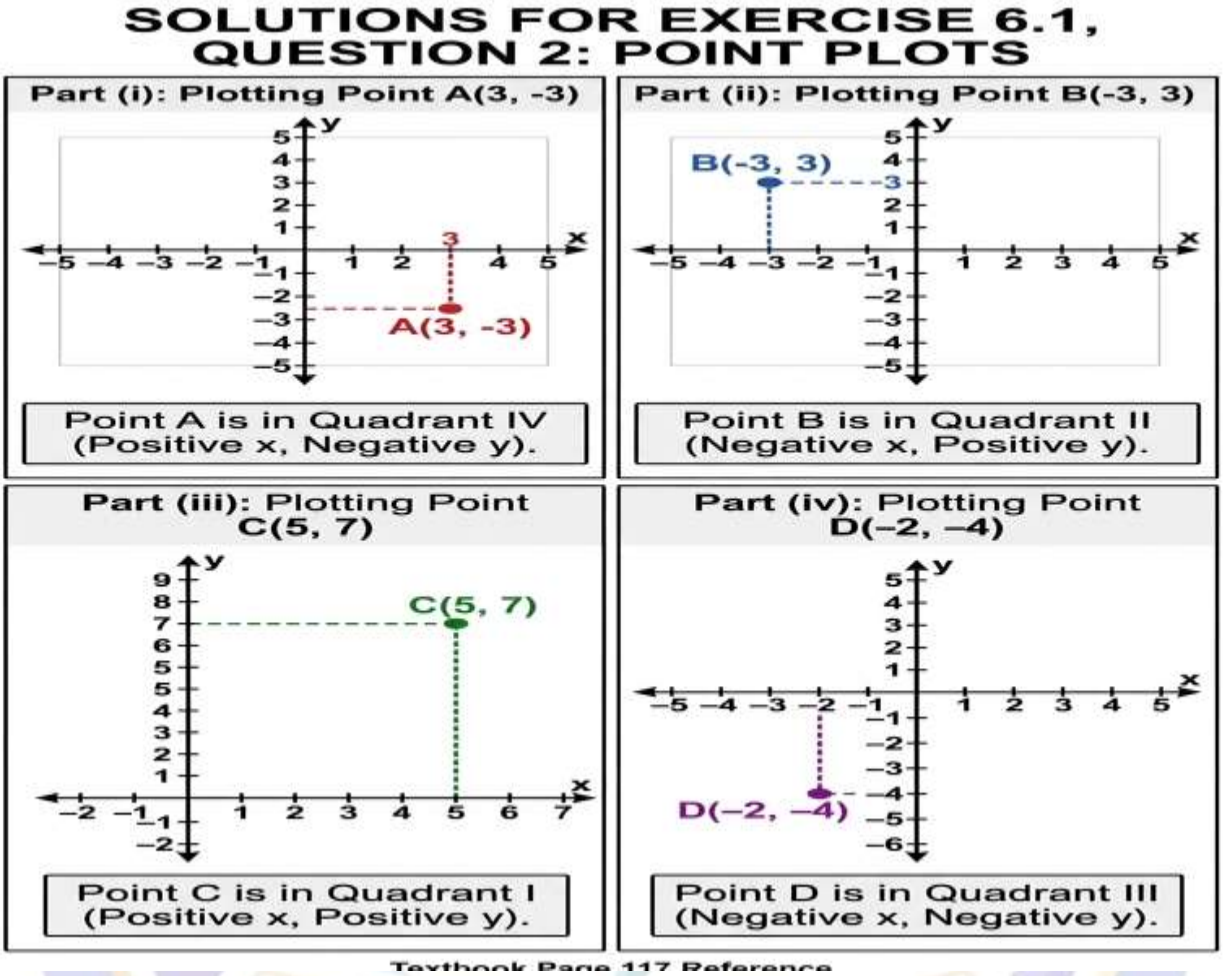
Exercise 6.1

1. Name the quadrant in which each point lies:

- **(i)** $(4,3)$
  - Both $x > 0$ and $y > 0$.
  - **Answer: Quadrant I**
- **(ii)** $(5,-4)$
  - $x > 0$ and $y < 0$.
  - **Answer: Quadrant IV**
- **(iii)** $(-6,2)$
  - $x < 0$ and $y > 0$.
  - **Answer: Quadrant II**
- **(iv)** $(-4,-4)$
  - Both $x < 0$ and $y < 0$.
  - **Answer: Quadrant III**

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2. Plot the following points on the coordinate plane:



Textbook Page 117 Reference

3. Name the tail and tip of the following vectors:

*Note: The **tail** is the initial point (starting point of the arrow), and the **tip** is the terminal point (arrowhead).*

- **(i)** Vector goes from B to A
  - **Tail:** B, **Tip:** A
- **(ii)** Vector goes from P to Q
  - **Tail:** P, **Tip:** Q
- **(iii)** Vector goes from S to R
  - **Tail:** S, **Tip:** R
- **(iv)** Vector goes from M to N
  - **Tail:** M, **Tip:** N
- **(v)** Vector goes from A to B

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- **Tail:** A, **Tip:** B

- **(vi)** Vector goes from D to C
  - **Tail:** D, **Tip:** C

4. Write the vector $\overrightarrow{AB}$ in the form of $x\hat{\imath} + y\hat{\jmath}$:

Formula: $\overrightarrow{AB} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$

- **(i)** $A(1,-7), B(-2,4)$

$$\overrightarrow{AB} = (-2-1)\hat{\imath} + \big(4-(-7)\big)\hat{\jmath} = \mathbf{-3\hat{\imath} + 11\hat{\jmath}}$$

- **(ii)** $A(8,9), B(12,3)$

$$\overrightarrow{AB} = (12-8)\hat{\imath} + (3-9)\hat{\jmath} = \mathbf{4\hat{\imath} - 6\hat{\jmath}}$$

5. Find the magnitude of the vector $\underline{a}$:

Formula: $|\underline{a}| = \sqrt{x^2 + y^2}$

- **(i)** $\underline{a} = -3\hat{\imath} + 2\hat{\jmath}$

$$|\underline{a}| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9+4} = \mathbf{\sqrt{13}}$$

- **(ii)** $\underline{a} = 4\hat{\imath} - 3\hat{\jmath}$

$$|\underline{a}| = \sqrt{(4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = \mathbf{5}$$

- **(iii)** $\underline{a} = \frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath}$

$$|\underline{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{10}{4}} = \mathbf{\frac{\sqrt{10}}{2}}$$

6. Find a unit vector in the direction of the vector given below:

Formula: Unit Vector $\hat{a} = \frac{\underline{a}}{|\underline{a}|}$

- **(i)** $\underline{a} = -4\hat{\imath} + 5\hat{\jmath}$
  - $|\underline{a}| = \sqrt{(-4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41}$
  - $\hat{a} = \mathbf{-\frac{4}{\sqrt{41}}\hat{\imath} + \frac{5}{\sqrt{41}}\hat{\jmath}}$

- **(ii)** $\underline{a} = 6\hat{\imath} + 8\hat{\jmath}$
  - $|\underline{a}| = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$
  - $\hat{a} = \frac{6\hat{\imath}+8\hat{\jmath}}{10} = \mathbf{\frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}}$

- **(iii)** $\underline{a} = \frac{1}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath}$
  - $|\underline{a}| = \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2} = \sqrt{\frac{1}{6} + \frac{1}{6}} = \sqrt{\frac{2}{6}} = \frac{1}{\sqrt{3}}$
  - $\hat{a} = \frac{\frac{1}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath}}{\frac{1}{\sqrt{3}}} = \sqrt{3}\left(\frac{1}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath}\right) = \mathbf{\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}}$

- **(iv)** $\underline{a} = \frac{1}{2}\hat{\imath} - \frac{3}{4}\hat{\jmath}$

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- $|\underline{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{3}{4}\right)^2} = \sqrt{\frac{1}{4} + \frac{9}{16}} = \sqrt{\frac{4+9}{16}} = \frac{\sqrt{13}}{4}$
- $\hat{a} = \frac{\frac{1}{2}\hat{\imath} - \frac{3}{4}\hat{\jmath}}{\frac{\sqrt{13}}{4}} = \frac{4}{\sqrt{13}}\left(\frac{1}{2}\hat{\imath} - \frac{3}{4}\hat{\jmath}\right) = \frac{\mathbf{2}}{\sqrt{\mathbf{13}}}\hat{\mathbf{i}} - \frac{\mathbf{3}}{\sqrt{\mathbf{13}}}\hat{\mathbf{j}}$

7. If $\underline{a} = 5\hat{\imath} - 7\hat{\jmath}$, $\underline{b} = -\hat{\imath} - \hat{\jmath}$ and $\underline{c} = 2\hat{\imath} + 3\hat{\jmath}$, then find unit vector parallel to $\underline{a} + \underline{b} - 3\underline{c}$.

