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Class	10th	Chapter	01
Subject	Maths	Board	Punjab

Chapter 01: Complex Numbers

EXERCISE 1.1

1. Simplify the following:

(i) i^5

$$i^5 = i^4 \times i = (i^2)^2 \times i = (-1)^2 \times i = 1 \times i = i$$

$$i^5 = i^4 \times i = (i^2)^2 \times i = (-1)^2 \times i = 1 \times i = i$$

(ii) i^{16}

$$i^{16} = (i^2)^8 = (-1)^8 = 1$$

$$i^{16} = (i^2)^8 = (-1)^8 = 1$$

(iii) $(-i)^{-19}$

$$(-i)^{-19} = \frac{1}{(-i)^{19}} = \frac{1}{(-1)^{19} \cdot i^{19}} = \frac{1}{-1 \cdot (i^{18} \cdot i)} = \frac{1}{-1 \cdot ((i^2)^9 \cdot i)} = \frac{1}{-1 \cdot ((-1)^9 \cdot i)}$$
$$= \frac{1}{-1 \cdot (-1) \cdot i} = \frac{1}{i}$$

$$(-i)^{-19} = \frac{1}{(-i)^{19}} = \frac{1}{(-1)^{19} \cdot i^{19}} = \frac{1}{-1 \cdot (i^{18} \cdot i)} = \frac{1}{-1 \cdot ((i^2)^9 \cdot i)} = \frac{1}{-1 \cdot ((-1)^9 \cdot i)}$$
$$= \frac{1}{-1 \cdot (-1) \cdot i} = \frac{1}{i}$$

Multiply numerator and denominator by i:

$$\frac{1}{i} = \frac{1 \times i}{i \times i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

$$\frac{1}{i} = \frac{1 \times i}{i \times i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

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(iv) $27i^{-26}$

$$27i^{-26} = \frac{27}{i^{26}} = \frac{27}{(i^2)^{13}} = \frac{27}{(-1)^{13}} = \frac{27}{-1} = -27$$

$$27i^{-26} = \frac{27}{i^{26}} = \frac{27}{(i^2)^{13}} = \frac{27}{(-1)^{13}} = \frac{27}{-1} = -27$$

(v) $i^{11} + i^5$

- $i^{11} = i^{10} \times i = (i^2)^5 \times i = (-1)^5 \times i = -i$
- $i^5 = i^4 \times i = (i^2)^2 \times i = (-1)^2 \times i = i$

$$i^{11} + i^5 = -i + i = 0$$

$$i^{11} + i^5 = -i + i = 0$$

(vi) $(i^4 + i^3 + i^2 + i)^2$

Simplify the terms inside the parentheses first:

- $i^4 = (i^2)^2 = (-1)^2 = 1$
- $i^3 = i^2 \cdot i = -1 \cdot i = -i$
- $i^2 = -1$
- $i = i$

Inner expression:

$$1 + (-i) + (-1) + i = 1 - i - 1 + i = 0$$

$$1 + (-i) + (-1) + i = 1 - i - 1 + i = 0$$

Therefore:

$$(0)^2 = 0$$

$$(0)^2 = 0$$

(vii) $\left(\frac{i^8}{i^5}\right)^{-5}$

Simplify inside the parentheses:

$$\frac{i^8}{i^5} = i^{8-5} = i^3 = -i$$

$$\frac{i^8}{i^5} = i^{8-5} = i^3 = -i$$

Now evaluate the power:

$$(-i)^{-5} = \frac{1}{(-i)^5} = \frac{1}{(-1)^5 \cdot i^5} = \frac{1}{-1 \cdot (i^4 \cdot i)} = \frac{1}{-1 \cdot (1 \cdot i)} = \frac{1}{-i} = -\frac{1}{i}$$

$$(-i)^{-5} = \frac{1}{(-i)^5} = \frac{1}{(-1)^5 \cdot i^5} = \frac{1}{-1 \cdot (i^4 \cdot i)} = \frac{1}{-1 \cdot (1 \cdot i)} = \frac{1}{-i} = -\frac{1}{i}$$

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Multiply numerator and denominator by i:

$$-\frac{1 \cdot i}{i \cdot i} = -\frac{i}{i^2} = -\frac{i}{-1} = i$$

$$-\frac{1 \cdot i}{i \cdot i} = -\frac{i}{i^2} = -\frac{i}{-1} = i$$

(viii) $i^{13} \times i^{29}$

Using exponent rules:

$$i^{13} \times i^{29} = i^{13+29} = i^{42}$$

$$i^{13} \times i^{29} = i^{13+29} = i^{42}$$

$$i^{42} = (i^2)^{21} = (-1)^{21} = -1$$

$$i^{42} = (i^2)^{21} = (-1)^{21} = -1$$

2. Write in terms of i:

(i) $2 + \sqrt{-4}$

$$2 + \sqrt{4 \times (-1)} = 2 + \sqrt{4}\sqrt{-1} = 2 + 2i$$

$$2 + \sqrt{4 \times (-1)} = 2 + \sqrt{4}\sqrt{-1} = 2 + 2i$$

(ii) $3 - \sqrt{-7}$

$$3 - \sqrt{7 \times (-1)} = 3 - \sqrt{7}\sqrt{-1} = 3 - i\sqrt{7}$$

$$3 - \sqrt{7 \times (-1)} = 3 - \sqrt{7}\sqrt{-1} = 3 - i\sqrt{7}$$

(iii) $\frac{2}{5} + \frac{\sqrt{-16}}{5}$

$$\frac{2}{5} + \frac{\sqrt{16 \times (-1)}}{5} = \frac{2}{5} + \frac{4i}{5}$$

$$\frac{2}{5} + \frac{\sqrt{16 \times (-1)}}{5} = \frac{2}{5} + \frac{4i}{5}$$

(iv) $\sqrt{2} - \sqrt{-3}$

$$\sqrt{2} - \sqrt{3 \times (-1)} = \sqrt{2} - i\sqrt{3}$$

$$\sqrt{2} - \sqrt{3 \times (-1)} = \sqrt{2} - i\sqrt{3}$$

3. Find the values of $x$and y:

(i) $(2x + 5) + (y - 3)i = 1 + 2i$

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Equating real and imaginary parts:

- **Real parts:**

$$2x + 5 = 1 \Longrightarrow 2x = 1 - 5 = -4 \Longrightarrow x = -2$$

$$2x + 5 = 1 \Longrightarrow 2x = 1 - 5 = -4 \Longrightarrow x = -2$$

- **Imaginary parts:**

$$y - 3 = 2 \Longrightarrow y = 2 + 3 \Longrightarrow y = 5$$

$$y - 3 = 2 \Longrightarrow y = 2 + 3 \Longrightarrow y = 5$$

$$x = -2,\ y = 5$$

$$\mathbf{x = -2,\quad y = 5}$$

(ii) $\boldsymbol{(3x + 2) - (4 - y)i = 5 + 3i}$

Rearrange the imaginary term: $(3x + 2) + (y - 4)i = 5 + 3i$Equating real and imaginary parts:

