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Class	10th	Chapter	07
Subject	Mathematics	Board	Punjab

CHAPTER 07: TRIGNOMETRY

Exercise 7.1

Question 1: Find the signs of the following:

Quadrant Rules for Signs:

- **1st Quadrant** ($0° < \theta < 90°$)**:** All trigonometric functions are **Positive (+)**.
- **2nd Quadrant** ($90° < \theta < 180°$)**:** sin and csc are **Positive (+)**; others are Negative (–).
- **3rd Quadrant** ($180° < \theta < 270°$)**:** tan and cot are **Positive (+)**; others are Negative (–).
- **4th Quadrant** ($270° < \theta < 360°$)**:** cos and sec are **Positive (+)**; others are Negative (–).

(i) $\sin 55°$

- **Quadrant:** $55°$ lies in the **1st Quadrant** ($0° < 55° < 90°$).
- **Sign:** Since sine is positive in the 1st Quadrant:

Positive $(+)$

(ii) $\cos 145°$

- **Quadrant:** $145°$ lies in the **2nd Quadrant** ($90° < 145° < 180°$).
- **Sign:** Since cosine is negative in the 2nd Quadrant:

Negative $(-)$

(iii) $\tan 111°$

- **Quadrant:** $111°$ lies in the **2nd Quadrant** ($90° < 111° < 180°$).
- **Sign:** Since tangent is negative in the 2nd Quadrant:

Negative $(-)$

(iv) $\sec 179°$

- **Quadrant:** $179°$ lies in the **2nd Quadrant** ($90° < 179° < 180°$).
- **Sign:** Since secant is negative in the 2nd Quadrant:

Negative $(-)$

(v) $\csc 88°$

- **Quadrant:** $88°$ lies in the **1st Quadrant** ($0° < 88° < 90°$).
- **Sign:** Since cosecant is positive in the 1st Quadrant:

Positive $(+)$

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(vi) $\cot 14^\circ$

- **Quadrant:** 14° lies in the **1st Quadrant** ($0^\circ < 14^\circ < 90^\circ$).
- **Sign:** Since cotangent is positive in the 1st Quadrant:

Positive $(+)$

Question 2: Fill in the blanks:

Using the reduction formulas for trigonometric functions:

(i) $\tan(180^\circ - \theta) = \cdots \tan\theta$

- $(180^\circ - \theta)$ lies in the 2nd Quadrant where tangent is negative.
- $\tan(180^\circ - \theta) = -\tan\theta$
- **Blank:** -1 (or simply $-$)

(ii) $\sin(180^\circ - \theta) = \cdots \sin\theta$

- $(180^\circ - \theta)$ lies in the 2nd Quadrant where sine is positive.
- $\sin(180^\circ - \theta) = \sin\theta$
- **Blank:** $+1$ (or simply $+$)

(iii) $\tan(90^\circ + \theta) = \cdots \cot\theta$

- $(90^\circ + \theta)$ lies in the 2nd Quadrant where tangent is negative, and due to co-function identity at 90°, tangent changes to cotangent.
- $\tan(90^\circ + \theta) = -\cot\theta$
- **Blank:** -1 (or simply $-$)

(iv) $\cos(90^\circ + \theta) = \cdots \sin\theta$

- $(90^\circ + \theta)$ lies in the 2nd Quadrant where cosine is negative, and due to co-function identity at 90°, cosine changes to sine.
- $\cos(90^\circ + \theta) = -\sin\theta$
- **Blank:** -1 (or simply $-$)

(v) $\csc(180^\circ - \theta) = \cdots \csc\theta$

- $(180^\circ - \theta)$ lies in the 2nd Quadrant where cosecant is positive.
- $\csc(180^\circ - \theta) = \csc\theta$
- **Blank:** $+1$ (or simply $+$)

(vi) $\sec(90^\circ - \theta) = \cdots \csc\theta$

- $(90^\circ - \theta)$ lies in the 1st Quadrant where all functions are positive, and due to co-function identity at 90°, secant changes to cosecant.
- $\sec(90^\circ - \theta) = \csc\theta$
- **Blank:** $+1$ (or simply $+$)

Question 3: Without using calculator, find the exact values of the following trigonometric functions:

Angles with Reference Angle 30° ($150^\circ = 180^\circ - 30^\circ$)

(i) $\sin 150^\circ$

$$\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

(ii) $\tan 150^\circ$

$$\tan 150^\circ = \tan(180^\circ - 30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$$

(iii) $\sec 150^\circ$

$$\sec 150^\circ = \sec(180^\circ - 30^\circ) = -\sec 30^\circ = -\frac{2}{\sqrt{3}}$$

Angles with Reference Angle 60° ($120^\circ = 180^\circ - 60^\circ$)

(iv) $\csc 120^\circ$

$$\csc 120^\circ = \csc(180^\circ - 60^\circ) = \csc 60^\circ = \frac{2}{\sqrt{3}}$$

(v) $\cos 120^\circ$

$$\cos 120^\circ = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$$

(vi) $\cot 120^\circ$

$$\cot 120^\circ = \cot(180^\circ - 60^\circ) = -\cot 60^\circ = -\frac{1}{\sqrt{3}}$$

Angles with Reference Angle 45° ($135^\circ = 180^\circ - 45^\circ$)

(vii) $\sin 135^\circ$

$$\sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

(viii) $\sec 135^\circ$

$$\sec 135^\circ = \sec(180^\circ - 45^\circ) = -\sec 45^\circ = -\sqrt{2}$$

(ix) $\cot 135^\circ$

$$\cot 135^\circ = \cot(180^\circ - 45^\circ) = -\cot 45^\circ = -1$$

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Exercise 7.2.

Key Formulas Used

1. **Law of Sines:**

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

2. **Sum of Angles of a Triangle:**

$$\alpha + \beta + \gamma = 180^\circ$$

3. **Area Formulas for Triangle ABC:**
   - **Two sides and included angle:**

$$\text{Area} = \frac{1}{2}ab\sin\gamma = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ac\sin\beta$$

   - **One side and three angles:**

$$\text{Area} = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \frac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$$

   - **Three sides (Heron's Formula):**

$$s = \frac{a+b+c}{2}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Question 1: Solve the triangle ABC

(i) $a = 6.1$ cm, $b = 8.4$ cm, $\alpha = 42^\circ$

1. **Find β using the Law of Sines:**

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$$

$$\sin\beta = \frac{b\sin\alpha}{a} = \frac{8.4\times\sin 42^\circ}{6.1} \approx \frac{8.4\times 0.6691}{6.1} \approx 0.9214$$

$$\beta = \sin^{-1}(0.9214) \approx 67.13^\circ \quad \text{or} \quad \beta' = 180^\circ - 67.13^\circ = 112.87^\circ$$

   - **Case 1 ($\beta \approx 67.13^\circ$):**
     - **Find γ:**

$$\gamma = 180^\circ - (42^\circ + 67.13^\circ) = 70.87^\circ$$

     - **Find c:**

$$c = \frac{a\sin\gamma}{\sin\alpha} = \frac{6.1\times\sin 70.87^\circ}{\sin 42^\circ} \approx \frac{6.1\times 0.9448}{0.6691} \approx 8.61 \text{ cm}$$

   - **Case 2 ($\beta' \approx 112.87^\circ$):**
     - **Find γ':**

$$\gamma' = 180^\circ - (42^\circ + 112.87^\circ) = 25.13^\circ$$

     - **Find c':**

$$c' = \frac{a\sin\gamma'}{\sin\alpha} = \frac{6.1\times\sin 25.13^\circ}{\sin 42^\circ} \approx \frac{6.1\times 0.4247}{0.6691} \approx 3.87 \text{ cm}$$

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(ii) $a = 12.2$ cm, $c = 15.8$ cm, $\gamma = 50°$

1. **Find α using the Law of Sines:**

$$\sin\alpha = \frac{a\sin\gamma}{c} = \frac{12.2 \times \sin 50°}{15.8} \approx \frac{12.2 \times 0.7660}{15.8} \approx 0.5915$$
$$\alpha = \sin^{-1}(0.5915) \approx 36.26°$$

(Note: Since $c > a$, the ambiguous case $180° - \alpha$ is not possible because $\gamma > \alpha$.)

2. **Find β:**

$$\beta = 180° - (36.26° + 50°) = 93.74°$$

3. **Find b:**

$$b = \frac{c\sin\beta}{\sin\gamma} = \frac{15.8 \times \sin 93.74°}{\sin 50°} \approx \frac{15.8 \times 0.9978}{0.7660} \approx 20.58 \text{ cm}$$

(iii) $b = 5.2$ cm, $c = 5$ cm, $\gamma = 48°$

1. **Find β using the Law of Sines:**

$$\sin\beta = \frac{b\sin\gamma}{c} = \frac{5.2 \times \sin 48°}{5} \approx \frac{5.2 \times 0.7431}{5} \approx 0.7729$$
$$\beta = \sin^{-1}(0.7729) \approx 50.62° \quad \text{or} \quad \beta' = 180° - 50.62° = 129.38°$$

- **Case 1 ($\beta \approx 50.62°$):**
  - **Find α:**

$$\alpha = 180° - (50.62° + 48°) = 81.38°$$

  - **Find a:**

$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{5 \times \sin 81.38°}{\sin 48°} \approx \frac{5 \times 0.9887}{0.7431} \approx 6.65 \text{ cm}$$

- **Case 2 ($\beta' \approx 129.38°$):**
  - **Find α':**

$$\alpha' = 180° - (129.38° + 48°) = 2.62°$$

  - **Find a':**

$$a' = \frac{c\sin\alpha'}{\sin\gamma} = \frac{5 \times \sin 2.62°}{\sin 48°} \approx \frac{5 \times 0.0457}{0.7431} \approx 0.31 \text{ cm}$$

(iv) $b = 4.8$ cm, $a = 4$ cm, $\beta = 71°$

1. **Find α using the Law of Sines:**

$$\sin\alpha = \frac{a\sin\beta}{b} = \frac{4 \times \sin 71°}{4.8} \approx \frac{4 \times 0.9455}{4.8} \approx 0.7879$$
$$\alpha = \sin^{-1}(0.7879) \approx 51.99°$$

(Since $b > a$, $\beta > \alpha$, so the obtuse angle option is invalid.)