1. Let $\underline{v} = \underline{a} + \underline{b} - 3\underline{c}$:

$$\underline{v} = (5\hat{\imath} - 7\hat{\jmath}) + (-\hat{\imath} - \hat{\jmath}) - 3(2\hat{\imath} + 3\hat{\jmath})$$
$$\underline{v} = (5 - 1 - 6)\hat{\imath} + (-7 - 1 - 9)\hat{\jmath} = -2\hat{\imath} - 17\hat{\jmath}$$

2. Find magnitude $|\underline{v}|$:

$$|\underline{v}| = \sqrt{(-2)^2 + (-17)^2} = \sqrt{4 + 289} = \sqrt{293}$$

3. Unit vector parallel to $\underline{v}$:

$$\hat{v} = \frac{\mathbf{-2\hat{i} - 17\hat{j}}}{\sqrt{\mathbf{293}}} \quad \text{or} \quad -\frac{\mathbf{2}}{\sqrt{\mathbf{293}}}\hat{\mathbf{i}} - \frac{\mathbf{17}}{\sqrt{\mathbf{293}}}\hat{\mathbf{j}}$$

8. If $\underline{a} = 3\hat{\imath} - \hat{\jmath}$, $\underline{b} = -2\hat{\imath} + 4\hat{\jmath}$ and $\underline{c} = \hat{\imath} + 2\hat{\jmath}$, then find unit vector parallel to $3\underline{a} - 2\underline{c} + 4\underline{b}$.

1. Let $\underline{v} = 3\underline{a} - 2\underline{c} + 4\underline{b}$:

$$\underline{v} = 3(3\hat{\imath} - \hat{\jmath}) - 2(\hat{\imath} + 2\hat{\jmath}) + 4(-2\hat{\imath} + 4\hat{\jmath})$$
$$\underline{v} = (9\hat{\imath} - 3\hat{\jmath}) - (2\hat{\imath} + 4\hat{\jmath}) + (-8\hat{\imath} + 16\hat{\jmath})$$
$$\underline{v} = (9 - 2 - 8)\hat{\imath} + (-3 - 4 + 16)\hat{\jmath} = -\hat{\imath} + 9\hat{\jmath}$$

2. Find magnitude $|\underline{v}|$:

$$|\underline{v}| = \sqrt{(-1)^2 + (9)^2} = \sqrt{1 + 81} = \sqrt{82}$$

3. Unit vector parallel to $\underline{v}$:

$$\hat{v} = \frac{\mathbf{-\hat{i} + 9\hat{j}}}{\sqrt{\mathbf{82}}} \quad \text{or} \quad -\frac{\mathbf{1}}{\sqrt{\mathbf{82}}}\hat{\mathbf{i}} + \frac{\mathbf{9}}{\sqrt{\mathbf{82}}}\hat{\mathbf{j}}$$

9. Which of the following vectors are parallel?

Two vectors $\underline{a}$ and $\underline{b}$ are parallel if $\underline{b} = k\underline{a}$ for some scalar $k \neq 0$.

- **(i)** $\underline{a} = 6\hat{\imath} + \hat{\jmath}, \underline{b} = 12\hat{\imath} + 2\hat{\jmath}$

$$\underline{b} = 2(6\hat{\imath} + \hat{\jmath}) = 2\underline{a} \Longrightarrow \text{Parallel}$$

- **(ii)** $\underline{a} = -2\hat{\imath} + 3\hat{\jmath}, \underline{b} = 6\hat{\imath} - 9\hat{\jmath}$

$$\underline{b} = -3(-2\hat{\imath} + 3\hat{\jmath}) = -3\underline{a} \Longrightarrow \text{Parallel}$$

- **(iii)** $\underline{a} = 5\hat{\imath} - 4\hat{\jmath}, \underline{b} = 6\hat{\imath} - 3\hat{\jmath}$

No constant k exists such that $\underline{b} = k\underline{a} \Longrightarrow$ Not Parallel

- **(iv)** $\underline{a} = 3\hat{\imath} - 7\hat{\jmath}, \underline{b} = 6\hat{\imath} - 14\hat{\jmath}$

$$\underline{b} = 2(3\hat{\imath} - 7\hat{\jmath}) = 2\underline{a} \Longrightarrow \text{Parallel}$$

10. Find a vector thrice in length of $3\hat{\imath} - 2\hat{\jmath}$, but opposite in direction.

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- Let $\underline{v} = 3\hat{\imath} - 2\hat{\jmath}$.
- For a vector **thrice in length** ($3\times$) and **opposite in direction** (multiply by -1):

Required Vector $= -3\underline{v} = -3(3\hat{\imath} - 2\hat{\jmath}) = \mathbf{-9\hat{\imath} + 6\hat{\jmath}}$

11. Find two vectors that are double in magnitude of $3\hat{\imath} - 5\hat{\jmath}$, one in the same direction of it and other in its opposite direction.

Let $\underline{v} = 3\hat{\imath} - 5\hat{\jmath}$.

1. **Double in magnitude and same direction:**

$\text{Vector}_1 = 2\underline{v} = 2(3\hat{\imath} - 5\hat{\jmath}) = \mathbf{6\hat{\imath} - 10\hat{\jmath}}$

2. **Double in magnitude and opposite direction:**

$\text{Vector}_2 = -2\underline{v} = -2(3\hat{\imath} - 5\hat{\jmath}) = \mathbf{-6\hat{\imath} + 10\hat{\jmath}}$

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EXERCISE 6.2:

Question 1

Given:

$\underline{a} = 7\underline{i} - 3\underline{j}$ and $\underline{b} = \underline{i} + 5\underline{j}$

(i) $\underline{a} + \underline{b}$

$$\underline{a} + \underline{b} = \left(7\underline{i} - 3\underline{j}\right) + \left(\underline{i} + 5\underline{j}\right)$$
$$\underline{a} + \underline{b} = (7 + 1)\underline{i} + (-3 + 5)\underline{j}$$
$$\mathbf{\underline{a} + \underline{b} = 8\underline{i} + 2\underline{j}}$$

(ii) $\underline{a} + 3\underline{b}$

$$\underline{a} + 3\underline{b} = \left(7\underline{i} - 3\underline{j}\right) + 3\left(\underline{i} + 5\underline{j}\right)$$
$$\underline{a} + 3\underline{b} = 7\underline{i} - 3\underline{j} + 3\underline{i} + 15\underline{j}$$
$$\underline{a} + 3\underline{b} = (7 + 3)\underline{i} + (-3 + 15)\underline{j}$$
$$\mathbf{\underline{a} + 3\underline{b} = 10\underline{i} + 12\underline{j}}$$

(iii) $3\underline{a} + \frac{1}{2}\underline{b}$

$$3\underline{a} + \frac{1}{2}\underline{b} = 3\left(7\underline{i} - 3\underline{j}\right) + \frac{1}{2}\left(\underline{i} + 5\underline{j}\right)$$
$$3\underline{a} + \frac{1}{2}\underline{b} = 21\underline{i} - 9\underline{j} + \frac{1}{2}\underline{i} + \frac{5}{2}\underline{j}$$
$$3\underline{a} + \frac{1}{2}\underline{b} = \left(21 + \frac{1}{2}\right)\underline{i} + \left(-9 + \frac{5}{2}\right)\underline{j}$$
$$3\underline{a} + \frac{1}{2}\underline{b} = \left(\frac{42 + 1}{2}\right)\underline{i} + \left(\frac{-18 + 5}{2}\right)\underline{j}$$
$$\mathbf{3\underline{a} + \frac{1}{2}\underline{b} = \frac{43}{2}\underline{i} - \frac{13}{2}\underline{j}}$$