- **Real parts:**

$$3x + 2 = 5 \Longrightarrow 3x = 3 \Longrightarrow x = 1$$

$$3x + 2 = 5 \Longrightarrow 3x = 3 \Longrightarrow x = 1$$

- **Imaginary parts:**

$$y - 4 = 3 \Longrightarrow y = 7$$

$$y - 4 = 3 \Longrightarrow y = 7$$

$$x = 1,\ y = 7$$

$$\mathbf{x = 1,\quad y = 7}$$

(iii) $\boldsymbol{(2 + i)x + (1 - 2i)y = 3 + 4i}$

Expand the left side:

$$2x + xi + y - 2yi = 3 + 4i$$

$$2x + xi + y - 2yi = 3 + 4i$$

$$(2x + y) + (x - 2y)i = 3 + 4i$$

$$(2x + y) + (x - 2y)i = 3 + 4i$$

Equating real and imaginary parts:

1. $2x + y = 3 \Longrightarrow y = 3 - 2x$--- (Equation 1)
2. $x - 2y = 4$--- (Equation 2)

Substitute Eq. 1 into Eq. 2:

$$x - 2(3 - 2x) = 4$$

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$$x - 2(3 - 2x) = 4$$
$$x - 6 + 4x = 4 \Longrightarrow 5x = 10 \Longrightarrow x = 2$$

$$x - 6 + 4x = 4 \Longrightarrow 5x = 10 \Longrightarrow x = 2$$

Substitute $x = 2$back into Eq. 1:

$$y = 3 - 2(2) = 3 - 4 = -1$$

$$y = 3 - 2(2) = 3 - 4 = -1$$
$$x = 2,\ y = -1$$

$$\mathbf{x = 2,\quad y = -1}$$

(iv) $\boldsymbol{(3x - 1) + (2y - 3)i = 8 + 7i}$

Equating real and imaginary parts:

- **Real parts:**

$$3x - 1 = 8 \Longrightarrow 3x = 9 \Longrightarrow x = 3$$

$$3x - 1 = 8 \Longrightarrow 3x = 9 \Longrightarrow x = 3$$

- **Imaginary parts:**

$$2y - 3 = 7 \Longrightarrow 2y = 10 \Longrightarrow y = 5$$

$$2y - 3 = 7 \Longrightarrow 2y = 10 \Longrightarrow y = 5$$
$$x = 3,\ y = 5$$

$$\mathbf{x = 3,\quad y = 5}$$

(v) $\boldsymbol{(1 - i)x + (2 + i)y = 4 - i}$

Expand the left side:

$$x - xi + 2y + yi = 4 - i$$

$$x - xi + 2y + yi = 4 - i$$
$$(x + 2y) + (-x + y)i = 4 - i$$

$$(x + 2y) + (-x + y)i = 4 - i$$

Equating real and imaginary parts:

1. $x + 2y = 4$--- (Equation 1)
2. $-x + y = -1 \Longrightarrow y = x - 1$--- (Equation 2)

Substitute Eq. 2 into Eq. 1:

$$x + 2(x - 1) = 4$$

$$x + 2(x - 1) = 4$$

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$$x + 2x - 2 = 4 \Longrightarrow 3x = 6 \Longrightarrow x = 2$$

$$x + 2x - 2 = 4 \Longrightarrow 3x = 6 \Longrightarrow x = 2$$

Substitute $x = 2$back into Eq. 2:

$$y = 2 - 1 = 1$$

$$y = 2 - 1 = 1$$

$$x = 2,\ y = 1$$

$$\mathbf{x = 2,\quad y = 1}$$

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EXERCISE 1.2

1. Simplify and write in the form $a + bi$:

(i) $(2 + 5i) + (3 - zi)$

$$(2 + 3) + (5 - z)i = 5 + (5 - z)i$$

$$(2 + 3) + (5 - z)i = 5 + (5 - z)i$$

(ii) $(16 - 3i) + (9 + 2i)$

$$(16 + 9) + (-3 + 2)i = 25 - i$$

$$(16 + 9) + (-3 + 2)i = 25 - i$$

(iii) $(9 - 2i) - (7 - 3i)$

$$9 - 2i - 7 + 3i = (9 - 7) + (-2 + 3)i = 2 + i$$

$$9 - 2i - 7 + 3i = (9 - 7) + (-2 + 3)i = 2 + i$$

(iv) $(11 + 9i) - (9 - 7i)$

$$11 + 9i - 9 + 7i = (11 - 9) + (9 + 7)i = 2 + 16i$$

$$11 + 9i - 9 + 7i = (11 - 9) + (9 + 7)i = 2 + 16i$$

(v) $(3 + 4i)(2 - 3i)$

$$= 3(2 - 3i) + 4i(2 - 3i) = 6 - 9i + 8i - 12i^2$$

$$= 3(2 - 3i) + 4i(2 - 3i) = 6 - 9i + 8i - 12i^2$$

Since $i^2 = -1$:

$$= 6 - i - 12(-1) = 6 - i + 12 = 18 - i$$

$$= 6 - i - 12(-1) = 6 - i + 12 = 18 - i$$

(vi) $(5 - 2i)(3 - 4i)$

$$= 5(3 - 4i) - 2i(3 - 4i) = 15 - 20i - 6i + 8i^2$$

$$= 5(3 - 4i) - 2i(3 - 4i) = 15 - 20i - 6i + 8i^2$$

$$= 15 - 26i + 8(-1) = 15 - 8 - 26i = 7 - 26i$$

$$= 15 - 26i + 8(-1) = 15 - 8 - 26i = 7 - 26i$$

(vii) $(3 - 5i) \div (2 - 4i)$

$$\frac{3 - 5i}{2 - 4i} \times \frac{2 + 4i}{2 + 4i} = \frac{6 + 12i - 10i - 20i^2}{2^2 - (4i)^2} = \frac{6 + 2i - 20(-1)}{4 - 16(-1)} = \frac{6 + 20 + 2i}{4 + 16} = \frac{26 + 2i}{20}$$

$$\frac{3 - 5i}{2 - 4i} \times \frac{2 + 4i}{2 + 4i} = \frac{6 + 12i - 10i - 20i^2}{2^2 - (4i)^2} = \frac{6 + 2i - 20(-1)}{4 - 16(-1)} = \frac{6 + 20 + 2i}{4 + 16} = \frac{26 + 2i}{20}$$