2. **Find γ:**

$$\gamma = 180° - (51.99° + 71°) = 57.01°$$

3. **Find c:**

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$$c = \frac{b \sin \gamma}{\sin \beta} = \frac{4.8 \times \sin 57.01^\circ}{\sin 71^\circ} \approx \frac{4.8 \times 0.8388}{0.9455} \approx 4.26 \text{ cm}$$

(v) $\boldsymbol{\beta = 70^\circ, b = 8 \text{ cm}, \alpha = 100^\circ}$

1. **Find γ:**

$$\gamma = 180^\circ - (100^\circ + 70^\circ) = 10^\circ$$

2. **Find a:**

$$a = \frac{b \sin \alpha}{\sin \beta} = \frac{8 \times \sin 100^\circ}{\sin 70^\circ} \approx \frac{8 \times 0.9848}{0.9397} \approx 8.38 \text{ cm}$$

3. **Find c:**

$$c = \frac{b \sin \gamma}{\sin \beta} = \frac{8 \times \sin 10^\circ}{\sin 70^\circ} \approx \frac{8 \times 0.1736}{0.9397} \approx 1.48 \text{ cm}$$

(vi) $\boldsymbol{a = 6 \text{ cm}, \alpha = 55^\circ, \gamma = 60^\circ}$

1. **Find β:**

$$\beta = 180^\circ - (55^\circ + 60^\circ) = 65^\circ$$

2. **Find b:**

$$b = \frac{a \sin \beta}{\sin \alpha} = \frac{6 \times \sin 65^\circ}{\sin 55^\circ} \approx \frac{6 \times 0.9063}{0.8192} \approx 6.64 \text{ cm}$$

3. **Find c:**

$$c = \frac{a \sin \gamma}{\sin \alpha} = \frac{6 \times \sin 60^\circ}{\sin 55^\circ} \approx \frac{6 \times 0.8660}{0.8192} \approx 6.34 \text{ cm}$$

(vii) $\boldsymbol{c = 7 \text{ cm}, \beta = 34^\circ, \gamma = 64^\circ}$

1. **Find α:**

$$\alpha = 180^\circ - (34^\circ + 64^\circ) = 82^\circ$$

2. **Find a:**

$$a = \frac{c \sin \alpha}{\sin \gamma} = \frac{7 \times \sin 82^\circ}{\sin 64^\circ} \approx \frac{7 \times 0.9903}{0.8988} \approx 7.71 \text{ cm}$$

3. **Find b:**

$$b = \frac{c \sin \beta}{\sin \gamma} = \frac{7 \times \sin 34^\circ}{\sin 64^\circ} \approx \frac{7 \times 0.5592}{0.8988} \approx 4.36 \text{ cm}$$

(viii) $\boldsymbol{b = 12 \text{ cm}, \alpha = 92^\circ, \beta = 77^\circ}$

1. **Find γ:**

$$\gamma = 180^\circ - (92^\circ + 77^\circ) = 11^\circ$$

2. **Find a:**

$$a = \frac{b \sin \alpha}{\sin \beta} = \frac{12 \times \sin 92^\circ}{\sin 77^\circ} \approx \frac{12 \times 0.9994}{0.9744} \approx 12.31 \text{ cm}$$

3. **Find c:**

$$c = \frac{b \sin \gamma}{\sin \beta} = \frac{12 \times \sin 11^\circ}{\sin 77^\circ} \approx \frac{12 \times 0.1908}{0.9744} \approx 2.35 \text{ cm}$$

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Question 2: Calculate area of each triangle ABC

(i) $a = 7$ cm, $b = 8$ cm, $\gamma = 38°$

$$\text{Area} = \frac{1}{2}ab\sin\gamma = \frac{1}{2} \times 7 \times 8 \times \sin 38° = 28 \times 0.6157 \approx 17.24 \text{ cm}^2$$

(ii) $a = 11$ cm, $c = 14$ cm, $\beta = 51°$

$$\text{Area} = \frac{1}{2}ac\sin\beta = \frac{1}{2} \times 11 \times 14 \times \sin 51° = 77 \times 0.7771 \approx 59.84 \text{ cm}^2$$

(iii) $b = 3$ cm, $c = 9$ cm, $\alpha = 78°$

$$\text{Area} = \frac{1}{2}bc\sin\alpha = \frac{1}{2} \times 3 \times 9 \times \sin 78° = 13.5 \times 0.9781 \approx 13.20 \text{ cm}^2$$

(iv) $a = 10$ cm, $\alpha = 62°, \beta = 69°$

1. **Find γ:**

$$\gamma = 180° - (62° + 69°) = 49°$$

2. **Calculate Area:**

$$\text{Area} = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \frac{10^2 \times \sin 69° \times \sin 49°}{2 \times \sin 62°}$$

$$\text{Area} = \frac{100 \times 0.9336 \times 0.7547}{2 \times 0.8829} = \frac{70.46}{1.7658} \approx 39.90 \text{ cm}^2$$

(v) $c = 4$ cm, $\beta = 36°, \gamma = 80°$

1. **Find α:**

$$\alpha = 180° - (36° + 80°) = 64°$$

2. **Calculate Area:**

$$\text{Area} = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \frac{4^2 \times \sin 64° \times \sin 36°}{2 \times \sin 80°}$$

$$\text{Area} = \frac{16 \times 0.8988 \times 0.5878}{2 \times 0.9848} = \frac{8.453}{1.9696} \approx 4.29 \text{ cm}^2$$

(vi) $c = 6.6$ cm, $\alpha = 23°, \gamma = 89°$

1. **Find β:**

$$\beta = 180° - (23° + 89°) = 68°$$

2. **Calculate Area:**

$$\text{Area} = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \frac{(6.6)^2 \times \sin 23° \times \sin 68°}{2 \times \sin 89°}$$

$$\text{Area} = \frac{43.56 \times 0.3907 \times 0.9272}{2 \times 0.9998} = \frac{15.78}{1.9996} \approx 7.89 \text{ cm}^2$$

(vii) $a = 5.3$ cm, $b = 4.7$ cm, $c = 8.2$ cm

1. **Find s:**

$$s = \frac{5.3 + 4.7 + 8.2}{2} = \frac{18.2}{2} = 9.1 \text{ cm}$$

2. **Calculate Area using Heron's Formula:**

$$s - a = 9.1 - 5.3 = 3.8$$

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$$s - b = 9.1 - 4.7 = 4.4$$
$$s - c = 9.1 - 8.2 = 0.9$$
$$\text{Area} = \sqrt{9.1 \times 3.8 \times 4.4 \times 0.9} = \sqrt{136.9368} \approx 11.70 \text{ cm}^2$$

(viii) $a = 6.12$ cm, $b = 8.34$ cm, $c = 7.12$ cm

1. **Find s:**

$$s = \frac{6.12 + 8.34 + 7.12}{2} = \frac{21.58}{2} = 10.79 \text{ cm}$$

2. **Calculate Area using Heron's Formula:**

$$s - a = 10.79 - 6.12 = 4.67$$
$$s - b = 10.79 - 8.34 = 2.45$$
$$s - c = 10.79 - 7.12 = 3.67$$
$$\text{Area} = \sqrt{10.79 \times 4.67 \times 2.45 \times 3.67} = \sqrt{452.88} \approx 21.28 \text{ cm}^2$$

NOTE:

This figure sheet includes diagrams for:

- **Q1 (i):** The complex **ambiguous case (SSA)**, where we find two possible triangles based on the initial constraints.
- **Q1 (v):** A standard triangle solution where all parameters define a single unique shape.
- **Q2 (i):** An area calculation illustrating the **SAS (Side-Angle-Side)** method.
- **Q2 (vii):** An area calculation illustrating the **SSS (Side-Side-Side)** method using Heron's formula.

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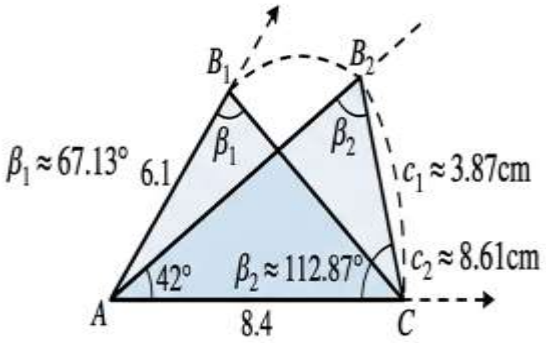
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EXERCISE 7.2 DIAGRAMS

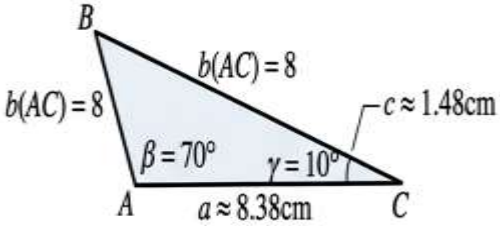
Solve the triangle ABC.