(iv) $\underline{b} - \underline{a}$

$$\underline{b} - \underline{a} = \left(\underline{i} + 5\underline{j}\right) - \left(7\underline{i} - 3\underline{j}\right)$$
$$\underline{b} - \underline{a} = \underline{i} + 5\underline{j} - 7\underline{i} + 3\underline{j}$$
$$\underline{b} - \underline{a} = (1 - 7)\underline{i} + (5 + 3)\underline{j}$$
$$\mathbf{\underline{b} - \underline{a} = -6\underline{i} + 8\underline{j}}$$

(v) $4\underline{b} - 5\underline{a}$

$$4\underline{b} - 5\underline{a} = 4\left(\underline{i} + 5\underline{j}\right) - 5\left(7\underline{i} - 3\underline{j}\right)$$
$$4\underline{b} - 5\underline{a} = 4\underline{i} + 20\underline{j} - 35\underline{i} + 15\underline{j}$$
$$4\underline{b} - 5\underline{a} = (4 - 35)\underline{i} + (20 + 15)\underline{j}$$
$$\mathbf{4\underline{b} - 5\underline{a} = -31\underline{i} + 35\underline{j}}$$

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(vi) $\frac{3}{2}\underline{a} - \underline{b}$

$$\frac{3}{2}\underline{a} - \underline{b} = \frac{3}{2}\left(7\underline{i} - 3\underline{j}\right) - \left(\underline{i} + 5\underline{j}\right)$$
$$\frac{3}{2}\underline{a} - \underline{b} = \frac{21}{2}\underline{i} - \frac{9}{2}\underline{j} - \underline{i} - 5\underline{j}$$
$$\frac{3}{2}\underline{a} - \underline{b} = \left(\frac{21}{2} - 1\right)\underline{i} + \left(-\frac{9}{2} - 5\right)\underline{j}$$
$$\frac{3}{2}\underline{a} - \underline{b} = \left(\frac{21 - 2}{2}\right)\underline{i} + \left(\frac{-9 - 10}{2}\right)\underline{j}$$
$$\mathbf{\frac{3}{2}\underline{a} - \underline{b} = \frac{19}{2}\underline{i} - \frac{19}{2}\underline{j}}$$

Question 2

Given:

$\underline{a} = 6\underline{i} - \underline{j}$, $\underline{b} = \underline{i} + 5\underline{j}$, and $\underline{c} = 3\underline{i} + 5\underline{j}$

Recall that for a vector $\underline{v} = x\underline{i} + y\underline{j}$, its magnitude is $|\underline{v}| = \sqrt{x^2 + y^2}$.

(i) $|\underline{b} - \underline{c}|$?

First, find the vector $(\underline{b} - \underline{c})$:

$$\underline{b} - \underline{c} = \left(\underline{i} + 5\underline{j}\right) - \left(3\underline{i} + 5\underline{j}\right) = (1 - 3)\underline{i} + (5 - 5)\underline{j} = -2\underline{i} + 0\underline{j}$$

Now, find its magnitude:

$$|\underline{b} - \underline{c}| = \sqrt{(-2)^2 + 0^2} = \sqrt{4 + 0} = \sqrt{4} = \mathbf{2}$$

(ii) $|\underline{a} - 2\underline{b} + \underline{c}|$?

First, calculate vector $\underline{a} - 2\underline{b} + \underline{c}$:

$$\underline{a} - 2\underline{b} + \underline{c} = \left(6\underline{i} - \underline{j}\right) - 2\left(\underline{i} + 5\underline{j}\right) + \left(3\underline{i} + 5\underline{j}\right)$$
$$= 6\underline{i} - \underline{j} - 2\underline{i} - 10\underline{j} + 3\underline{i} + 5\underline{j}$$
$$= (6 - 2 + 3)\underline{i} + (-1 - 10 + 5)\underline{j}$$
$$= 7\underline{i} - 6\underline{j}$$

Now, find its magnitude:

$$|\underline{a} - 2\underline{b} + \underline{c}| = \sqrt{7^2 + (-6)^2} = \sqrt{49 + 36} = \sqrt{\mathbf{85}}$$

(iii) $|\underline{c} - \underline{b} - \underline{a}|$?

First, calculate vector $\underline{c} - \underline{b} - \underline{a}$:

$$\underline{c} - \underline{b} - \underline{a} = \left(3\underline{i} + 5\underline{j}\right) - \left(\underline{i} + 5\underline{j}\right) - \left(6\underline{i} - \underline{j}\right)$$
$$= 3\underline{i} + 5\underline{j} - \underline{i} - 5\underline{j} - 6\underline{i} + \underline{j}$$
$$= (3 - 1 - 6)\underline{i} + (5 - 5 + 1)\underline{j}$$
$$= -4\underline{i} + 1\underline{j}$$

Now, find its magnitude:

$$|\underline{c} - \underline{b} - \underline{a}| = \sqrt{(-4)^2 + 1^2} = \sqrt{16 + 1} = \sqrt{\mathbf{17}}$$

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Question 3

Find the values of x and y in each equation by equating corresponding $\underline{i}$ and $\underline{j}$ components:

(i) $\left(x\underline{i} + y\underline{j}\right) + \left(2\underline{i} + 3\underline{j}\right) = 7\underline{i} + 6\underline{j}$

$$(x+2)\underline{i} + (y+3)\underline{j} = 7\underline{i} + 6\underline{j}$$

Equating corresponding components:

- $\underline{i}$-component: $x + 2 = 7 \Longrightarrow \mathbf{x = 5}$
- $\underline{j}$-component: $y + 3 = 6 \Longrightarrow \mathbf{y = 3}$

(ii) $\left(x\underline{i} - 5\underline{j}\right) + \left(3\underline{i} + 5\underline{j}\right) = -8\underline{i} + y\underline{j}$

$$(x+3)\underline{i} + (-5+5)\underline{j} = -8\underline{i} + y\underline{j}$$

$$(x+3)\underline{i} + 0\underline{j} = -8\underline{i} + y\underline{j}$$

Equating corresponding components:

- $\underline{i}$-component: $x + 3 = -8 \Longrightarrow x = -8 - 3 \Longrightarrow \mathbf{x = -11}$
- $\underline{j}$-component: $\mathbf{y = 0}$

(iii) $\left(y\underline{i} + 3\underline{j}\right) + \left(-5\underline{i} + 2x\underline{j}\right) = 9\underline{i} + 7\underline{j}$

$$(y-5)\underline{i} + (3+2x)\underline{j} = 9\underline{i} + 7\underline{j}$$

Equating corresponding components:

- $\underline{i}$-component: $y - 5 = 9 \Longrightarrow y = 9 + 5 \Longrightarrow \mathbf{y = 14}$
- $\underline{j}$-component: $3 + 2x = 7 \Longrightarrow 2x = 4 \Longrightarrow \mathbf{x = 2}$

Question 4

Given:

$\underline{a} = \underline{i} + 3\underline{j}$, $\underline{c} = 2\underline{i} + \underline{j}$, and $\underline{a} + 2\underline{b} = \underline{c}$

Find: $|\underline{b}|$

Step 1: Express $\underline{b}$ in terms of $\underline{a}$ and $\underline{c}$.

$$2\underline{b} = \underline{c} - \underline{a}$$

$$2\underline{b} = \left(2\underline{i} + \underline{j}\right) - \left(\underline{i} + 3\underline{j}\right)$$

$$2\underline{b} = (2-1)\underline{i} + (1-3)\underline{j}$$

$$2\underline{b} = \underline{i} - 2\underline{j}$$

$$\underline{b} = \frac{1}{2}\underline{i} - \underline{j}$$

Step 2: Find magnitude $|\underline{b}|$.

$$|\underline{b}| = \sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2} = \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

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Question 5

Given:

$\underline{a} = -2\underline{i} + 7\underline{j}$ and $\underline{b} = 3\underline{i} - 5\underline{j}$

Find: Vector components of $\underline{a} + 5\underline{b}$

Step 1: Calculate vector $\underline{a} + 5\underline{b}$.

$$\underline{a} + 5\underline{b} = \left(-2\underline{i} + 7\underline{j}\right) + 5\left(3\underline{i} - 5\underline{j}\right)$$
$$= -2\underline{i} + 7\underline{j} + 15\underline{i} - 25\underline{j}$$
$$= (-2 + 15)\underline{i} + (7 - 25)\underline{j}$$
$$= 13\underline{i} - 18\underline{j}$$

Step 2: State the vector components.