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$$\frac{26}{20}+\frac{2}{20}i=\frac{13}{10}+\frac{1}{10}i$$

$$\frac{26}{20}+\frac{2}{20}i=\frac{13}{10}+\frac{1}{10}i$$

(viii) $(5+2i)\div(6-3i)$

$$\frac{5+2i}{6-3i}\times\frac{6+3i}{6+3i}=\frac{30+15i+12i+6i^2}{6^2-(3i)^2}=\frac{30+27i+6(-1)}{36-9(-1)}=\frac{24+27i}{45}$$

$$\frac{5+2i}{6-3i}\times\frac{6+3i}{6+3i}=\frac{30+15i+12i+6i^2}{6^2-(3i)^2}=\frac{30+27i+6(-1)}{36-9(-1)}=\frac{24+27i}{45}$$

$$\frac{24}{45}+\frac{27}{45}i=\frac{8}{15}+\frac{3}{5}i$$

$$\frac{24}{45}+\frac{27}{45}i=\frac{8}{15}+\frac{3}{5}i$$

2. Write additive inverse for each complex number:

The additive inverse of $z=x+iy$is $-z=-x-iy$.

(i) $3+2i$

$$\text{Additive Inverse}=-(3+2i)=-3-2i$$

$$\text{Additive Inverse}=-(3+2i)=\mathbf{-3-2i}$$

(ii) $4-3i$

$$\text{Additive Inverse}=-(4-3i)=-4+3i$$

$$\text{Additive Inverse}=-(4-3i)=\mathbf{-4+3i}$$

(iii) $5-7i$

$$\text{Additive Inverse}=-(5-7i)=-5+7i$$

$$\text{Additive Inverse}=-(5-7i)=\mathbf{-5+7i}$$

(iv) $-\frac{2}{3}+\frac{5}{4}i$

$$\text{Additive Inverse}=-\left(-\frac{2}{3}+\frac{5}{4}i\right)=\frac{2}{3}-\frac{5}{4}i$$

$$\text{Additive Inverse}=-\left(-\frac{2}{3}+\frac{5}{4}i\right)=\mathbf{\frac{2}{3}-\frac{5}{4}i}$$

3. Find multiplicative inverse for each complex number:

The multiplicative inverse of $z$is $z^{-1}=\frac{1}{z}=\frac{\bar{z}}{|z|^2}$.

(i) $z=4+5i$

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$$z^{-1} = \frac{1}{4+5i} \times \frac{4-5i}{4-5i} = \frac{4-5i}{4^2-(5i)^2} = \frac{4-5i}{16-(-25)} = \frac{4-5i}{41} = \frac{4}{41} - \frac{5}{41}i$$

$$z^{-1} = \frac{1}{4+5i} \times \frac{4-5i}{4-5i} = \frac{4-5i}{4^2-(5i)^2} = \frac{4-5i}{16-(-25)} = \frac{4-5i}{41} = \mathbf{\frac{4}{41} - \frac{5}{41}i}$$

(ii) $\boldsymbol{z = 6 + 2i}$

$$z^{-1} = \frac{1}{6+2i} \times \frac{6-2i}{6-2i} = \frac{6-2i}{6^2-(2i)^2} = \frac{6-2i}{36-(-4)} = \frac{6-2i}{40} = \frac{6}{40} - \frac{2}{40}i = \frac{3}{20} - \frac{1}{20}i$$

$$z^{-1} = \frac{1}{6+2i} \times \frac{6-2i}{6-2i} = \frac{6-2i}{6^2-(2i)^2} = \frac{6-2i}{36-(-4)} = \frac{6-2i}{40} = \frac{6}{40} - \frac{2}{40}i = \mathbf{\frac{3}{20} - \frac{1}{20}i}$$

(iii) $\boldsymbol{z = 7 - 3i}$

$$z^{-1} = \frac{1}{7-3i} \times \frac{7+3i}{7+3i} = \frac{7+3i}{7^2-(3i)^2} = \frac{7+3i}{49-(-9)} = \frac{7+3i}{58} = \frac{7}{58} + \frac{3}{58}i$$

$$z^{-1} = \frac{1}{7-3i} \times \frac{7+3i}{7+3i} = \frac{7+3i}{7^2-(3i)^2} = \frac{7+3i}{49-(-9)} = \frac{7+3i}{58} = \mathbf{\frac{7}{58} + \frac{3}{58}i}$$

(iv) $\boldsymbol{z = \sqrt{5} - 4i}$

$$z^{-1} = \frac{1}{\sqrt{5}-4i} \times \frac{\sqrt{5}+4i}{\sqrt{5}+4i} = \frac{\sqrt{5}+4i}{\left(\sqrt{5}\right)^2-(4i)^2} = \frac{\sqrt{5}+4i}{5-(-16)} = \frac{\sqrt{5}+4i}{21} = \frac{\sqrt{5}}{21} + \frac{4}{21}i$$

$$z^{-1} = \frac{1}{\sqrt{5}-4i} \times \frac{\sqrt{5}+4i}{\sqrt{5}+4i} = \frac{\sqrt{5}+4i}{\left(\sqrt{5}\right)^2-(4i)^2} = \frac{\sqrt{5}+4i}{5-(-16)} = \frac{\sqrt{5}+4i}{21} = \mathbf{\frac{\sqrt{5}}{21} + \frac{4}{21}i}$$

4. If $\boldsymbol{z_1 = 2 + 5i, z_2 = 1 - 3i, z_3 = 2 + i}$**, then verify that:**

(i) $\boldsymbol{z_1 + z_2 = z_2 + z_1}$

- **LHS:** $z_1 + z_2 = (2 + 5i) + (1 - 3i) = (2 + 1) + (5 - 3)i = 3 + 2i$
- **RHS:** $z_2 + z_1 = (1 - 3i) + (2 + 5i) = (1 + 2) + (-3 + 5)i = 3 + 2i$

LHS = RHS (Verified)

LHS = RHS (Verified)

(ii) $\boldsymbol{(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)}$

- **LHS:** $(z_1 + z_2) + z_3 = (3 + 2i) + (2 + i) = (3 + 2) + (2 + 1)i = 5 + 3i$
- **RHS:** $z_2 + z_3 = (1 - 3i) + (2 + i) = 3 - 2i$

$$z_1 + (z_2 + z_3) = (2 + 5i) + (3 - 2i) = (2 + 3) + (5 - 2)i = 5 + 3i$$

$$z_1 + (z_2 + z_3) = (2 + 5i) + (3 - 2i) = (2 + 3) + (5 - 2)i = 5 + 3i$$

LHS = RHS (Verified)

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LHS = RHS (Verified)

(iii) $z_1 z_2 = z_2 z_1$

- **LHS:** $z_1 z_2 = (2+5i)(1-3i) = 2-6i+5i-15i^2 = 2-i+15 = 17-i$
- **RHS:** $z_2 z_1 = (1-3i)(2+5i) = 2+5i-6i-15i^2 = 2-i+15 = 17-i$