Q1 (i) $a = 6.1\text{cm}, b = 8.4\text{ cm}, \alpha = 42°$



Q1 (i) Ambiguous Case: Two Possible Triangles

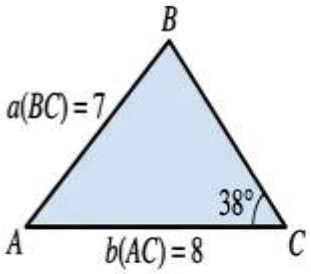
Q1 (v) $\beta = 70°, b = 8\text{cm}, \alpha = 100°$



Q1 (v) Single Solution

Calculate area of each triangle ABC.

Q2 (i) $a = 7\text{ cm}, b = 8\text{ cm}, \gamma = 38°$

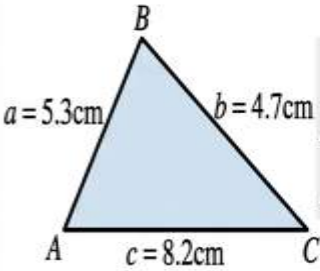


$$\text{Area} = \frac{1}{2}ab\sin(\gamma)$$

$$\text{Area} \approx 17.24\text{ cm}^2$$

Q2 (i) SAS Triangle Area

Q2 (vii) $a = 5.3\text{ cm}, b = 4.7\text{ cm}, c = 8.2\text{ cm}$



$$s = 9.1$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area} \approx 11.70\text{ cm}^2$$

Q2 (vii) SSS Triangle Area (Heron's Formula)

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EXERCISE 7.3

Question 1

Given:

- Right-angled triangular prism.
- $m\overline{BC} = 3$ cm, $m\overline{CE} = 4$ cm, $m\overline{FE} = 5$ cm.
- Base $\triangle CDE$ is right-angled at D (where $m\overline{CD} = 3$ cm or corresponding side dimensions apply).
- In right-angled triangle $\triangle CDE$: $m\overline{CE} = 4$ cm, $m\overline{DE} = 3$ cm (or $CD \perp DE$).

(i) Find the length $\overline{CF}$

In the right-angled triangle $\triangle CFE$ (or using the base right triangle $\triangle CDE$ where $FE = 5$ cm is the hypotenuse of the base): Using Pythagorean theorem in right triangle $\triangle CDE$:

$$m\overline{CF}^2 = m\overline{CE}^2 + m\overline{FE}^2$$

$$m\overline{CF}^2 = 4^2 + 5^2$$

$$m\overline{CF}^2 = 16 + 25 = 41$$

$$m\overline{CF} = \sqrt{41} \approx 6.40 \text{ cm} \quad \text{(or 6.4 cm)}$$

(ii) Find the length $\overline{BF}$

In vertical right triangle $\triangle BCF$ (where $\angle BCF = 90°$ and $m\overline{BC} = 3$ cm):

$$m\overline{BF}^2 = m\overline{BC}^2 + m\overline{CF}^2$$

$$m\overline{BF}^2 = 3^2 + 41$$

$$m\overline{BF}^2 = 9 + 41 = 50$$

$$m\overline{BF} = \sqrt{50} = 5\sqrt{2} \approx 7.07 \text{ cm} \quad \text{(or 7.1 cm)}$$

(iii) Find the angle BFC, correct to 1 decimal place

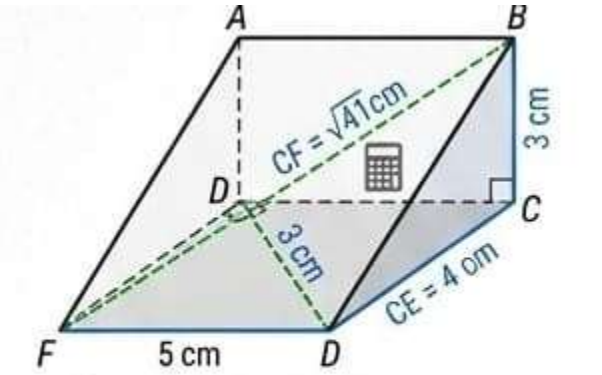
In right-angled triangle $\triangle BCF$:

$$\tan(\angle BFC) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{m\overline{BC}}{m\overline{CF}}$$

$$\tan(\angle BFC) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{m\overline{BC}}{m\overline{CF}}$$

$$\tan(\angle BFC) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{m\overline{BC}}{m\overline{CF}}$$

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(1) **Problem 1:** Triangular Prism.

Q1 Sol (i): CF=√16 + 25

Question 2

Given:

- Rectangular base $PQRS$ with $m\overline{PQ} = 16$ cm and $m\overline{QR} = 10$ cm.
- M is the midpoint of diagonal PR.
- $m\overline{MT} = 15$ cm (vertical height).

Steps:

1. **Find length of diagonal PR:** Using Pythagorean theorem in right-angled triangle $\triangle PQR$:

$$m\overline{PR}^2 = m\overline{PQ}^2 + m\overline{QR}^2 = 16^2 + 10^2 = 256 + 100 = 356$$
$$m\overline{PR} = \sqrt{356} \approx 18.868 \text{ cm}$$

2. **Find length PM:** Since M is the midpoint of PR:

$$m\overline{PM} = \frac{m\overline{PR}}{2} = \frac{\sqrt{356}}{2} = \sqrt{\frac{356}{4}} = \sqrt{89} \approx 9.434 \text{ cm}$$

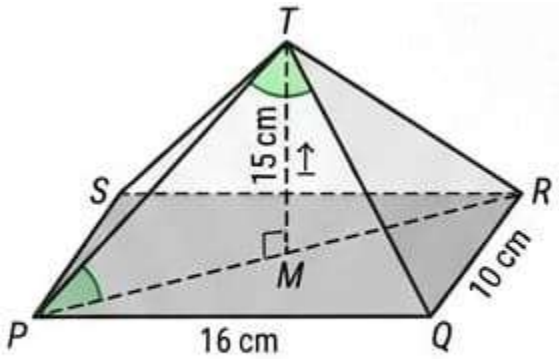
3. **Find the angle between TP and base $PQRS$ (i.e., angle $\angle TPM$):** In right-angled triangle $\triangle TMP$:

$$\tan(\angle TPM) = \frac{m\overline{TM}}{m\overline{PM}} = \frac{15}{\sqrt{89}} \approx \frac{15}{9.434} \approx 1.590$$
$$\angle TPM = \tan^{-1}(1.590) \approx 57.8^\circ$$

Answer: 57.8°

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(2) **Problem 2:** Triangular Pyramid.

Q2 Sol: TPM = 57.8°

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Question 3

Given:

- Square base $PQRS$ of side 10 cm.
- M is the center (intersection of diagonals) of the base.
- Height $m\overline{VM} = 12$ cm.

Steps:

1. **Find length of diagonal PR:**

$$m\overline{PR} = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2} \text{ cm}$$

2. **Find length PM:**

$$m\overline{PM} = \frac{m\overline{PR}}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \approx 7.071 \text{ cm}$$

3. **Find angle $\angle VPM$:** In right triangle $\triangle VPM$:

$$\tan(\angle VPM) = \frac{m\overline{VM}}{m\overline{PM}} = \frac{12}{5\sqrt{2}} \approx \frac{12}{7.071} \approx 1.697$$

$$\angle VPM = \tan^{-1}(1.697) \approx 59.5^\circ$$

Answer: 59.5° (or $59^\circ/60^\circ$ depending on rounding requirement).

Question 4

Given:

- Square base $ABCD$ with side $m\overline{AB} = 15$ cm.
- Slant edges $m\overline{AE} = m\overline{BE} = m\overline{CE} = m\overline{DE} = 12$ cm.
- Need to find angle $\angle DEB$.

Steps:

1. **Find diagonal DB of the square base:**

$$m\overline{DB} = \sqrt{15^2 + 15^2} = \sqrt{225 + 225} = \sqrt{450} = 15\sqrt{2} \approx 21.213 \text{ cm}$$

2. **Use the Law of Cosines in triangle $\triangle DEB$:** In $\triangle DEB$, sides are $m\overline{DE} = 12$ cm, $m\overline{BE} = 12$ cm, and $m\overline{DB} = 15\sqrt{2}$ cm.

$$\cos(\angle DEB) = \frac{m\overline{DE}^2 + m\overline{BE}^2 - m\overline{DB}^2}{2 \cdot m\overline{DE} \cdot m\overline{BE}}$$

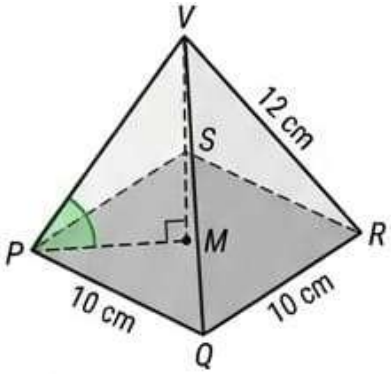
$$\cos(\angle DEB) = \frac{12^2 + 12^2 - \left(\sqrt{450}\right)^2}{2 \cdot 12 \cdot 12}$$

$$\cos(\angle DEB) = \frac{144 + 144 - 450}{288} = \frac{288 - 450}{288} = \frac{-162}{288} = -0.5625$$

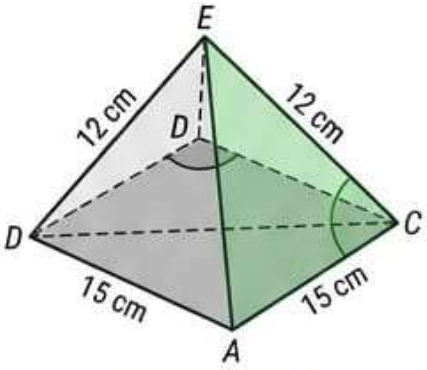
$$\angle DEB = \cos^{-1}(-0.5625) \approx 124.23^\circ$$

To the nearest whole degree: 124°.