- The $\underline{i}$-component is $13\underline{i}$ (or scalar component 13)
- The $\underline{j}$-component is $-18\underline{j}$ (or scalar component -18)

Thus, the vector components are $13\underline{i}$ **and** $-18\underline{j}$ (or expressed as a pair: $(13, -18)$).

Question 6

Given:

$$5\underline{i} - 3\underline{j} = m\left(\underline{i} - 10\underline{j}\right) + n\left(4\underline{i} - 3\underline{j}\right)$$

Find: m and n

Step 1: Expand the right-hand side.

$$5\underline{i} - 3\underline{j} = m\underline{i} - 10m\underline{j} + 4n\underline{i} - 3n\underline{j}$$
$$5\underline{i} - 3\underline{j} = m\underline{i} - 10m\underline{j} + 4n\underline{i} - 3n\underline{j}$$

Step 2: Equate coefficients of $\underline{i}$ and $\underline{j}$ to set up simultaneous equations.

1. $m + 4n = 5$ --- (Equation 1)
2. $-10m - 3n = -3$ --- (Equation 2)

Step 3: Solve the linear system.

From Equation 1, express m in terms of n:

$$m = 5 - 4n$$

Substitute m into Equation 2:

$$-10(5 - 4n) - 3n = -3$$
$$-10(5 - 4n) - 3n = -3$$
$$-50 + 40n - 3n = -3$$
$$37n = -3 + 50$$
$$37n = 47$$

Substitute n back into $m = 5 - 4n$:

$$m = 5 - 4\left(\frac{47}{37}\right)$$
$$m = 5 - 4\left(\frac{47}{37}\right)$$

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$$m = 5 - \frac{188}{37}$$

$$m = \frac{185 - 188}{37}$$

Final Values:

$$\mathbf{m = -\frac{3}{37}, \quad n = \frac{47}{37}}$$

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EXERCISE 6.3

1. Plot $A(-5,4)$, $B(-6,1)$, and $C(-3,1)$to form a ΔABC. Also translate $\Delta ABC$to $\Delta A'B'C'$by translation vector $8\underline{i}-6\underline{j}$.

Step 1: Write down the original coordinates as position vectors

- $\vec{r}_A = -5\underline{i}+4\underline{j}$
- $\vec{r}_B = -6\underline{i}+1\underline{j}$
- $\vec{r}_C = -3\underline{i}+1\underline{j}$

Translation vector, $\vec{v} = 8\underline{i}-6\underline{j}$.

Step 2: Find the translated coordinates

To translate a point by a vector $\vec{v}$, add the translation vector to each position vector:

- **For A':**

$$\vec{r}_{A'} = \vec{r}_A + \vec{v} = (-5\,\underline{i}+4\,\underline{j}) + (8\,\underline{i}-6\,\underline{j}) = (-5+8)\,\underline{i}+(4-6)\,\underline{j} = 3\,\underline{i}-2\,\underline{j}$$

$$\vec{r}_{A'} = \vec{r}_A + \vec{v} = (-5\underline{i}+4\underline{j}) + (8\underline{i}-6\underline{j}) = (-5+8)\underline{i}+(4-6)\underline{j} = 3\underline{i}-2\underline{j}$$

$$A'(3,-2)$$

$$\mathbf{A'(3,-2)}$$

- **For B':**

$$\vec{r}_{B'} = \vec{r}_B + \vec{v} = (-6\,\underline{i}+1\,\underline{j}) + (8\,\underline{i}-6\,\underline{j}) = (-6+8)\,\underline{i}+(1-6)\,\underline{j} = 2\,\underline{i}-5\,\underline{j}$$

$$\vec{r}_{B'} = \vec{r}_B + \vec{v} = (-6\underline{i}+1\underline{j}) + (8\underline{i}-6\underline{j}) = (-6+8)\underline{i}+(1-6)\underline{j} = 2\underline{i}-5\underline{j}$$

$$B'(2,-5)$$

$$\mathbf{B'(2,-5)}$$

- **For C':**

$$\vec{r}_{C'} = \vec{r}_C + \vec{v} = (-3\,\underline{i}+1\,\underline{j}) + (8\,\underline{i}-6\,\underline{j}) = (-3+8)\,\underline{i}+(1-6)\,\underline{j} = 5\,\underline{i}-5\,\underline{j}$$

$$\vec{r}_{C'} = \vec{r}_C + \vec{v} = (-3\underline{i}+1\underline{j}) + (8\underline{i}-6\underline{j}) = (-3+8)\underline{i}+(1-6)\underline{j} = 5\underline{i}-5\underline{j}$$

$$C'(5,-5)$$

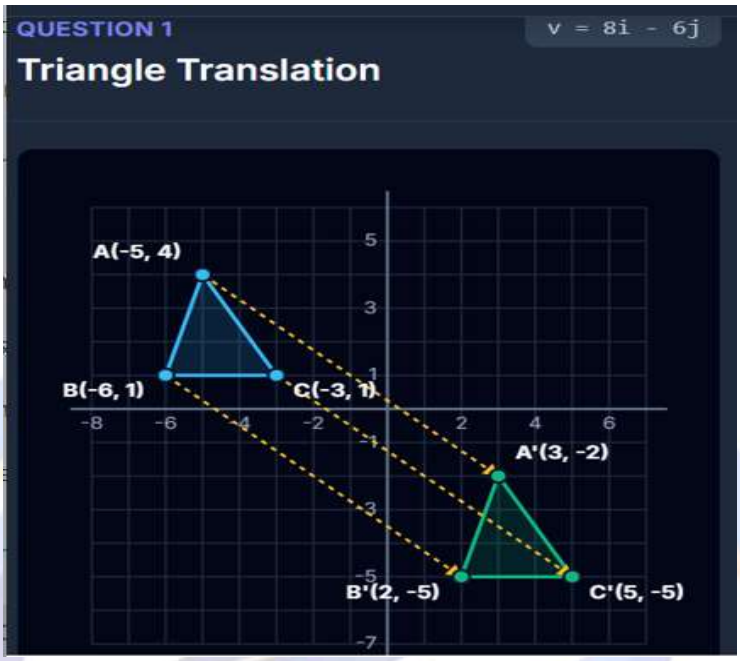
$$\mathbf{C'(5,-5)}$$

Answer:

- Original vertices: $A(-5,4)$, $B(-6,1)$, $C(-3,1)$
- Translated vertices: $\mathbf{A'(3,-2)}$, $\mathbf{B'(2,-5)}$, $\mathbf{C'(5,-5)}$

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2. Plot $A(-6,-2)$, $B(-6,-5)$, $C(-3,-5)$, and $D(-3,-2)$to form a square $ABCD$. Also translate square $ABCD$to square $A'B'C'D'$by translation vector $9\underline{i}+7\underline{j}$.

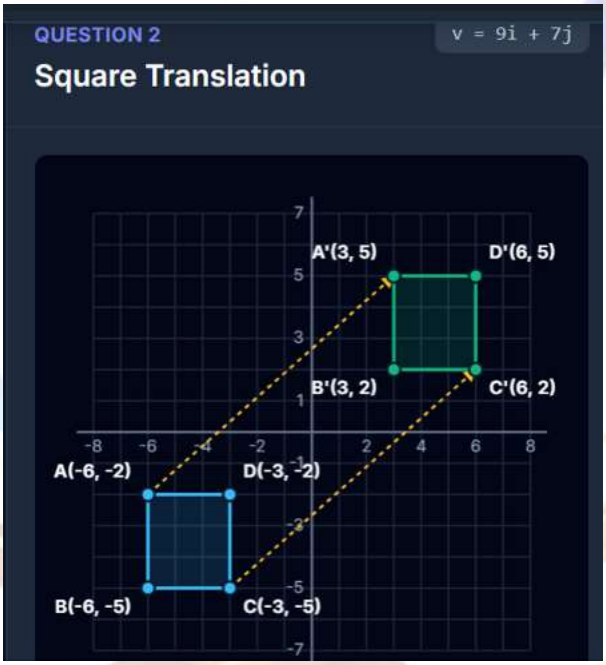
Step 1: Write down position vectors

Translation vector $\vec{v}=9\underline{i}+7\underline{j}$.