LHS = RHS (Verified)

LHS = RHS (Verified)

(iv) $(z_1 z_2) z_3 = z_1 (z_2 z_3)$

- **LHS:** $(z_1 z_2) z_3 = (17-i)(2+i) = 34+17i-2i-i^2 = 34+15i-(-1) = 35+15i$
- **RHS:** $z_2 z_3 = (1-3i)(2+i) = 2+i-6i-3i^2 = 2-5i+3 = 5-5i$

$z_1(z_2 z_3) = (2+5i)(5-5i) = 10-10i+25i-25i^2 = 10+15i+25 = 35+15i$

$z_1(z_2 z_3) = (2+5i)(5-5i) = 10-10i+25i-25i^2 = 10+15i+25 = 35+15i$

LHS = RHS (Verified)

LHS = RHS (Verified)

(v) $z_1 + (-z_1) = (-z_1) + z_1 = 0$

Here $-z_1 = -2-5i$.

- $z_1 + (-z_1) = (2+5i) + (-2-5i) = (2-2) + (5-5)i = 0+0i = 0$
- $(-z_1) + z_1 = (-2-5i) + (2+5i) = (-2+2) + (-5+5)i = 0+0i = 0$

Hence verified.

Hence verified.

5. If $\frac{(1+i)^2}{2-i} = x + iy$**, then find the values of** x**and** y**:**

First, simplify numerator $(1+i)^2$:

$$(1+i)^2 = 1+2i+i^2 = 1+2i-1 = 2i$$

$$(1+i)^2 = 1+2i+i^2 = 1+2i-1 = 2i$$

So the expression becomes:

$$\frac{2i}{2-i}$$

$$\frac{2i}{2-i}$$

Multiply numerator and denominator by conjugate $(2+i)$:

$$\frac{2i(2+i)}{(2-i)(2+i)} = \frac{4i+2i^2}{2^2-i^2} = \frac{4i+2(-1)}{4-(-1)} = \frac{-2+4i}{5} = -\frac{2}{5}+\frac{4}{5}i$$

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$$\frac{2i(2+i)}{(2-i)(2+i)} = \frac{4i+2i^2}{2^2-i^2} = \frac{4i+2(-1)}{4-(-1)} = \frac{-2+4i}{5} = -\frac{2}{5}+\frac{4}{5}i$$

Since $\frac{(1+i)^2}{2-i} = x+iy$:

$$x+iy = -\frac{2}{5}+\frac{4}{5}i$$

$$x+iy = -\frac{2}{5}+\frac{4}{5}i$$

Equating real and imaginary parts:

$$x = -\frac{2}{5},\ y = \frac{4}{5}$$

$$\mathbf{x = -\frac{2}{5}, \quad y = \frac{4}{5}}$$

6. If $(2x+iy)(1-i) = 4+2i$, then find the values of $x$and y:

Expand the left side:

$$(2x+iy)(1-i) = 2x-2xi+iy-iy^2 = 2x-2xi+iy+y = (2x+y)+(-2x+y)i$$

$$(2x+iy)(1-i) = 2x-2xi+iy-iy^2 = 2x-2xi+iy+y = (2x+y)+(-2x+y)i$$

Equating real and imaginary parts with $4+2i$:

1. $2x+y = 4 \Longrightarrow y = 4-2x$--- (Equation 1)
2. $-2x+y = 2$--- (Equation 2)

Substitute Eq. 1 into Eq. 2:

$$-2x+(4-2x) = 2$$

$$-2x+(4-2x) = 2$$

$$-4x+4 = 2 \Longrightarrow -4x = -2 \Longrightarrow x = \frac{1}{2}$$

$$-4x+4 = 2 \Longrightarrow -4x = -2 \Longrightarrow x = \frac{1}{2}$$

Substitute $x = \frac{1}{2}$into Eq. 1:

$$y = 4-2\left(\frac{1}{2}\right) = 4-1 = 3$$

$$y = 4-2\left(\frac{1}{2}\right) = 4-1 = 3$$

$$x = \frac{1}{2},\ y = 3$$

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$$\mathbf{x = \frac{1}{2}, \quad y = 3}$$

7. Find the values of $a$and b, if $(a + bi)(1 + 3i) = -8 + 11i$:

Expand the left side:

$$(a + bi)(1 + 3i) = a + 3ai + bi + 3bi^2 = (a - 3b) + (3a + b)i$$

$$(a + bi)(1 + 3i) = a + 3ai + bi + 3bi^2 = (a - 3b) + (3a + b)i$$

Equating real and imaginary parts with $-8 + 11i$:

1. $a - 3b = -8 \Longrightarrow a = 3b - 8$--- (Equation 1)
2. $3a + b = 11$--- (Equation 2)

Substitute Eq. 1 into Eq. 2:

$$3(3b - 8) + b = 11$$

$$3(3b - 8) + b = 11$$

$$9b - 24 + b = 11 \Longrightarrow 10b = 35 \Longrightarrow b = \frac{35}{10} = \frac{7}{2}$$

$$9b - 24 + b = 11 \Longrightarrow 10b = 35 \Longrightarrow b = \frac{35}{10} = \frac{7}{2}$$

Substitute $b = \frac{7}{2}$into Eq. 1:

$$a = 3\left(\frac{7}{2}\right) - 8 = \frac{21}{2} - 8 = \frac{21 - 16}{2} = \frac{5}{2}$$

$$a = 3\left(\frac{7}{2}\right) - 8 = \frac{21}{2} - 8 = \frac{21 - 16}{2} = \frac{5}{2}$$

$$a = \frac{5}{2}, \; b = \frac{7}{2}$$

$$\mathbf{a = \frac{5}{2}, \quad b = \frac{7}{2}}$$

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EXERCISE 1.3

1. Find the modulus of the following complex numbers:

Formula: $|z| = |x + iy| = \sqrt{x^2 + y^2}$

(i) $z = 4 + 3i$

$$| z | = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$|z| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = \mathbf{5}$$

(ii) $z = -5 - 4i$

$$| z | = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25 + 16} = \sqrt{41}$$

$$|z| = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25 + 16} = \sqrt{\mathbf{41}}$$

(iii) $z = \frac{3}{5} - \frac{4}{5}i$

$$| z | = \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$$

$$|z| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = \mathbf{1}$$

(iv) $z = -\sqrt{2} - \sqrt{3}i$

$$| z | = \sqrt{\left(-\sqrt{2}\right)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{2 + 3} = \sqrt{5}$$

$$|z| = \sqrt{\left(-\sqrt{2}\right)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{2 + 3} = \sqrt{\mathbf{5}}$$