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(3) **Problem 4:** Square Base Pyramid (DEB Angle).



(4) Q4 Sol: ∠DEB ≈ 124°

Question 5

Given:

- Base $ABCD$ with $m\overline{AB} = 10$ cm, $m\overline{BC} = 7$ cm.
- M is the midpoint of AD. So $m\overline{AM} = \frac{7}{2} = 3.5$ cm.
- Height $m\overline{TM} = 6$ cm.

Steps:

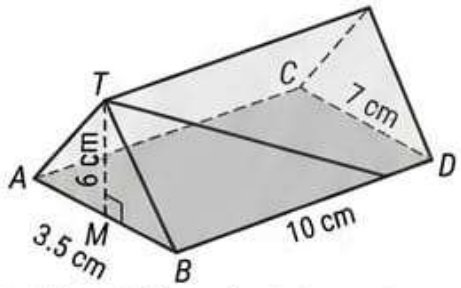
1. **Find length MB on the base:** In right triangle $\triangle MAB$ (where $\angle MAB = 90°$):

$$m\overline{MB}^2 = m\overline{AM}^2 + m\overline{AB}^2 = (3.5)^2 + 10^2 = 12.25 + 100 = 112.25$$
$$m\overline{MB} = \sqrt{112.25} \approx 10.595 \text{ cm}$$

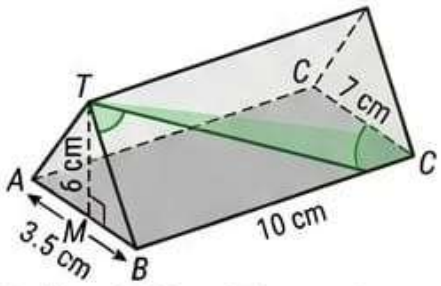
2. **Find angle between TB and the base (i.e., angle $\angle TBM$):** In right triangle $\triangle TMB$:

$$\tan(\angle TBM) = \frac{m\overline{TM}}{m\overline{MB}} = \frac{6}{10.595} \approx 0.5663$$
$$\angle TBM = \tan^{-1}(0.5663) \approx 29.5°$$

Answer: 29.5°



(5) **Problem 5:** Triangular Prism on Rectangle.



(5) **Problem 5:** TriangulPrism on Rectangle.

Question 6

Given:

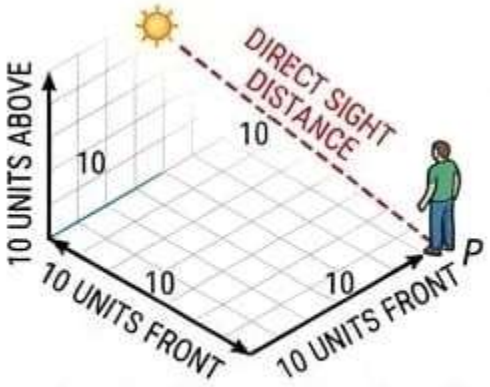
- Distance horizontal (front) = 10 units.
- Distance vertical (above) = 10 units.

Formula:

Direct line-of-sight distance $= \sqrt{x^2 + y^2}$

$$\text{Distance} = \sqrt{10^2 + 10^2} = \sqrt{100 + 100} = \sqrt{200} = 10\sqrt{2} \approx 14.14 \text{ units}$$

Answer: 14.14 units (or $10\sqrt{2}$ units)



(6) **Problem 6:** Sight Distance (Isometric).

Q6 Sol: $\sqrt{10^2 + 10^2} \approx 14.14$ UNITS

Question 7

Given:

- Angle of elevation $\theta = 28°$.
- Distance from base $= 60$ m.

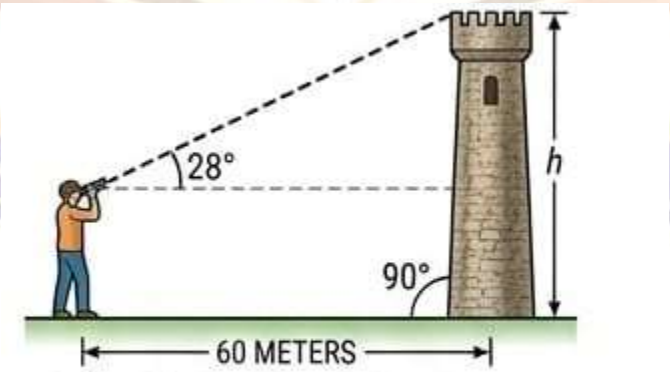
Formula:

$$\tan(28°) = \frac{\text{Height}}{\text{Distance}} = \frac{h}{60}$$

$$\tan(28°) = \frac{\text{Height}}{\text{Distance}} = \frac{h}{60}$$

$$\tan(28°) = \frac{\text{Height}}{\text{Distance}} = \frac{h}{60}$$

Answer: 31.9 m



(7) **Problem 7:** Tower Elevation.

Q2 Sol: TPM = 57.8°

Question 8

Given:

- Width of river (opposite side) $= 80$ m.

- Angle to current = 60°.

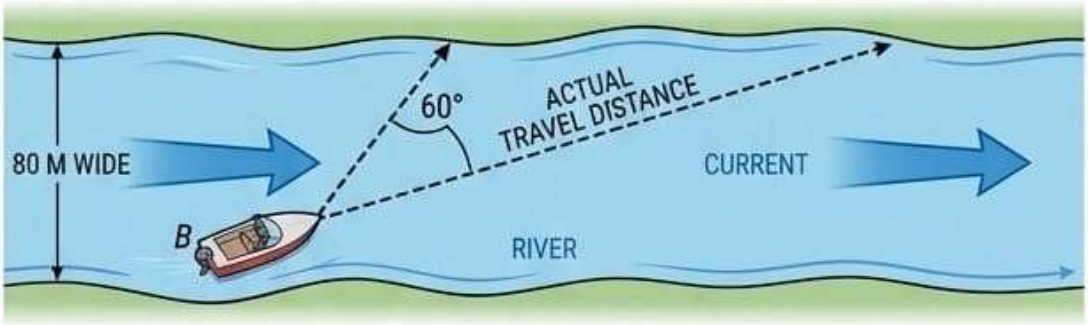
Formula:

$$\sin(60°) = \frac{\text{Width of river}}{\text{Actual distance traveled}}$$

$$\sin(60°) = \frac{\text{Width of river}}{\text{Actual distance traveled}}$$

$$\sin(60°) = \frac{\text{Width of river}}{\text{Actual distance traveled}}$$

Answer: 92.38 m (or $\frac{160}{\sqrt{3}}$ m)



(8) **Problem 8:** Boat River Current.
Q3 Sol: ∠TPM = 128°

Question 9

Given:

- Speaker 1: 6 m directly ahead (0°).
- Speaker 2: 6 m away at an angle of 30°.

Using Law of Cosines:

In the triangle formed by listener and two speakers:

$$d^2 = a^2 + b^2 - 2ab\cos(30°)$$

$$d^2 = a^2 + b^2 - 2ab\cos(30°)$$

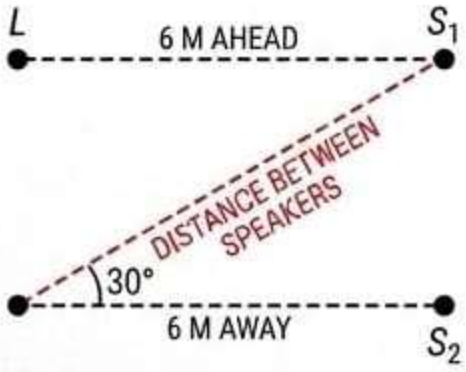
$$d^2 = a^2 + b^2 - 2ab\cos(30°)$$

$$d^2 = 6^2 + 6^2 - 2(6)(6)\cos(30°)$$

$$d^2 = 36 + 36 - 72\left(\frac{\sqrt{3}}{2}\right)$$

Answer: 3.11 m

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(9) **Problem 9:** Speaker Distance.
Q4 Sol: ∠DEB ≈ 124°

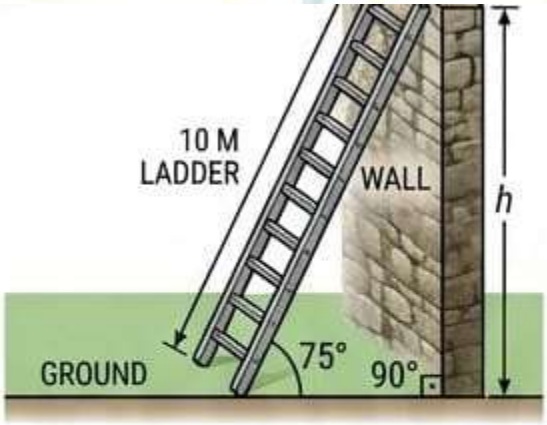
Question 10
Given:

- Length of ladder (hypotenuse) $= 10$ m.
- Angle with ground $\theta = 75°$.