Step 2: Translate each vertex

- A': $\left(-6\underline{i}-2\underline{j}\right)+\left(9\underline{i}+7\underline{j}\right)=3\underline{i}+5\underline{j}\Longrightarrow \mathbf{A'(3,5)}$
- B': $\left(-6\underline{i}-5\underline{j}\right)+\left(9\underline{i}+7\underline{j}\right)=3\underline{i}+2\underline{j}\Longrightarrow \mathbf{B'(3,2)}$
- C': $\left(-3\underline{i}-5\underline{j}\right)+\left(9\underline{i}+7\underline{j}\right)=6\underline{i}+2\underline{j}\Longrightarrow \mathbf{C'(6,2)}$
- D': $\left(-3\underline{i}-2\underline{j}\right)+\left(9\underline{i}+7\underline{j}\right)=6\underline{i}+5\underline{j}\Longrightarrow \mathbf{D'(6,5)}$

Answer:

- Translated vertices: $\mathbf{A'(3,5)}$, $\mathbf{B'(3,2)}$, $\mathbf{C'(6,2)}$, $\mathbf{D'(6,5)}$
- 

3. Plot $A(-6,3)$, $B(-4,0)$, $C(-2,3)$, and $D(-4,4)$to form a kite $ABCD$. Also translate kite $ABCD$to kite $A'B'C'D'$by translation vector $6\underline{i}-6\underline{j}$.

Step 1: Write down position vectors

Translation vector $\vec{v}=6\underline{i}-6\underline{j}$.

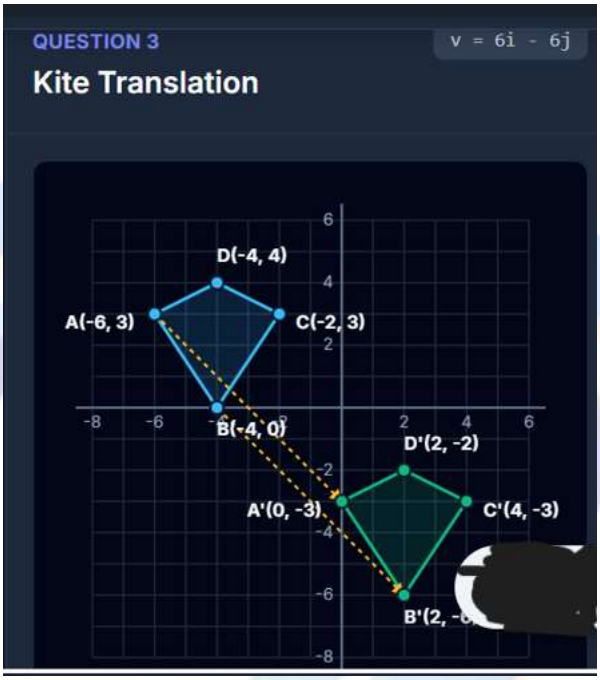
Step 2: Translate each vertex

- A': $\left(-6\underline{i}+3\underline{j}\right)+\left(6\underline{i}-6\underline{j}\right)=0\underline{i}-3\underline{j}\Longrightarrow \mathbf{A'(0,-3)}$
- B': $\left(-4\underline{i}+0\underline{j}\right)+\left(6\underline{i}-6\underline{j}\right)=2\underline{i}-6\underline{j}\Longrightarrow \mathbf{B'(2,-6)}$
- C': $\left(-2\underline{i}+3\underline{j}\right)+\left(6\underline{i}-6\underline{j}\right)=4\underline{i}-3\underline{j}\Longrightarrow \mathbf{C'(4,-3)}$

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- D': $\left(-4\underline{i}+4\underline{j}\right)+\left(6\underline{i}-6\underline{j}\right)=2\underline{i}-2\underline{j} \Longrightarrow \mathbf{D'(2,-2)}$

Answer:

- Translated vertices: $\mathbf{A'(0,-3)}$, $\mathbf{B'(2,-6)}$, $\mathbf{C'(4,-3)}$, $\mathbf{D'(2,-2)}$
- 

4. The coordinates of A, B, and $D$are $(1,2)$, $(6,3)$, and $(2,8)$respectively. Find the coordinates of $C$by using vector method if $ABCD$is a parallelogram.

Step 1: Use the property of a parallelogram

In a parallelogram $ABCD$, opposite vector sides are equal:

$$\overrightarrow{AB}=\overrightarrow{DC}$$

$$\overrightarrow{AB}=\overrightarrow{DC}$$

Step 2: Express vectors in terms of position vectors

Let $C=(x,y)$.

- $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\left(6\underline{i}+3\underline{j}\right)-\left(1\underline{i}+2\underline{j}\right)=5\underline{i}+1\underline{j}$
- $\overrightarrow{DC}=\overrightarrow{OC}-\overrightarrow{OD}=\left(x\underline{i}+y\underline{j}\right)-\left(2\underline{i}+8\underline{j}\right)=(x-2)\underline{i}+(y-8)\underline{j}$

Step 3: Equate components

$$(x-2)\,\underline{i}+(y-8)\,\underline{j}=5\,\underline{i}+1\,\underline{j}$$

$$(x-2)\underline{i}+(y-8)\underline{j}=5\underline{i}+1\underline{j}$$

Equating $\underline{i}$and $\underline{j}$components:

- $x-2=5 \Longrightarrow x=7$
- $y-8=1 \Longrightarrow y=9$

Answer:

- Coordinates of $C = (7, 9)$
- 

5. In parallelogram $ABCD$, the vectors representing two opposite sides are $\overrightarrow{AB} = 6\underline{i} + 2\underline{j}$, $\overrightarrow{DC} = -6\underline{i} - 2\underline{j}$. Show that the opposite sides are equal in magnitude and parallel.

Step 1: Find magnitudes

- $|\overrightarrow{AB}| = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$
- $|\overrightarrow{DC}| = \sqrt{(-6)^2 + (-2)^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

Since $|\overrightarrow{AB}| = |\overrightarrow{DC}|$, **the opposite sides are equal in magnitude**.

Step 2: Show that they are parallel

Notice that:

$$\overrightarrow{DC} = -1 \cdot (6\,\underline{i} + 2\,\underline{j}) = -1 \cdot \overrightarrow{AB}$$

$$\overrightarrow{DC} = -1 \cdot \left(6\underline{i} + 2\underline{j}\right) = -1 \cdot \overrightarrow{AB}$$

Since $\overrightarrow{DC} = k \cdot \overrightarrow{AB}$(where scalar $k = -1$), **the vectors are parallel** (pointing in opposite directions along parallel line segments). ■

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6. Points $A(1,2)$, $B(4,6)$, and $C(7,2)$form a triangle. Check whether triangle $ABC$is an isosceles by using vector magnitude.

Step 1: Find vectors representing the sides

- $\overrightarrow{AB} = (4-1)\underline{i} + (6-2)\underline{j} = 3\underline{i} + 4\underline{j}$
- $\overrightarrow{BC} = (7-4)\underline{i} + (2-6)\underline{j} = 3\underline{i} - 4\underline{j}$
- $\overrightarrow{AC} = (7-1)\underline{i} + (2-2)\underline{j} = 6\underline{i} + 0\underline{j}$

Step 2: Calculate magnitudes

- $|\overrightarrow{AB}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$
- $|\overrightarrow{BC}| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$
- $|\overrightarrow{AC}| = \sqrt{6^2 + 0^2} = 6$

Conclusion

Since $|\overrightarrow{AB}| = |\overrightarrow{BC}| = 5$, two sides are equal in length.

Answer:

- **Yes, triangle $ABC$is an isosceles triangle.**



7. Use vectors to show that $PQRS$is a parallelogram, where the points P, Q, R,and Shave coordinates $(1,2)$, $(5,2)$, $(7,6)$, and $(3,6)$respectively.