2. If $z_1 = 2 + 7i$ and $z_2 = 4 - 3i$, then verify that:

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

- $z_1 + z_2 = (2 + 7i) + (4 - 3i) = 6 + 4i$
- **LHS:** $\overline{z_1 + z_2} = 6 - 4i$
- $\overline{z_1} = 2 - 7i, \overline{z_2} = 4 + 3i$
- **RHS:** $\overline{z_1} + \overline{z_2} = (2 - 7i) + (4 + 3i) = 6 - 4i$

LHS = RHS (Verified)

LHS = RHS (Verified)

(ii) $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$

- $z_1 z_2 = (2 + 7i)(4 - 3i) = 8 - 6i + 28i - 21i^2 = 8 + 22i + 21 = 29 + 22i$

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- **LHS:** $\overline{z_1 z_2} = 29 - 22i$
- **RHS:** $\overline{z_1} \cdot \overline{z_2} = (2 - 7i)(4 + 3i) = 8 + 6i - 28i - 21i^2 = 8 - 22i + 21 = 29 - 22i$

LHS = RHS (Verified)

LHS = RHS (Verified)

(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

- $\frac{z_1}{z_2} = \frac{2+7i}{4-3i} \times \frac{4+3i}{4+3i} = \frac{8+6i+28i+21i^2}{16-(-9)} = \frac{-13+34i}{25} = -\frac{13}{25} + \frac{34}{25}i$
- **LHS:** $\overline{\left(\frac{z_1}{z_2}\right)} = -\frac{13}{25} - \frac{34}{25}i$
- **RHS:** $\frac{\overline{z_1}}{\overline{z_2}} = \frac{2-7i}{4+3i} \times \frac{4-3i}{4-3i} = \frac{8-6i-28i+21i^2}{16+9} = \frac{-13-34i}{25} = -\frac{13}{25} - \frac{34}{25}i$

LHS = RHS (Verified)

LHS = RHS (Verified)

3. If $z = 5 - 2i$, then verify that:

(i) $\overline{\overline{z}} = z$

- $\overline{z} = 5 + 2i$
- $\overline{\overline{z}} = \overline{5 + 2i} = 5 - 2i = z$ (Verified)

(ii) $|z| = |\overline{z}|$

- $|z| = \sqrt{5^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$
- $|\overline{z}| = |5 + 2i| = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29}$

Hence verified.

Hence verified.

(iii) $|z| = |-z|$

- $-z = -(5 - 2i) = -5 + 2i$
- $|-z| = \sqrt{(-5)^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29} = |z|$ (Verified)

(iv) $z\overline{z} = |z|^2$

- **LHS:** $z\overline{z} = (5 - 2i)(5 + 2i) = 5^2 - (2i)^2 = 25 - (-4) = 29$
- **RHS:** $|z|^2 = \left(\sqrt{29}\right)^2 = 29$

LHS = RHS (Verified)

LHS = RHS (Verified)

(v) $|z| = |-\overline{z}|$

- $-\overline{z} = -(5 + 2i) = -5 - 2i$
- $|-\overline{z}| = \sqrt{(-5)^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29} = |z|$ (Verified)

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4. If $z = 4 - 3i$, then verify that $|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}|$:

- $z = 4 - 3i \Longrightarrow |z| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$
- $-z = -4 + 3i \Longrightarrow |-z| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$
- $\bar{\bar{z}} = z = 4 - 3i \Longrightarrow |\bar{\bar{z}}| = 5$
- $\bar{z} = 4 + 3i \Longrightarrow -\bar{z} = -4 - 3i \Longrightarrow |-\bar{z}| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$

Hence $|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}| = 5$ (Verified)

Hence $|z| = |-z| = |\bar{\bar{z}}| = |-\bar{z}| = 5$ (Verified)

5. If $z_1 = 2 + 3i$, $z_2 = -1 + i$, then evaluate:

First compute $z_1 z_2$:

$$z_1 z_2 = (2 + 3i)(-1 + i) = -2 + 2i - 3i + 3i^2 = -2 - i + 3(-1) = -5 - i$$

$$z_1 z_2 = (2 + 3i)(-1 + i) = -2 + 2i - 3i + 3i^2 = -2 - i + 3(-1) = -5 - i$$

(i) $\mathbf{Re}(z_1 z_2)$

$$\mathrm{Re}(-5 - i) = -5$$

$$\mathrm{Re}(-5 - i) = \mathbf{-5}$$

(ii) $\mathbf{Im}(z_1 z_2)$

$$\mathrm{Im}(-5 - i) = -1$$

$$\mathrm{Im}(-5 - i) = \mathbf{-1}$$

EXERCISE 1.4

1. Find the real and imaginary parts of the following complex numbers:

(i) $(8 - 3i)^2$

$$(8 - 3i)^2 = 8^2 - 2(8)(3i) + (3i)^2 = 64 - 48i + 9i^2 = 64 - 48i - 9 = 55 - 48i$$

$$(8 - 3i)^2 = 8^2 - 2(8)(3i) + (3i)^2 = 64 - 48i + 9i^2 = 64 - 48i - 9 = 55 - 48i$$

- **Real part:** 55
- **Imaginary part:** -48

(ii) $(5 + 3i)^{-1}$

$$(5 + 3i)^{-1} = \frac{1}{5 + 3i} = \frac{5 - 3i}{(5 + 3i)(5 - 3i)} = \frac{5 - 3i}{5^2 - (3i)^2} = \frac{5 - 3i}{25 + 9} = \frac{5 - 3i}{34} = \frac{5}{34} - \frac{3}{34}i$$

$$(5 + 3i)^{-1} = \frac{1}{5 + 3i} = \frac{5 - 3i}{(5 + 3i)(5 - 3i)} = \frac{5 - 3i}{5^2 - (3i)^2} = \frac{5 - 3i}{25 + 9} = \frac{5 - 3i}{34} = \frac{5}{34} - \frac{3}{34}i$$

- **Real part:** $\frac{5}{34}$
- **Imaginary part:** $-\frac{3}{34}$

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(iii) $(4-5i)^{-1}$

$$(4-5i)^{-1}=\frac{1}{4-5i}=\frac{4+5i}{4^2-(5i)^2}=\frac{4+5i}{16+25}=\frac{4+5i}{41}=\frac{4}{41}+\frac{5}{41}i$$

$$(4-5i)^{-1}=\frac{1}{4-5i}=\frac{4+5i}{4^2-(5i)^2}=\frac{4+5i}{16+25}=\frac{4+5i}{41}=\frac{4}{41}+\frac{5}{41}i$$