Formula:

$$\sin(75°) = \frac{\text{Height}}{10}$$
$$\sin(75°) = \frac{\text{Height}}{10}$$
$$\sin(75°) = \frac{\text{Height}}{10}$$

Answer: 9.66 m



(10) **Problem 10:** Ladder Angle.
Q6 Sol: $\sqrt{10^2 + 10^2} \approx 14$ UNITS

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Exercise 7.4

Definition of Bearing

A **three-figure bearing** is an angle measured:

1. Clockwise starting from the **North (N) direction** ($000°$).
2. Written as a **3-digit number** (e.g., $045°$, $120°$, $300°$).

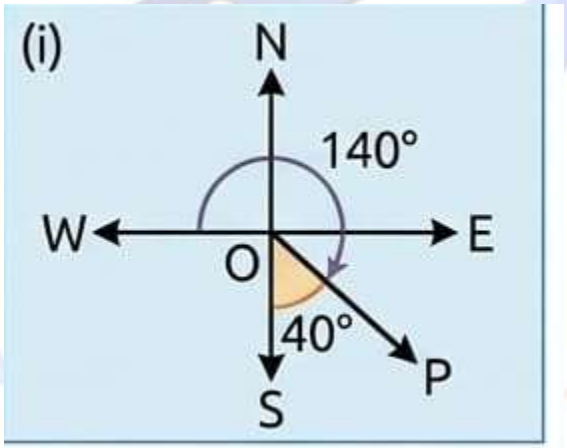
Question 1: Find the bearing of point *P* in each of the following:

(i)

- **Diagram analysis:** Point *P* is in the fourth quadrant (South-East direction). The angle given is $40°$ measured clockwise from the South ray ($180°$).
- **Calculation:**

$$\text{Bearing} = 180° - 40° = 140°$$

$$\text{Bearing} = 180° - 40° = 140°$$

- **Answer:** $140°$
- 

(ii)

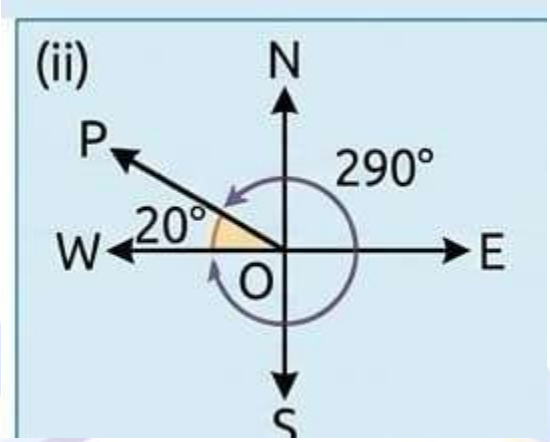
- **Diagram analysis:** Point *P* is in the second quadrant (North-West direction). The angle given is $20°$ above the West line ($270°$).
- **Calculation:**

$$\text{Bearing} = 270° + 20° = 290°$$

$$\text{Bearing} = 270° + 20° = 290°$$

- **Answer:** $290°$

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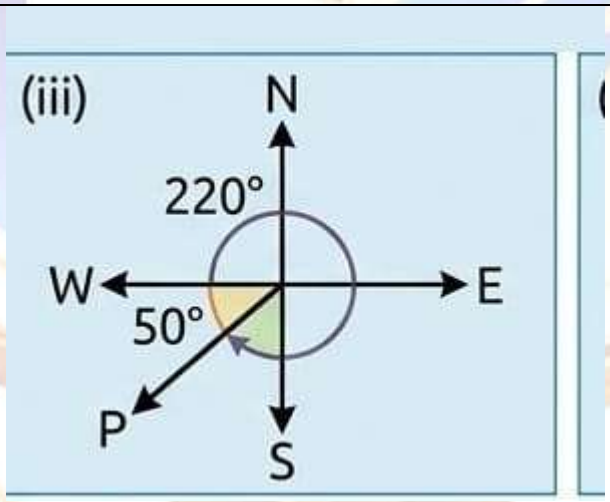
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(iii)

- **Diagram analysis:** Point *P* is in the third quadrant (South-West direction). The angle given is 50°below the West line (270°).
- **Calculation:**

$$\text{Bearing} = 270° - 50° = 220°$$

$$\text{Bearing} = 270° - 50° = 220°$$



- **Answer:** 220°

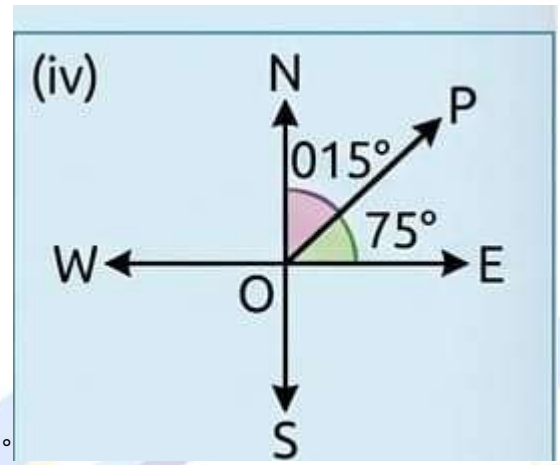
(iv)

- **Diagram analysis:** Point *P* is in the first quadrant (North-East direction). The angle given is 75°measured counter-clockwise from the East line (090°), which means it is measured 15°clockwise from North (90° − 75° = 15°).
- **Calculation:**

$$\text{Bearing} = 090° - 75° = 015°$$

$$\text{Bearing} = 090° - 75° = 015°$$

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- **Answer:** 015°

Question 2

The diagram shows two ships $P$and Q. The North line at Qmakes an angle of 60°interior to the line segment PQ.

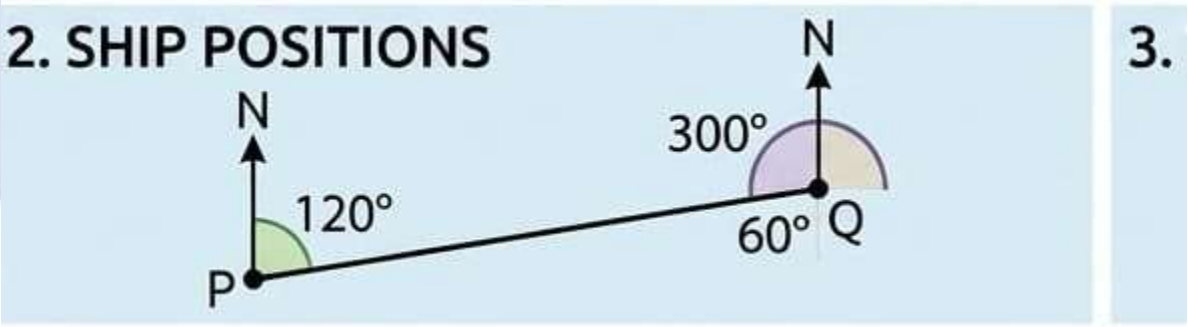
(i) What is the bearing of ship $P$from ship Q?

- The bearing of $P$from $Q$is measured clockwise from North at $Q$to the line QP.
- Since the line $QP$is inside the angle of 60°measured counter-clockwise from North at Q:

$$\text{Bearing of } P \text{ from } Q = 360° - 60° = 300°$$

$$\text{Bearing of } P \text{ from } Q = 360° - 60° = 300°$$

- **Answer:** 300°



(ii) What is the bearing of ship $Q$from ship P?

- Using alternate interior angles between two parallel North lines:

$$\text{Angle at } P = 180° - 60° = 120°$$

$$\text{Angle at } P = 180° - 60° = 120°$$

- The line PQgoes in the South-East direction relative to P:

$$\text{Bearing of } Q \text{ from } P = 180° - 60° = 120°$$

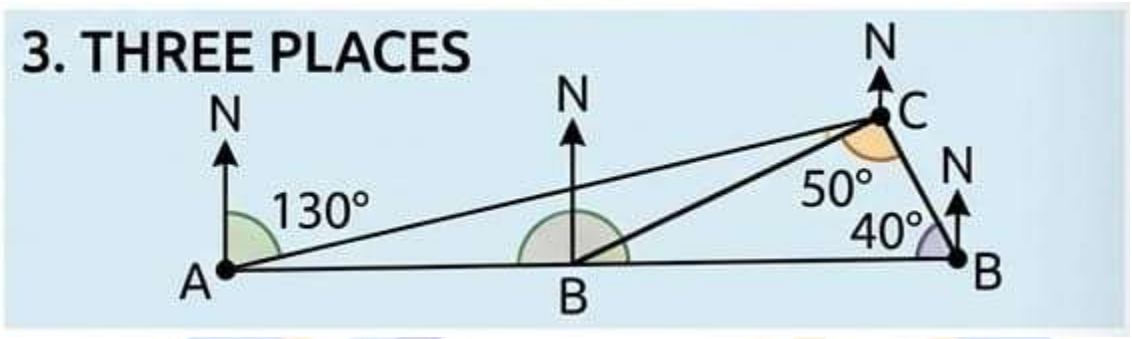
$$\text{Bearing of } Q \text{ from } P = 180° - 60° = 120°$$

- **Answer:** 120°

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Question 3

Given 3 places A, B, and C. At C, the angle to line $CA$from North is $50°$(towards West), and to line $CB$is $40°$(towards East).