Step 1: Find vector $\overrightarrow{PQ}$and vector $\overrightarrow{SR}$

- $\overrightarrow{PQ} = (5-1)\underline{i} + (2-2)\underline{j} = 4\underline{i} + 0\underline{j}$
- $\overrightarrow{SR} = (7-3)\underline{i} + (6-6)\underline{j} = 4\underline{i} + 0\underline{j}$

Since $\overrightarrow{PQ} = \overrightarrow{SR}$, side $PQ$is equal in magnitude and parallel to side SR.

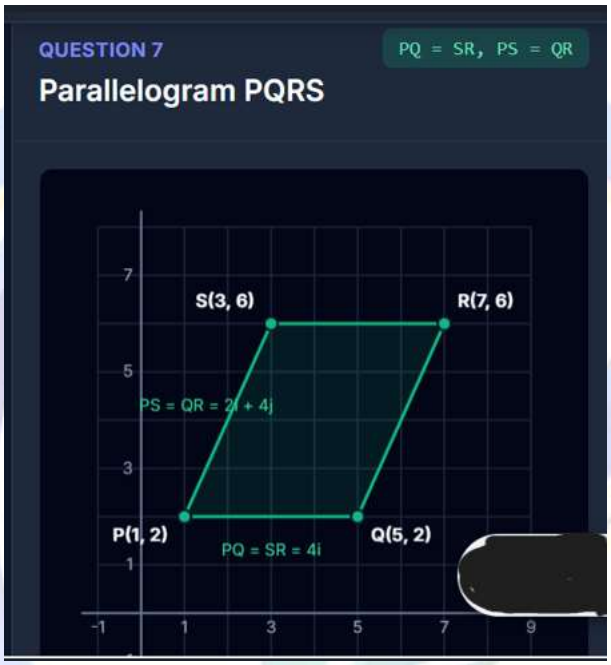
Step 2: Find vector $\overrightarrow{PS}$and vector $\overrightarrow{QR}$

- $\overrightarrow{PS} = (3-1)\underline{i} + (6-2)\underline{j} = 2\underline{i} + 4\underline{j}$
- $\overrightarrow{QR} = (7-5)\underline{i} + (6-2)\underline{j} = 2\underline{i} + 4\underline{j}$

Since $\overrightarrow{PS} = \overrightarrow{QR}$, side $PS$is equal in magnitude and parallel to side QR.

Conclusion

Since opposite vector sides are equal ($\overrightarrow{PQ} = \overrightarrow{SR}$ and $\overrightarrow{PS} = \overrightarrow{QR}$), $PQRS$**is a parallelogram.**



8. Use vectors to show that triangle $XYZ$is an isosceles, where the points X, Y, Zhave the coordinates $(0, 0)$, $(2, 0)$, and $(1, 3)$respectively.

Step 1: Find side vectors

- $\overrightarrow{XY} = (2-0)\underline{i} + (0-0)\underline{j} = 2\underline{i}$
- $\overrightarrow{YZ} = (1-2)\underline{i} + (3-0)\underline{j} = -1\underline{i} + 3\underline{j}$
- $\overrightarrow{XZ} = (1-0)\underline{i} + (3-0)\underline{j} = 1\underline{i} + 3\underline{j}$

Step 2: Calculate side lengths (magnitudes)

- $|\overrightarrow{XY}| = \sqrt{2^2 + 0^2} = 2$
- $|\overrightarrow{YZ}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$
- $|\overrightarrow{XZ}| = \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$

Conclusion

Since $|\overrightarrow{YZ}| = |\overrightarrow{XZ}| = \sqrt{10}$, two sides are equal in length.

Answer:

- **Triangle $XYZ$is an isosceles triangle.** ■

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9. A ball is projected with an initial velocity vector $\underline{v}_0 = 10\underline{i} + 20\underline{j}$. The horizontal component is in the x-direction and gravity is $\underline{g} = 0\underline{i} - 10\underline{j}$. Find the maximum height and horizontal range.

Given Data:

- Initial horizontal velocity component: $v_{0x} = 10$ m/s
- Initial vertical velocity component: $v_{0y} = 20$ m/s
- Acceleration due to gravity magnitude: $g = 10$ m/s^2

Part 1: Maximum Height (H)

Using kinematic formula for vertical motion ($v_y^2 = v_{0y}^2 - 2gH$ with final vertical velocity $v_y = 0$):

$$H = \frac{v_{0y}^2}{2g}$$

$$H = \frac{v_{0y}^2}{2g}$$

$$H = \frac{(20)^2}{2(10)} = \frac{400}{20} = 20 \text{ units}$$

$$H = \frac{(20)^2}{2(10)} = \frac{400}{20} = \mathbf{20} \text{ units}$$

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Part 2: Horizontal Range (R)

1. **Time of flight (T):**

$$T = \frac{2v_{0y}}{g} = \frac{2(20)}{10} = 4 \text{ seconds}$$

$$T = \frac{2v_{0y}}{g} = \frac{2(20)}{10} = 4 \text{ seconds}$$

2. **Horizontal Range (R):**

$$R = v_{0x} \times T = 10 \times 4 = 40 \text{ units}$$

$$R = v_{0x} \times T = 10 \times 4 = \mathbf{40} \text{ units}$$

Answer:

- Maximum Height = **20 units**
- Horizontal Range = **40 units**

10. A car enters a loop with velocity vector $\underline{v} = -30\underline{j}$ and exits with velocity $\underline{v}' = 30\underline{i}$. What is the change in velocity vector?

Given:

- Initial velocity: $\vec{v}_i = -30\underline{j}$
- Final velocity: $\vec{v}_f = 30\underline{i}$

Step 1: Calculate change in velocity vector ($\Delta\vec{v}$)

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

$$\Delta\vec{v} = 30\,\underline{i} - (-30\,\underline{j}) = 30\,\underline{i} + 30\,\underline{j}$$

$$\Delta\vec{v} = 30\underline{i} - (-30\underline{j}) = \mathbf{30\underline{i} + 30\underline{j}}$$

Answer:

- Change in velocity vector = $\mathbf{30\underline{i} + 30\underline{j}}$

11. An aeroplane has airspeed $\underline{v}_a = 20\underline{i}$ and there is a crosswind $\underline{v}_w = 50\underline{j}$. Find the resultant velocity and its magnitude.

Given:

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- Aeroplane velocity: $\vec{v}_a = 20\underline{i}$
- Wind velocity: $\vec{v}_w = 50\underline{j}$

Part 1: Resultant Velocity Vector ($\vec{v}_r$)

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$$\vec{v}_r = \vec{v}_a + \vec{v}_w = 20\,\underline{i} + 50\,\underline{j}$$

$$\vec{v}_r = \vec{v}_a + \vec{v}_w = \mathbf{20\underline{i} + 50\underline{j}}$$

Part 2: Magnitude of Resultant Velocity ($|\vec{v}_r|$)

$$|\,\vec{v}_r\,| = \sqrt{(20)^2 + (50)^2} = \sqrt{400 + 2500} = \sqrt{2900} = 10\sqrt{29} \approx 53.85 \text{ units}$$

$$|\vec{v}_r| = \sqrt{(20)^2 + (50)^2} = \sqrt{400 + 2500} = \sqrt{2900} = \mathbf{10\sqrt{29}} \approx \mathbf{53.85} \text{ units}$$

Answer:

- Resultant velocity vector = $\mathbf{20\underline{i} + 50\underline{j}}$
- Magnitude = $\mathbf{10\sqrt{29}}$(or approximately **53.85**)

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Review Exercise 6.

1. Multiple Choice Questions (MCQs)

Four possible answers are given for the following questions. Choose the correct answer.

(i) x-axis and y-axis divide a coordinate plane into _____ parts.

- **Options:** (a) one (b) two (c) three (d) four
- **Correct Answer: (d) four**

(ii) P(4,-4) lies in _____ quadrant.

- **Options:** (a) first (b) second (c) third (d) fourth
- **Correct Answer: (d) fourth**

(iii) A vector having magnitude 1, is called:

- **Options:** (a) equal vector (b) parallel vector (c) unit vector (d) zero vector
- **Correct Answer: (c) unit vector**

(iv) What is the value of |3i+4j|?