- **Real part:** $\frac{4}{41}$
- **Imaginary part:** $\frac{5}{41}$

(iv) $(4-3i)^{-2}$

$$(4-3i)^{-2}=\frac{1}{(4-3i)^2}=\frac{1}{16-24i+9i^2}=\frac{1}{7-24i}$$

$$(4-3i)^{-2}=\frac{1}{(4-3i)^2}=\frac{1}{16-24i+9i^2}=\frac{1}{7-24i}$$

Multiply numerator and denominator by $(7+24i)$:

$$\frac{7+24i}{7^2-(24i)^2}=\frac{7+24i}{49-576(-1)}=\frac{7+24i}{625}=\frac{7}{625}+\frac{24}{625}i$$

$$\frac{7+24i}{7^2-(24i)^2}=\frac{7+24i}{49-576(-1)}=\frac{7+24i}{625}=\frac{7}{625}+\frac{24}{625}i$$

- **Real part:** $\frac{7}{625}$
- **Imaginary part:** $\frac{24}{625}$

(v) $\left(\frac{3+2i}{4+3i}\right)^{-1}$

$$\left(\frac{3+2i}{4+3i}\right)^{-1}=\frac{4+3i}{3+2i}=\frac{(4+3i)(3-2i)}{(3+2i)(3-2i)}=\frac{12-8i+9i-6i^2}{3^2-(2i)^2}=\frac{12+i+6}{9+4}=\frac{18+i}{13}$$
$$=\frac{18}{13}+\frac{1}{13}i$$

$$\left(\frac{3+2i}{4+3i}\right)^{-1}=\frac{4+3i}{3+2i}=\frac{(4+3i)(3-2i)}{(3+2i)(3-2i)}=\frac{12-8i+9i-6i^2}{3^2-(2i)^2}=\frac{12+i+6}{9+4}=\frac{18+i}{13}$$
$$=\frac{18}{13}+\frac{1}{13}i$$

- **Real part:** $\frac{18}{13}$
- **Imaginary part:** $\frac{1}{13}$

(vi) $\left(\frac{2-i}{2+i}\right)^{-2}$

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$$\left(\frac{2-i}{2+i}\right)^{-2} = \left(\frac{2+i}{2-i}\right)^{2}$$

$$\left(\frac{2-i}{2+i}\right)^{-2} = \left(\frac{2+i}{2-i}\right)^{2}$$

First simplify $\frac{2+i}{2-i}$:

$$\frac{(2+i)(2+i)}{(2-i)(2+i)} = \frac{4+4i+i^2}{4-i^2} = \frac{3+4i}{5} = \frac{3}{5}+\frac{4}{5}i$$

$$\frac{(2+i)(2+i)}{(2-i)(2+i)} = \frac{4+4i+i^2}{4-i^2} = \frac{3+4i}{5} = \frac{3}{5}+\frac{4}{5}i$$

Now square it:

$$\left(\frac{3+4i}{5}\right)^2 = \frac{9+24i+16i^2}{25} = \frac{9+24i-16}{25} = \frac{-7+24i}{25} = -\frac{7}{25}+\frac{24}{25}i$$

$$\left(\frac{3+4i}{5}\right)^2 = \frac{9+24i+16i^2}{25} = \frac{9+24i-16}{25} = \frac{-7+24i}{25} = -\frac{7}{25}+\frac{24}{25}i$$

- **Real part:** $-\frac{7}{25}$
- **Imaginary part:** $\frac{24}{25}$

(vii) $\left(\frac{1-2i}{1+i}\right)^2$

First simplify $\frac{1-2i}{1+i}$:

$$\frac{(1-2i)(1-i)}{(1+i)(1-i)} = \frac{1-i-2i+2i^2}{1-i^2} = \frac{1-3i-2}{2} = \frac{-1-3i}{2}$$

$$\frac{(1-2i)(1-i)}{(1+i)(1-i)} = \frac{1-i-2i+2i^2}{1-i^2} = \frac{1-3i-2}{2} = \frac{-1-3i}{2}$$

Now square it:

$$\left(\frac{-1-3i}{2}\right)^2 = \frac{(-1)^2+2(-1)(-3i)+(-3i)^2}{4} = \frac{1+6i-9}{4} = \frac{-8+6i}{4} = -2+\frac{3}{2}i$$

$$\left(\frac{-1-3i}{2}\right)^2 = \frac{(-1)^2+2(-1)(-3i)+(-3i)^2}{4} = \frac{1+6i-9}{4} = \frac{-8+6i}{4} = -2+\frac{3}{2}i$$

- **Real part:** -2
- **Imaginary part:** $\frac{3}{2}$

---

2. Solve the following simultaneous linear equations with complex coefficients for *w*and *z*:

(i)

---

1. $3z+(2+i)w = 11-i$--- (Eq. 1)

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2. $(2-i)z-w=-1+i$--- (Eq. 2)

From Eq. 2, express $w$in terms of z:

$$w=(2-i)z+1-i$$

$$w=(2-i)z+1-i$$

--- (Eq. 3)

Substitute $w$into Eq. 1:

$$3z+(2+i)[(2-i)z+1-i]=11-i$$

$$3z+(2+i)[(2-i)z+1-i]=11-i$$

Note that $(2+i)(2-i)=4-i^2=5$, and $(2+i)(1-i)=2-2i+i-i^2=3-i$:

$$3z+[5z+3-i]=11-i$$

$$3z+[5z+3-i]=11-i$$

$$8z+3-i=11-i$$

$$8z+3-i=11-i$$

$$8z=11-3=8\Longrightarrow z=1$$

$$8z=11-3=8\Longrightarrow \mathbf{z=1}$$

Substitute $z=1$into Eq. 3:

$$w=(2-i)(1)+1-i=2-i+1-i=3-2i$$

$$w=(2-i)(1)+1-i=2-i+1-i=\mathbf{3-2i}$$

Solution: $\mathbf{z=1}$, $\mathbf{w=3-2i}$

(ii)

1. $2z+(3+i)w=9-i$--- (Eq. 1)
2. $-iz-iw=-1+i$--- (Eq. 2)

Divide Eq. 2 by $-i$:

$$z+w=\frac{-1+i}{-i}=\frac{(-1+i)i}{-i^2}=\frac{-i+i^2}{1}=-1-i$$

$$z+w=\frac{-1+i}{-i}=\frac{(-1+i)i}{-i^2}=\frac{-i+i^2}{1}=-1-i$$

$$z=-1-i-w$$

$$z=-1-i-w$$

--- (Eq. 3)

Substitute $z$into Eq. 1:

$$2(-1-i-w)+(3+i)w=9-i$$

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$$2(-1-i-w)+(3+i)w=9-i$$
$$-2-2i-2w+(3+i)w=9-i$$

$$-2-2i-2w+(3+i)w=9-i$$
$$(-2+3+i)w=9-i+2+2i$$

$$(-2+3+i)w=9-i+2+2i$$
$$(1+i)w=11+i$$

$$(1+i)w=11+i$$
$$w=\frac{11+i}{1+i}=\frac{(11+i)(1-i)}{(1+i)(1-i)}=\frac{11-11i+i-i^2}{1-(-1)}=\frac{12-10i}{2}=6-5i$$

$$w=\frac{11+i}{1+i}=\frac{(11+i)(1-i)}{(1+i)(1-i)}=\frac{11-11i+i-i^2}{1-(-1)}=\frac{12-10i}{2}=\mathbf{6-5i}$$