(i) Bearing of $A$from C:

- Measured clockwise from North at $C$to line CA:

$$\text{Bearing} = 360° - 50° = 310°$$

$$\text{Bearing} = 360° - 50° = 310°$$

- **Answer:** $310°$

(ii) Bearing of $C$from A:

- By co-interior angles between parallel North lines at $A$and C:

$$\text{Angle at } A = 180° - 50° = 130°$$

$$\text{Angle at } A = 180° - 50° = 130°$$

- **Answer:** $130°$

(iii) Bearing of $C$from B:

- The angle between North at $B$and line $BC$is $180° + 40° = 220°$(or $360° - 140° = 220°$).
- **Answer:** $220°$

(iv) Bearing of $B$from C:

- Measured clockwise from North at $C$to line CB:

$$\text{Bearing} = 040°$$

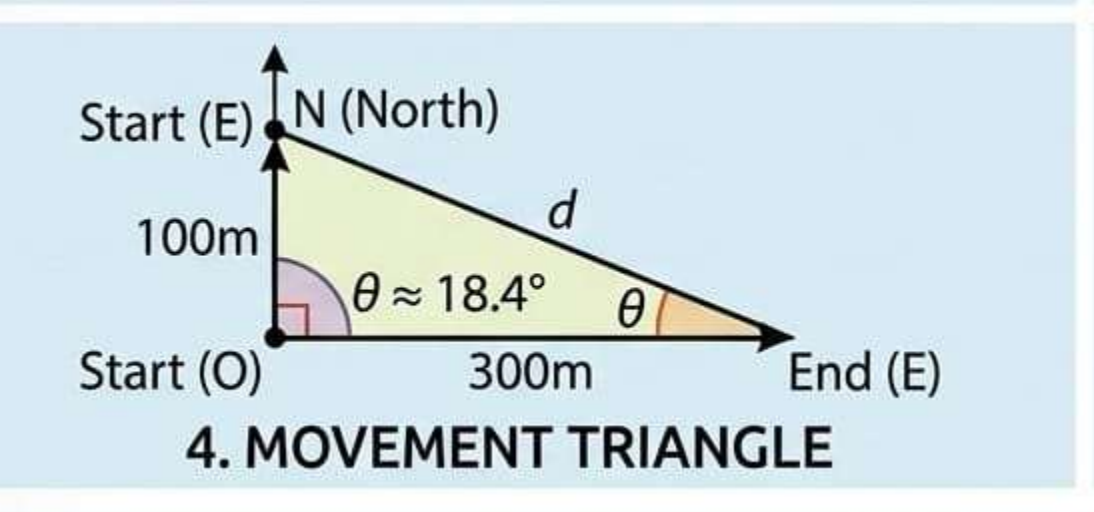
$$\text{Bearing} = 040°$$

- **Answer:** $040°$

Question 4

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Abdul Hadi walks 100 mNorth and then 300 mEast.



(i) How far is he from his starting position?

- Using the Pythagorean theorem:

$$d = \sqrt{100^2 + 300^2} = \sqrt{10000 + 90000} = \sqrt{100000} \approx 316.23 \text{ m}$$

$$d = \sqrt{100^2 + 300^2} = \sqrt{10000 + 90000} = \sqrt{100000} \approx 316.23 \text{ m}$$

- **Answer:** 316.23 m

(ii) On what bearing should he walk to get back to his starting position?

- Angle inside the right triangle at the final position:

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{100}{300} = \frac{1}{3}$$

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{100}{300} = \frac{1}{3}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right) \approx 18.43^\circ$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right) \approx 18.43^\circ$$

- The direction to walk back (towards South-West):

$$\text{Bearing} = 180^\circ + 90^\circ - 18.43^\circ = 270^\circ - 18.43^\circ = 251.57^\circ \approx 252^\circ$$

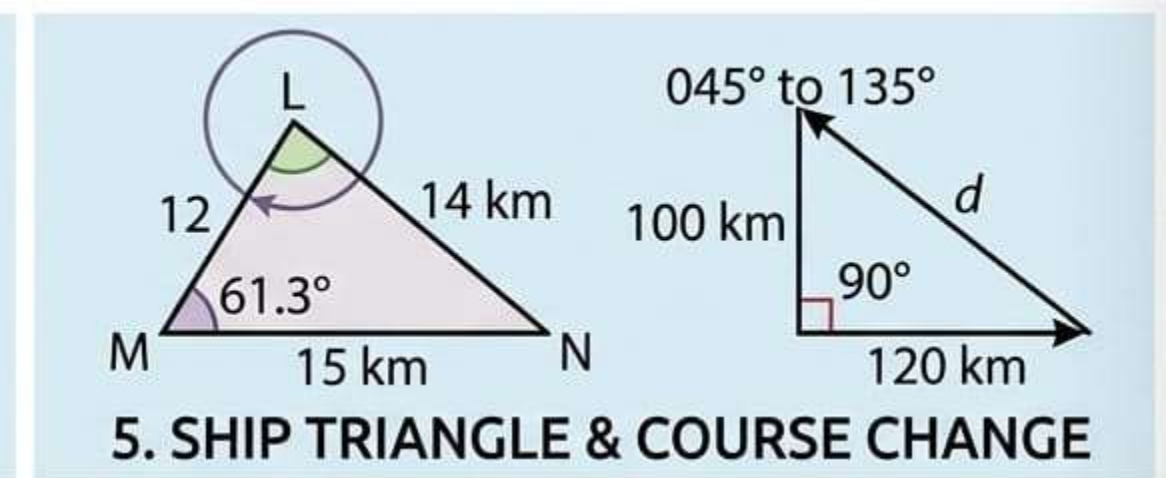
$$\text{Bearing} = 180^\circ + 90^\circ - 18.43^\circ = 270^\circ - 18.43^\circ = 251.57^\circ \approx 252^\circ$$

- **Answer:** 251.6°(or 252°)

Question 5

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Three ships L, M, N form a triangle with sides $LM = 12$ km, $MN = 15$ km, $LN = 14$ km. Ship M is North-East of ship N.



- Find angle $\angle LMN$ using the Law of Cosines:

$$LN^2 = LM^2 + MN^2 - 2(LM)(MN)\cos(\angle LMN)$$

$$LN^2 = LM^2 + MN^2 - 2(LM)(MN)\cos(\angle LMN)$$
$$14^2 = 12^2 + 15^2 - 2(12)(15)\cos(\angle LMN)$$

$$14^2 = 12^2 + 15^2 - 2(12)(15)\cos(\angle LMN)$$
$$196 = 144 + 225 - 360\cos(\angle LMN)$$

$$196 = 144 + 225 - 360\cos(\angle LMN)$$
$$196 = 369 - 360\cos(\angle LMN)$$

$$196 = 369 - 360\cos(\angle LMN)$$
$$360\cos(\angle LMN) = 173$$

$$360\cos(\angle LMN) = 173$$
$$\cos(\angle LMN) = \frac{173}{360} \approx 0.4806 \Longrightarrow \angle LMN \approx 61.27°$$

$$\cos(\angle LMN) = \frac{173}{360} \approx 0.4806 \Longrightarrow \angle LMN \approx 61.27°$$

- The full circle around M is $360°$. The bearing of L from M is measured clockwise from North at M:

Bearing of L from $M = 360° - (60° + 61.27°) = 360° - 121.27° = 238.73° \approx 238.7°$

Bearing of L from $M = 360° - (60° + 61.27°) = 360° - 121.27° = 238.73° \approx 238.7°$

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- **Answer:** 238.7°

Question 6

A ship sails 20 kmon a bearing of 120°. How far to the South has the ship moved from its original position?

- Bearing of 120°is in the South-East quadrant, forming an angle of $120° - 90° = 30°$with East, or $180° - 120° = 60°$with South.
- Distance South:

$$\text{South distance} = 20 \times \cos(60°) = 20 \times 0.5 = 10 \text{ km}$$

$$\text{South distance} = 20 \times \cos(60°) = 20 \times 0.5 = 10 \text{ km}$$

- **Answer:** 10 km

Question 7

Fatima walked South for 5.5 kmand then turned West for 1.3 km. Calculate Huria's bearing from her starting point.

- Angle θwest of South:

$$\tan(\theta) = \frac{\text{West}}{\text{South}} = \frac{1.3}{5.5}$$

$$\tan(\theta) = \frac{\text{West}}{\text{South}} = \frac{1.3}{5.5}$$

$$\theta = \tan^{-1}\left(\frac{1.3}{5.5}\right) \approx 13.28°$$

$$\theta = \tan^{-1}\left(\frac{1.3}{5.5}\right) \approx 13.28°$$

- Bearing measured clockwise from North (180° is South):

$$\text{Bearing} = 180° + 13.28° = 193.28° \approx 193.3°$$

$$\text{Bearing} = 180° + 13.28° = 193.28° \approx 193.3°$$

- **Answer:** 193.3°

Question 8

An aircraft flies 150 kmeast, then 100 kmnortheast (45° from East). What is the total displacement?