- **Options:** (a) 3 (b) 4 (c) 5 (d) 7
- **Correct Answer: (c) 5**

(v) If a=b, then a and b are:

- **Options:** (a) equal (b) parallel (c) perpendicular (d) non-parallel
- **Correct Answer: (b) parallel**

(vi) If OA=a, OB=b, then AB is:

- **Options:** (a) b-a (b) a-b (c) a+b (d) b+a
- **Correct Answer: (a) b-a**

(vii) Translation vector shows:

- **Options:** (a) deformation (b) rotation (c) movement (d) enlargement
- **Correct Answer: (c) movement**

(viii) Sum of two vectors is:

- **Options:** (a) a triangle (b) a vector (c) a scalar (d) a length
- **Correct Answer: (b) a vector**

(ix) The position vector of point P(3,-2) with respect to O is:

- **Options:** (a) 3i+2j (b) 3i-2j (c) -3i+2j (d) 2i-3j

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- **Correct Answer: (b) 3i-2j**

(x) Vector from point P(3,4) to origin is:

- **Options:** (a) 3i+4j (b) -3i+4j (c) -3i-4j (d) 3i-4j
- **Correct Answer: (c) -3i-4j**

2. Find magnitude of the $\overrightarrow{AB}$:

Formula: $\overrightarrow{AB} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$, $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(i) $A(7,7)$, $B(-12,0)$

- **Vector $\overrightarrow{AB}$:**

$$\overrightarrow{AB} = (-12 - 7)\hat{\imath} + (0 - 7)\hat{\jmath} = -19\hat{\imath} - 7\hat{\jmath}$$

- **Magnitude:**

$$|\overrightarrow{AB}| = \sqrt{(-19)^2 + (-7)^2} = \sqrt{361 + 49} = \sqrt{\mathbf{410}} \text{ units}$$

(ii) $A(9,3)$, $B(2,11)$

- **Vector $\overrightarrow{AB}$:**

$$\overrightarrow{AB} = (2 - 9)\hat{\imath} + (11 - 3)\hat{\jmath} = -7\hat{\imath} + 8\hat{\jmath}$$

- **Magnitude:**

$$|\overrightarrow{AB}| = \sqrt{(-7)^2 + (8)^2} = \sqrt{49 + 64} = \sqrt{\mathbf{113}} \text{ units}$$

3. Find a unit vector in the direction of $\underline{a} = \frac{5}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath}$

1. **Calculate the magnitude $|\underline{a}|$:**

$$|\underline{a}| = \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{25}{9} + \frac{1}{9}} = \sqrt{\frac{26}{9}} = \frac{\sqrt{26}}{3}$$

2. **Calculate the unit vector $\hat{a} = \frac{\underline{a}}{|\underline{a}|}$:**

$$\hat{a} = \frac{\frac{5}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath}}{\frac{\sqrt{26}}{3}} = \frac{3}{\sqrt{26}}\left(\frac{5}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath}\right)$$

$$\hat{a} = \frac{\mathbf{5}}{\sqrt{\mathbf{26}}}\hat{\mathbf{\imath}} + \frac{\mathbf{1}}{\sqrt{\mathbf{26}}}\hat{\mathbf{\jmath}}$$

4. Given: $\underline{a} = 2\hat{\imath} - \hat{\jmath}$, $\underline{b} = 3\hat{\imath} + \hat{\jmath}$, $\underline{c} = 4\hat{\imath} + \hat{\jmath}$

(i) $5\underline{b} - \underline{a} + \underline{c}$

$$5\underline{b} = 5(3\hat{\imath} + \hat{\jmath}) = 15\hat{\imath} + 5\hat{\jmath}$$

$$5\underline{b} - \underline{a} + \underline{c} = (15\hat{\imath} + 5\hat{\jmath}) - (2\hat{\imath} - \hat{\jmath}) + (4\hat{\imath} + \hat{\jmath})$$

$$= (15 - 2 + 4)\hat{\imath} + (5 + 1 + 1)\hat{\jmath} = \mathbf{17\hat{\imath} + 7\hat{\jmath}}$$

(ii) $8\underline{a} + \underline{b} + 5\underline{c}$

$$8\underline{a} = 8(2\hat{\imath} - \hat{\jmath}) = 16\hat{\imath} - 8\hat{\jmath}$$

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$$5\underline{c} = 5(4\hat{\imath} + \hat{\jmath}) = 20\hat{\imath} + 5\hat{\jmath}$$
$$8\underline{a} + \underline{b} + 5\underline{c} = (16\hat{\imath} - 8\hat{\jmath}) + (3\hat{\imath} + \hat{\jmath}) + (20\hat{\imath} + 5\hat{\jmath})$$
$$= (16 + 3 + 20)\hat{\imath} + (-8 + 1 + 5)\hat{\jmath} = \mathbf{39\hat{\imath} - 2\hat{\jmath}}$$

(iii) $\underline{c} + \underline{b} - 4\underline{a}$

$$4\underline{a} = 4(2\hat{\imath} - \hat{\jmath}) = 8\hat{\imath} - 4\hat{\jmath}$$
$$\underline{c} + \underline{b} - 4\underline{a} = (4\hat{\imath} + \hat{\jmath}) + (3\hat{\imath} + \hat{\jmath}) - (8\hat{\imath} - 4\hat{\jmath})$$
$$= (4 + 3 - 8)\hat{\imath} + (1 + 1 + 4)\hat{\jmath} = \mathbf{-\hat{\imath} + 6\hat{\jmath}}$$

5. Find the values of x and y in the equation:

$$(2x\hat{\imath} + y\hat{\jmath}) + (-i + 5\hat{\jmath}) = \frac{1}{3}\hat{\imath} - 8\hat{\jmath}$$

1. **Group components on LHS:**

$$(2x - 1)\hat{\imath} + (y + 5)\hat{\jmath} = \frac{1}{3}\hat{\imath} - 8\hat{\jmath}$$

2. **Equate the $\hat{\imath}$ components:**

$$2x - 1 = \frac{1}{3}$$
$$2x = \frac{1}{3} + 1 = \frac{4}{3}$$
$$\mathbf{x = \frac{2}{3}}$$

3. **Equate the $\hat{\jmath}$ components:**

$$y + 5 = -8$$
$$y = -8 - 5$$
$$\mathbf{y = -13}$$

6. Translate ΔABC to $\Delta A'B'C'$ by translation vector $5\hat{\imath} - 2\hat{\jmath}$:

Given points: $A(-5,3)$, $B(-2,3)$, $C(-4,5)$

Translation vector: $\vec{T} = 5\hat{\imath} - 2\hat{\jmath}$ (meaning shift x by $+5$ and y by -2).

- **For $A(-5,3)$:**

$$A' = (-5 + 5, 3 - 2) = \mathbf{A'(0,1)}$$

- **For $B(-2,3)$:**

$$B' = (-2 + 5, 3 - 2) = \mathbf{B'(3,1)}$$

- **For $C(-4,5)$:**

$$C' = (-4 + 5, 5 - 2) = \mathbf{C'(1,3)}$$

(To plot: Draw the original triangle with vertices A, B, C and the shifted triangle with vertices A', B', C' on a coordinate plane).

7. Use vectors to show that $ABCD$ is a parallelogram:

Given vertices: $A(2,3)$, $B(6,3)$, $C(7,6)$, $D(3,6)$

A quadrilateral is a parallelogram if opposite sides are equal and parallel vectors (i.e., $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$).

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1. **Calculate $\overrightarrow{AB}$ and $\overrightarrow{DC}$:**

$$\overrightarrow{AB} = (6-2)\hat{\imath} + (3-3)\hat{\jmath} = 4\hat{\imath} + 0\hat{\jmath}$$
$$\overrightarrow{DC} = (7-3)\hat{\imath} + (6-6)\hat{\jmath} = 4\hat{\imath} + 0\hat{\jmath}$$

Since $\overrightarrow{AB} = \overrightarrow{DC}$, side AB is equal and parallel to side DC.