Substitute $w=6-5i$into Eq. 3:

$$z=-1-i-(6-5i)=-1-i-6+5i=-7+4i$$

$$z=-1-i-(6-5i)=-1-i-6+5i=\mathbf{-7+4i}$$

Solution: $\mathbf{z=-7+4i}$, $\mathbf{w=6-5i}$

(iii)

---

1. $z-4w=3i\Longrightarrow z=4w+3i$--- (Eq. 1)
2. $2z+3w=11-5i$--- (Eq. 2)

Substitute $z$into Eq. 2:

$$2(4w+3i)+3w=11-5i$$

$$2(4w+3i)+3w=11-5i$$
$$8w+6i+3w=11-5i$$

$$8w+6i+3w=11-5i$$
$$11w=11-11i\Longrightarrow w=1-i$$

$$11w=11-11i\Longrightarrow \mathbf{w=1-i}$$

Substitute $w=1-i$into Eq. 1:

$$z=4(1-i)+3i=4-4i+3i=4-i$$

$$z=4(1-i)+3i=4-4i+3i=\mathbf{4-i}$$

Solution: $\mathbf{z=4-i}$, $\mathbf{w=1-i}$

(iv)

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1. $z + w = 3i \Longrightarrow z = 3i - w$--- (Eq. 1)
2. $2z + 3w = 2$--- (Eq. 2)

Substitute $z$into Eq. 2:

$$2(3i - w) + 3w = 2$$

$$2(3i - w) + 3w = 2$$
$$6i - 2w + 3w = 2 \Longrightarrow w + 6i = 2 \Longrightarrow w = 2 - 6i$$

$$6i - 2w + 3w = 2 \Longrightarrow w + 6i = 2 \Longrightarrow \mathbf{w = 2 - 6i}$$

Substitute $w = 2 - 6i$into Eq. 1:

$$z = 3i - (2 - 6i) = 3i - 2 + 6i = -2 + 9i$$

$$z = 3i - (2 - 6i) = 3i - 2 + 6i = \mathbf{-2 + 9i}$$

Solution: $\mathbf{z = -2 + 9i}$, $\mathbf{w = 2 - 6i}$

(v)

1. $2z + (3 + i)w = 1$--- (Eq. 1)
2. $-z - (1 - i)w = 2 \Longrightarrow z = -2 - (1 - i)w$--- (Eq. 2)

Substitute $z$into Eq. 1:

$$2[-2 - (1 - i)w] + (3 + i)w = 1$$

$$2[-2 - (1 - i)w] + (3 + i)w = 1$$
$$-4 - 2(1 - i)w + (3 + i)w = 1$$

$$-4 - 2(1 - i)w + (3 + i)w = 1$$
$$-4 + [-2 + 2i + 3 + i]w = 1$$

$$-4 + [-2 + 2i + 3 + i]w = 1$$
$$(1 + 3i)w = 5$$

$$(1 + 3i)w = 5$$
$$w = \frac{5}{1 + 3i} = \frac{5(1 - 3i)}{1^2 - (3i)^2} = \frac{5(1 - 3i)}{10} = \frac{1}{2} - \frac{3}{2}i$$

$$w = \frac{5}{1 + 3i} = \frac{5(1 - 3i)}{1^2 - (3i)^2} = \frac{5(1 - 3i)}{10} = \mathbf{\frac{1}{2} - \frac{3}{2}i}$$

Substitute $w$into Eq. 2:

$$z = -2 - (1 - i)\left(\frac{1 - 3i}{2}\right)$$

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$$z = -2 - (1 - i)\left(\frac{1 - 3i}{2}\right)$$

$$(1 - i)(1 - 3i) = 1 - 3i - i + 3i^2 = -2 - 4i$$

$$(1 - i)(1 - 3i) = 1 - 3i - i + 3i^2 = -2 - 4i$$

$$z = -2 - \left(\frac{-2 - 4i}{2}\right) = -2 - (-1 - 2i) = -2 + 1 + 2i = -1 + 2i$$

$$z = -2 - \left(\frac{-2 - 4i}{2}\right) = -2 - (-1 - 2i) = -2 + 1 + 2i = \mathbf{-1 + 2i}$$

Solution: $\mathbf{z = -1 + 2i}$, $\mathbf{w = \frac{1}{2} - \frac{3}{2}i}$

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REVIEW EXERCISE 1

1. Choose the correct answer:

1. $i^2 + i^4 =$
   - $i^2 = -1, i^4 = 1 \Longrightarrow -1 + 1 = 0$
   - **Correct Answer: (b) 0**
2. Real part of $(2 - 3i)(2 + 3i)$is:
   - $2^2 - (3i)^2 = 4 - (-9) = 13 + 0i \Longrightarrow$ Real part $= 13$
   - **Correct Answer: (d) 13**
3. Imaginary part of $(2 - i)(2 + i)$is:
   - $2^2 - i^2 = 4 - (-1) = 5 + 0i \Longrightarrow$ Imaginary part $= 0$
   - **Correct Answer: (a) 0**
4. $x + iy$will be pure imaginary number, when:
   - A number is purely imaginary when its real part $x = 0$(and $y \neq 0$).
   - **Correct Answer: (b)** $x = 0$
5. What is additive inverse of $5 - 2i$?
   - $-(5 - 2i) = -5 + 2i$
   - **Correct Answer: (d)** $-5 + 2i$
6. What is multiplicative inverse of $z = 1 + i$?
   - $\frac{1}{1+i} = \frac{1-i}{1+1} = \frac{1}{2} - \frac{1}{2}i$
   - **Correct Answer: (c)** $\frac{1}{2} - \frac{1}{2}i$
7. If $z = 4 - 3i$, then $z\overline{z} =$
   - $z\overline{z} = |z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$
   - **Correct Answer: (d) 25**
8. Conjugate of $9 - 4i$is:
   - $\overline{9 - 4i} = 9 + 4i$
   - **Correct Answer: (b)** $9 + 4i$
9. If $z = 4 + 4i$, then $z + \overline{z} =$
   - $z + \overline{z} = (4 + 4i) + (4 - 4i) = 8$
   - **Correct Answer: (a) 8**
10. If $z = 5 + 4i$, then $|z| =$
    - $|z| = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$
    - **Correct Answer: (d)** $\sqrt{41}$

2. Short Answer Questions:

(i) Is "0" a complex number? Explain.