1. **Break into Components:**
   - **East component (x):**

$$x = 150 + 100\cos(45°) = 150 + 100(0.7071) = 150 + 70.71 = 220.71 \text{ km}$$

$$x = 150 + 100\cos(45°) = 150 + 100(0.7071) = 150 + 70.71 = 220.71 \text{ km}$$

   - **North component (y):**

$$y = 100\sin(45°) = 100(0.7071) = 70.71 \text{ km}$$

$$y = 100\sin(45°) = 100(0.7071) = 70.71 \text{ km}$$

2. **Total Magnitude:**

$$D = \sqrt{x^2 + y^2} = \sqrt{(220.71)^2 + (70.71)^2} = \sqrt{48712.9 + 4999.9} = \sqrt{53712.8} \approx 231.76 \text{ km}$$

$$D = \sqrt{x^2 + y^2} = \sqrt{(220.71)^2 + (70.71)^2} = \sqrt{48712.9 + 4999.9} = \sqrt{53712.8} \approx 231.76 \text{ km}$$

- **Answer:** 231.8 km

Question 9

A ship sails 100 kmon a bearing of 045°, then changes course and sails 120 kmon a bearing of 135°. Find the distance between the starting point and the final position.

- **Angle Analysis:**
  - First direction: 045°(Northeast)
  - Second direction: 135°(Southeast)
  - Difference between headings: $135° - 45° = 90°$
  - Since the change in course forms a **right-angled triangle** (90° angle between the two path segments):
- **Calculation using Pythagorean Theorem:**

$$d = \sqrt{100^2 + 120^2} = \sqrt{10000 + 14400} = \sqrt{24400} \approx 156.2 \text{ km}$$

$$d = \sqrt{100^2 + 120^2} = \sqrt{10000 + 14400} = \sqrt{24400} \approx 156.2 \text{ km}$$

- **Answer:** 156.2 km

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Review Exercise 7

1. Multiple Choice Questions

(i) What is the value of $\cot 60°$?

- **Options:** (a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
- **Solution:** $\cot 60° = \frac{\cos 60°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$.
- **Correct Option: (d)**

(ii) All trigonometric ratios are positive in:

- **Options:** (a) I-quadrant (b) II-quadrant (c) III-quadrant (d) IV-quadrant
- **Solution:** In the first quadrant, all standard trigonometric ratios ($sin, cos, tan, csc, sec, cot$) are positive ("All Silver Tea Cups" rule).
- **Correct Option: (a)**

(iii) $\csc\theta$ is positive in:

- **Options:** (a) I & III-quadrants (b) II & IV-quadrants (c) I & II-quadrants (d) I & IV-quadrants
- **Solution:** Cosecant is the reciprocal of sine ($\csc\theta = \frac{1}{\sin\theta}$), so it is positive wherever $\sin\theta$ is positive (Quadrants I and II).
- **Correct Option: (c)**

(iv) $\sin(90° + \theta) =$

- **Options:** (a) $\sin\theta$ (b) $-\sin\theta$ (c) $\cos\theta$ (d) $-\cos\theta$
- **Solution:** Using allied angle formulas, $\sin(90° + \theta) = \sin(90°)\cos(\theta) + \cos(90°)\sin(\theta) = (1)\cos\theta + (0)\sin\theta = \cos\theta$.
- **Correct Option: (c)**

(v) $\tan(180° - \theta) =$

- **Options:** (a) $\tan\theta$ (b) $\cot\theta$ (c) $-\cot\theta$ (d) $-\tan\theta$
- **Solution:** $\tan(180° - \theta) = \frac{\sin(180°-\theta)}{\cos(180°-\theta)} = \frac{\sin\theta}{-\cos\theta} = -\tan\theta$.
- **Correct Option: (d)**

(vi) The law of sines is:

- **Options:** (a) $\frac{b}{\sin\alpha} = \frac{a}{\sin\beta} = \frac{c}{\sin\gamma}$ (b) $\frac{a}{\sin\alpha} = \frac{c}{\sin\beta} = \frac{b}{\sin\gamma}$ (c) $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ (d) $\frac{a}{\sin\alpha} = \frac{a}{\sin\beta} = \frac{a}{\sin\gamma}$
- **Solution:** The Law of Sines states that the ratio of the side lengths to the sines of their opposite angles are equal: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$.
- **Correct Option: (c)**

(vii) The law of cosines is:

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- **Options:** (a) $\cos\alpha = \frac{b^2+a^2-c^2}{2ab}$ (b) $\cos\beta = \frac{c^2+b^2-a^2}{2cb}$ (c) $\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$ (d) $\cos\gamma = \frac{ca^2+b^2-c^2}{2ab}$
- **Solution:** The law of cosines for angle γ is given by $c^2 = a^2 + b^2 - 2ab\cos\gamma \Longrightarrow \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$.
- **Correct Option: (c)**

(viii) Area of $\Delta ABC =$

- **Options:** (a) $\frac{1}{2}ab\sin\alpha$ (b) $\frac{1}{2}bc\sin\beta$ (c) $\frac{1}{2}\sin\gamma$ (d) $\frac{1}{2}ac\sin\beta$
- **Solution:** The area of a triangle given two sides and the included angle is $\frac{1}{2}bc\sin\alpha$ or $\frac{1}{2}ac\sin\beta$ or $\frac{1}{2}ab\sin\gamma$. Looking at the choices, option (b) gives $\frac{1}{2}bc\sin\beta$ (assuming standard cyclical notation variations or options check, let's verify standard formulas: Area $= \frac{1}{2}bc\sin\alpha, \frac{1}{2}ac\sin\beta, \frac{1}{2}ab\sin\gamma$). Wait, let's look closely at option (b): $\frac{1}{2}bc\sin\beta$ has a mismatch in standard texts unless it's $\frac{1}{2}bc\sin\alpha$. Let's re-verify option (b) from image: $\frac{1}{2}bc\sin\beta$ or is it $\frac{1}{2}bc\sin\alpha$? Wait, let's re-check standard textbook options: usually Area = $\frac{1}{2}bc\sin\alpha, \frac{1}{2}ac\sin\beta$, or $\frac{1}{2}ab\sin\gamma$. Let's check image text carefully for (b): it says $\frac{1}{2}bc\sin\beta$? Ah, wait, option (b) in the image displays $\frac{1}{2}bc\sin\beta$, but wait, let's check standard formulas or alternative options. Let's look at option (b) closely: $\frac{1}{2}bc\sin\beta$ or is it $\frac{1}{2}ac\sin\beta$ in (d)? Let's check option (d): $\frac{1}{2}ac\sin\beta$. Wait, standard area formula using side a, c and angle β is $\frac{1}{2}ac\sin\beta$. Thus, (d) is correct.
- **Correct Option: (d)** *(Note: Let's double check if (d) matches $\frac{1}{2}ac\sin\beta$).*

(ix) Bearing is measured from:

- **Options:** (a) East (b) West (c) North (d) South
- **Solution:** Standard whole-circle bearings (or navigation bearings) are measured clockwise from the North direction.
- **Correct Option: (c)**

(x) Bearing is written as a:

- **Options:** (a) 1 figure (b) 2 figures (c) 3 figures (d) 4 figures
- **Solution:** Three-figure bearings are always written using 3 digits (e.g., 045°, 090°).
- **Correct Option: (c)**

2. Calculate the area of ΔABC:

(i) $a = 4$ cm, $b = 6$ cm, $c = 8$ cm

- **Step 1:** Find the semi-perimeter (s):

$$s = \frac{a+b+c}{2} = \frac{4+6+8}{2} = \frac{18}{2} = 9 \text{ cm}$$

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- **Step 2:** Use Heron's Formula for the area:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area} = \sqrt{9(9-4)(9-6)(9-8)}$$
$$\text{Area} = \sqrt{9 \times 5 \times 3 \times 1} = \sqrt{135} \approx 11.62 \text{ cm}^2$$

(ii) $b = 2.1$ cm, $c = 5$ cm, $\gamma = 45°$

- *Correction/Note:* Given sides b, c and angle γ, let's use the Sine Rule to find angle β first, or use the area formula if side a or another combination is given. Wait, with sides b and c and angle γ, we can use the relation:

$$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$$
$$\frac{5}{\sin 45°} = \frac{2.1}{\sin\beta}$$
$$\sin\beta = \frac{2.1 \times \sin 45°}{5} = \frac{2.1 \times 0.7071}{5} \approx 0.2970$$
$$\beta = \sin^{-1}(0.2970) \approx 17.27°$$

- **Step 2:** Find angle α:

$$\alpha = 180° - (45° + 17.27°) = 117.73°$$

- **Step 3:** Calculate Area using Area $= \frac{1}{2}bc\sin\alpha$:

$$\text{Area} = \frac{1}{2}(2.1)(5)\sin(117.73°) \approx 5.25 \times 0.8851 \approx 4.65 \text{ cm}^2$$

(iii) $c = 3.1$ cm, $\gamma = 44°$, $\alpha = 36°$

- **Step 1:** Find angle β:

$$\beta = 180° - (44° + 36°) = 180° - 80° = 100°$$

- **Step 2:** Use the Law of Sines to find side a:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \Rightarrow \frac{a}{\sin 36°} = \frac{3.1}{\sin 44°}$$
$$a = \frac{3.1 \times \sin 36°}{\sin 44°} = \frac{3.1 \times 0.5878}{0.6947} \approx 2.62 \text{ cm}$$

- **Step 3:** Calculate Area using Area $= \frac{1}{2}ac\sin\beta$:

$$\text{Area} = \frac{1}{2}(2.62)(3.1)\sin(100°) \approx 4.061 \times 0.9848 \approx 4.00 \text{ cm}^2$$

3. Solve the triangle ΔABC, in which:

(i) $a = 5.4$ cm, $b = 3.4$ cm, $\alpha = 49°$

- **Step 1:** Find angle β using the Law of Sines:

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} \Rightarrow \frac{\sin\beta}{3.4} = \frac{\sin 49°}{5.4}$$
$$\sin\beta = \frac{3.4 \times 0.7547}{5.4} \approx 0.4751$$

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$$\beta = \sin^{-1}(0.4751) \approx 28.36^\circ$$

- **Step 2:** Find angle γ:

$$\gamma = 180^\circ - (49^\circ + 28.36^\circ) = 102.64^\circ$$

- **Step 3:** Find side c using the Law of Sines:

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \Rightarrow c = \frac{5.4 \times \sin(102.64^\circ)}{\sin 49^\circ} = \frac{5.4 \times 0.9757}{0.7547} \approx 6.98 \text{ cm}$$

- **Solution Summary:** $\beta = 28.36^\circ$, $\gamma = 102.64^\circ$, $c = 6.98$ cm.