2. **Calculate $\overrightarrow{AD}$ and $\overrightarrow{BC}$:**

$$\overrightarrow{AD} = (3-2)\hat{\imath} + (6-3)\hat{\jmath} = 1\hat{\imath} + 3\hat{\jmath}$$
$$\overrightarrow{BC} = (7-6)\hat{\imath} + (6-3)\hat{\jmath} = 1\hat{\imath} + 3\hat{\jmath}$$

Since $\overrightarrow{AD} = \overrightarrow{BC}$, side AD is equal and parallel to side BC.

Conclusion: Since opposite vector pairs are equal ($\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$), $ABCD$ **is a parallelogram**.

8. Use vectors to show that ΔABC is an isosceles triangle:

Given vertices: $A(1,2)$, $B(4,6)$, $C(7,2)$

An isosceles triangle has at least two sides of equal length.

1. **Side vector $\overrightarrow{AB}$ and its magnitude:**

$$\overrightarrow{AB} = (4-1)\hat{\imath} + (6-2)\hat{\jmath} = 3\hat{\imath} + 4\hat{\jmath}$$
$$|\overrightarrow{AB}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

2. **Side vector $\overrightarrow{BC}$ and its magnitude:**

$$\overrightarrow{BC} = (7-4)\hat{\imath} + (2-6)\hat{\jmath} = 3\hat{\imath} - 4\hat{\jmath}$$
$$|\overrightarrow{BC}| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

3. **Side vector $\overrightarrow{AC}$ and its magnitude:**

$$\overrightarrow{AC} = (7-1)\hat{\imath} + (2-2)\hat{\jmath} = 6\hat{\imath} + 0\hat{\jmath}$$
$$|\overrightarrow{AC}| = \sqrt{6^2 + 0^2} = 6$$

Conclusion: Since $|\overrightarrow{AB}| = |\overrightarrow{BC}| = 5$, two sides are equal in length, proving that ΔABC **is an isosceles triangle**.

9. A ball is projected with velocity vector $\underline{v} = 6\hat{\imath} + 8\hat{\jmath}$. What is the magnitude of velocity?

$$\text{Magnitude } |\underline{v}| = \sqrt{(6)^2 + (8)^2}$$
$$|\underline{v}| = \sqrt{36+64} = \sqrt{100} = \mathbf{10} \text{ units}$$

10. Aircraft flying east at 200 km/h, wind blowing north at 60 km/h. Find the resultant velocity and its magnitude.

Let:

- East direction be along the positive x-axis ($\hat{\imath}$)
- North direction be along the positive y-axis ($\hat{\jmath}$)

1. **Velocity vectors:**
   - Aircraft velocity: $\vec{v}_a = 200\hat{\imath}$
   - Wind velocity: $\vec{v}_w = 60\hat{\jmath}$

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2. **Resultant velocity vector ($\vec{v}_r$):**

$$\vec{v}_r = \vec{v}_a + \vec{v}_w = \mathbf{200\hat{i} + 60\hat{j}} \text{ km/h}$$

3. **Magnitude of resultant velocity ($|\vec{v}_r|$):**

$$|\vec{v}_r| = \sqrt{(200)^2 + (60)^2}$$

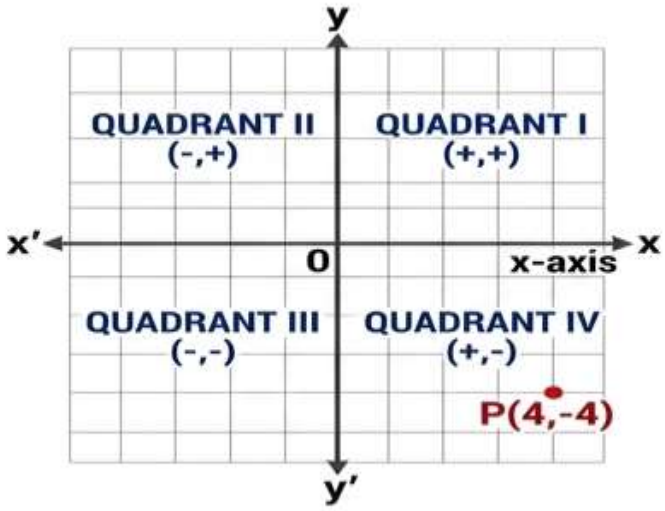
$$|\vec{v}_r| = \sqrt{40000 + 3600} = \sqrt{43600} = \mathbf{20\sqrt{109}} \approx \mathbf{208.81} \text{ km/h}$$

NOTE: The plots of the mentioned Questions are given in the next page. Thank you for the attention.

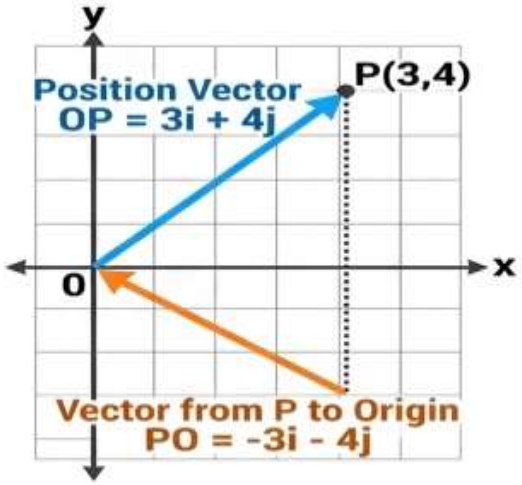
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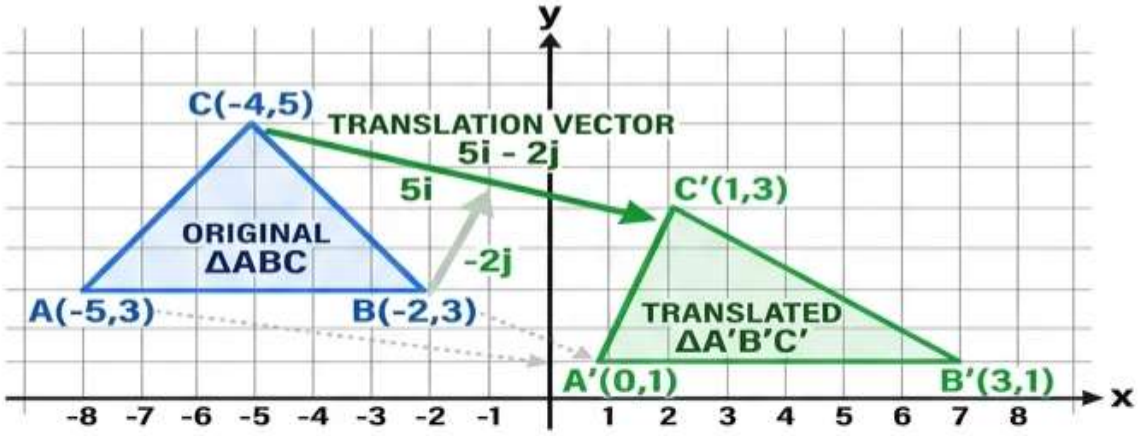
Quadrant System & Point P(4,-4) (Questions 1(i) & 1(ii))



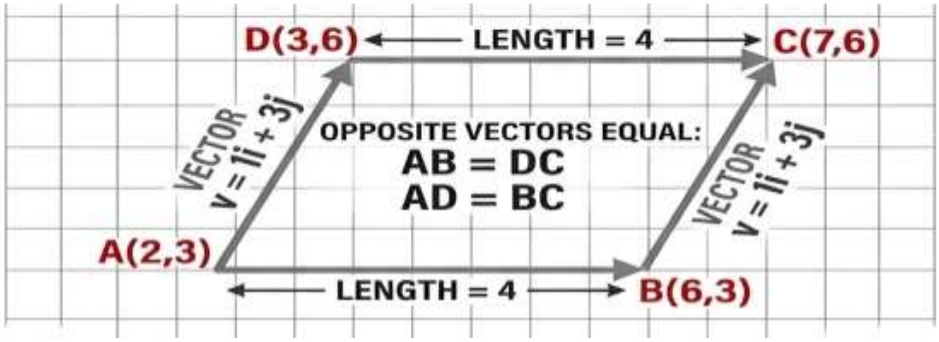
Position Vector to Origin (Question 1(x))



TRANSLATION OF TRIANGLE ABC (Question 6)



PARALLELOGRAM VERIFICATION (Question 7)



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