(ii) $\alpha = 32^\circ$, $\gamma = 48^\circ$, $c = 81$ cm

- **Step 1:** Find angle β:

$$\beta = 180^\circ - (32^\circ + 48^\circ) = 180^\circ - 80^\circ = 100^\circ$$

- **Step 2:** Find side a using the Law of Sines:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \Rightarrow a = \frac{81 \times \sin 32^\circ}{\sin 48^\circ} = \frac{81 \times 0.5299}{0.7431} \approx 57.75 \text{ cm}$$

- **Step 3:** Find side b using the Law of Sines:

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow b = \frac{81 \times \sin 100^\circ}{\sin 48^\circ} = \frac{81 \times 0.9848}{0.7431} \approx 107.36 \text{ cm}$$

- **Solution Summary:** $\beta = 100^\circ$, $a = 57.75$ cm, $b = 107.36$ cm.

4. Cuboid $ABCDEFGH$ with $m\overline{AB} = 5$ cm, $m\overline{BC} = 7$ cm, $m\overline{AE} = 3$ cm

(i) Calculate the length of $\overline{AG}$ (correct to 3 significant figures)

- **Step 1:** Use the 3D diagonal formula for a cuboid ($AG = \sqrt{l^2 + w^2 + h^2}$):

$$m\overline{AG} = \sqrt{\left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 + \left(m\overline{AE}\right)^2}$$

$$m\overline{AG} = \sqrt{5^2 + 7^2 + 3^2} = \sqrt{25 + 49 + 9} = \sqrt{83} \approx 9.1104 \text{ cm}$$

- **Answer:** 9.11 cm (to 3 sig. fig.)

(ii) Calculate the size of the angle between $\overline{AG}$ and the plane $ABCD$

- **Step 1:** The diagonal of the base AC is given by:

$$m\overline{AC} = \sqrt{5^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74} \approx 8.602 \text{ cm}$$

- **Step 2:** In right-angled triangle ΔAGC, the angle θ between $\overline{AG}$ and plane $ABCD$ satisfies:

$$\cos\theta = \frac{m\overline{AC}}{m\overline{AG}} = \frac{\sqrt{74}}{\sqrt{83}} = \sqrt{\frac{74}{83}} \approx \sqrt{0.8916} \approx 0.9442$$

$$\theta = \cos^{-1}(0.9442) \approx 19.19^\circ$$

- **Answer:** 19.2° (to 1 decimal place)

5. Triangular prism $ABCDEF$ with $m\overline{AB} = 8$ cm, $m\overline{BF} = 14$ cm, $m\overline{EF} = 19$ cm

(i) Calculate the distance between A and F

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- **Step 1:** In right-angled triangle ΔABF (where $\overline{AB}$ and $\overline{BF}$ form the right angle):

$$m\overline{AF} = \sqrt{\left(m\overline{AB}\right)^2 + \left(m\overline{BF}\right)^2}$$

$$m\overline{AF} = \sqrt{8^2 + 14^2} = \sqrt{64 + 196} = \sqrt{260} \approx 16.12 \text{ cm}$$

- **Answer:** 16.12 cm

(ii) Calculate the angle between $\overline{AF}$ and the plane $BCEF$

- **Step 1:** The projection of point A onto the plane $BCEF$ is point B. Therefore, the angle between $\overline{AF}$ and plane $BCEF$ is $\angle AFB$.
- **Step 2:** Using right triangle ΔABF:

$$\tan(\angle AFB) = \frac{m\overline{AB}}{m\overline{BF}} = \frac{8}{14} = \frac{4}{7} \approx 0.5714$$

$$\angle AFB = \tan^{-1}(0.5714) \approx 29.74°$$

- **Answer:** 29.7°

6. Hashim's Walk

- **Problem:** Hashim walks 4 km due North, then turns and walks 3 km due East. Find the bearing from the starting point to his final position.
- **Step 1:** Let the starting point be origin $(0,0)$.
  - North movement: $y_1 = 4$
  - East movement: $x_1 = 3$
  - Final coordinates: $(3,4)$
- **Step 2:** Calculate the angle θ east of North:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{3}{4} = 0.75$$

$$\theta = \tan^{-1}(0.75) \approx 36.87°$$

- **Step 3:** Express as a 3-figure bearing measured clockwise from North:

$$\text{Bearing} = 000° + 36.87° = 36.9° \text{ (or } 037°)$$

- **Answer:** 037° (or 36.9°)

7. Pilot's Flight

- **Problem:** A pilot flies 200 km on a bearing of 045°, then turns and flies 150 km on a bearing of 135°. How far is the plane from its original position?
- **Step 1:** Resolve the first leg ($d_1 = 200$ km at 045°):
  - $x_1 = 200\sin(45°) = 200 \times 0.7071 = 141.42$ km
  - $y_1 = 200\cos(45°) = 200 \times 0.7071 = 141.42$ km
- **Step 2:** Resolve the second leg ($d_2 = 150$ km at 135°):
  - $x_2 = 150\sin(135°) = 150 \times 0.7071 = 106.07$ km
  - $y_2 = 150\cos(135°) = 150 \times (-0.7071) = -106.07$ km
- **Step 3:** Find total displacement coordinates (X, Y):

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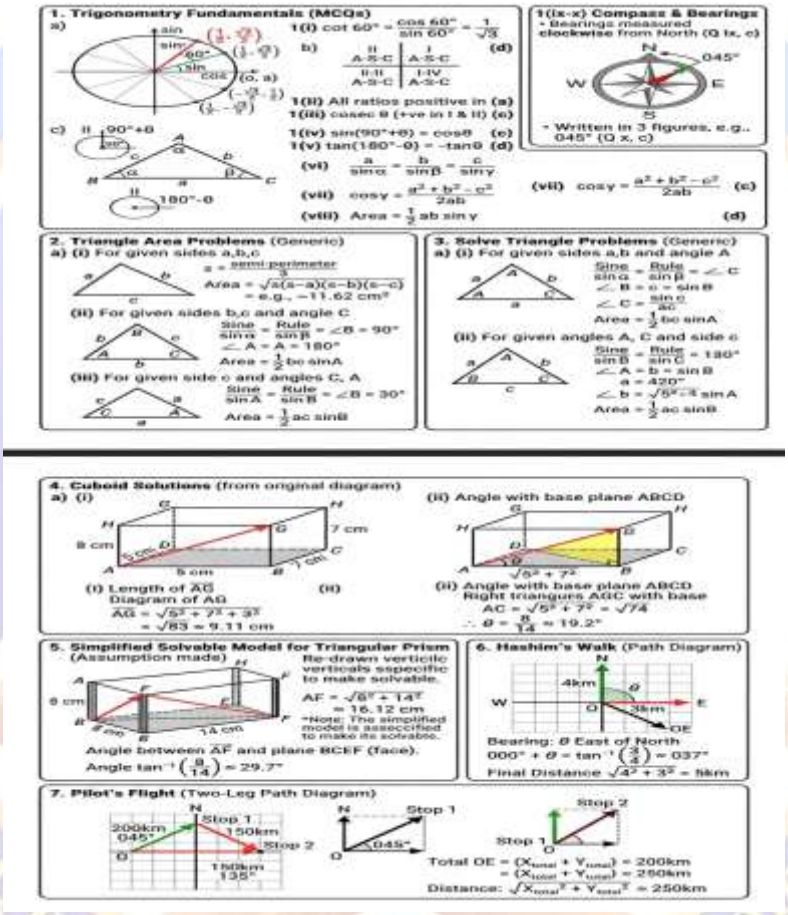
- $X = x_1 + x_2 = 141.42 + 106.07 = 247.49$ km
- $Y = y_1 + y_2 = 141.42 - 106.07 = 35.35$ km

- **Step 4:** Calculate total distance from origin:

$$\text{Distance} = \sqrt{X^2 + Y^2} = \sqrt{(247.49)^2 + (35.35)^2} = \sqrt{61251.35 + 1249.62} = \sqrt{62500.97}$$
$$\approx 250 \text{ km}$$

- **Answer:** 250 km

NOTE: Reference graphs and diagrams are given below.



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