Answer: Yes, "0" is a complex number. It can be written in standard rectangular form as $0 + 0i$, where both its real part and imaginary part are real numbers equal to 0.

(ii) What is the result of multiplying a complex number by its conjugate?

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Answer: Multiplying a complex number $z = x + iy$ by its conjugate $\bar{z} = x - iy$ yields a non-negative real number equal to the square of its modulus:

$$z\,\bar{z} = (x + iy)(x - iy) = x^2 + y^2 = |\,z\,|^2$$

$$z\bar{z} = (x + iy)(x - iy) = x^2 + y^2 = |z|^2$$

(iii) State the condition for two complex numbers to be equal.

Answer: Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are equal if and only if their real parts are equal and their imaginary parts are equal, i.e., $x_1 = x_2$ and $y_1 = y_2$.

3. Simplify:

(i) i^{37}

$$i^{37} = i^{36} \cdot i = (i^2)^{18} \cdot i = (-1)^{18} \cdot i = 1 \cdot i = i$$

$$i^{37} = i^{36} \cdot i = (i^2)^{18} \cdot i = (-1)^{18} \cdot i = 1 \cdot i = \mathbf{i}$$

(ii) $i^{13} \times i^{11}$

$$i^{13+11} = i^{24} = (i^2)^{12} = (-1)^{12} = 1$$

$$i^{13+11} = i^{24} = (i^2)^{12} = (-1)^{12} = \mathbf{1}$$

(iii) $(-i)^{-9}$

$$(-i)^{-9} = \frac{1}{(-i)^9} = \frac{1}{(-1)^9 \cdot i^9} = \frac{1}{-1 \cdot (i^8 \cdot i)} = \frac{1}{-1 \cdot (1 \cdot i)} = \frac{1}{-i} = i$$

$$(-i)^{-9} = \frac{1}{(-i)^9} = \frac{1}{(-1)^9 \cdot i^9} = \frac{1}{-1 \cdot (i^8 \cdot i)} = \frac{1}{-1 \cdot (1 \cdot i)} = \frac{1}{-i} = \mathbf{i}$$

(iv) $(3 - 4i)(5 - 6i)$

$$= 15 - 18i - 20i + 24i^2 = 15 - 38i - 24 = -9 - 38i$$

$$= 15 - 18i - 20i + 24i^2 = 15 - 38i - 24 = \mathbf{-9 - 38i}$$

(v) $(3 + 4i) \div (5 - 7i)$

$$\frac{3 + 4i}{5 - 7i} \times \frac{5 + 7i}{5 + 7i} = \frac{15 + 21i + 20i + 28i^2}{5^2 - (7i)^2} = \frac{15 + 41i - 28}{25 + 49} = \frac{-13 + 41i}{74} = -\frac{13}{74} + \frac{41}{74}i$$

$$\frac{3 + 4i}{5 - 7i} \times \frac{5 + 7i}{5 + 7i} = \frac{15 + 21i + 20i + 28i^2}{5^2 - (7i)^2} = \frac{15 + 41i - 28}{25 + 49} = \frac{-13 + 41i}{74} = -\frac{\mathbf{13}}{\mathbf{74}} + \frac{\mathbf{41}}{\mathbf{74}}\mathbf{i}$$

4. Find additive and multiplicative inverse of $z = 8 + 9i$:

- **Additive Inverse:**

$$-z = -(8 + 9i) = -8 - 9i$$

$$-z = -(8 + 9i) = \mathbf{-8 - 9i}$$

- **Multiplicative Inverse:**

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$$z^{-1} = \frac{1}{8+9i} \times \frac{8-9i}{8-9i} = \frac{8-9i}{8^2-(9i)^2} = \frac{8-9i}{64+81} = \frac{8-9i}{145} = \frac{8}{145} - \frac{9}{145}i$$

$$z^{-1} = \frac{1}{8+9i} \times \frac{8-9i}{8-9i} = \frac{8-9i}{8^2-(9i)^2} = \frac{8-9i}{64+81} = \frac{8-9i}{145} = \mathbf{\frac{8}{145} - \frac{9}{145}i}$$

5. If $z_1 = 3+4i$ and $z_2 = 2+3i$, then verify that:

(i) $\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$

- $z_1 + z_2 = (3+4i)+(2+3i) = 5+7i \Longrightarrow$ **LHS $= 5-7i$**
- $\overline{z_1} + \overline{z_2} = (3-4i)+(2-3i) =$ **$5-7i$ = RHS** (Verified)

(ii) $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$

- $z_1 z_2 = (3+4i)(2+3i) = 6+9i+8i+12i^2 = -6+17i \Longrightarrow$ **LHS $= -6-17i$**
- $\overline{z_1} \cdot \overline{z_2} = (3-4i)(2-3i) = 6-9i-8i+12i^2 =$ **$-6-17i$ = RHS** (Verified)

(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

- $\frac{z_1}{z_2} = \frac{3+4i}{2+3i} \times \frac{2-3i}{2-3i} = \frac{6-9i+8i+12}{13} = \frac{18-i}{13} = \frac{18}{13} - \frac{1}{13}i$
- **LHS:** $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{18}{13} + \frac{1}{13}i$
- **RHS:** $\frac{\overline{z_1}}{\overline{z_2}} = \frac{3-4i}{2-3i} \times \frac{2+3i}{2+3i} = \frac{6+9i-8i+12}{13} = \frac{18+i}{13} = \frac{18}{13} + \frac{1}{13}i$

LHS = RHS (Verified)

LHS = RHS (Verified)

(iv) $|z_1| = |-\overline{z_1}|$

- $|z_1| = \sqrt{3^2+4^2} = \sqrt{25} = 5$
- $-\overline{z_1} = -(3-4i) = -3+4i \Longrightarrow |-\overline{z_1}| = \sqrt{(-3)^2+4^2} = 5$

LHS = RHS (Verified)

LHS = RHS (Verified)

(v) $\overline{\overline{z_2}} = z_2$

- $\overline{z_2} = 2-3i \Longrightarrow \overline{\overline{z_2}} = \overline{2-3i} = 2+3i = z_2$ (Verified)

(vi) $z_1\overline{z_1} = |z_1|^2$

- **LHS:** $(3+4i)(3-4i) = 9-16i^2 = 9+16 = 25$
- **RHS:** $|z_1|^2 = 5^2 = 25$

LHS = RHS (Verified)

LHS = RHS (Verified)

6. If $z_1 = 5+4i$, $z_2 = 3+2i$, then find:

(i) $z_1 z_2$

$$z_1 z_2 = (5+4i)(3+2i) = 15+10i+12i+8i^2 = 15+22i-8 = 7+22i$$

$$z_1 z_2 = (5+4i)(3+2i) = 15+10i+12i+8i^2 = 15+22i-8 = \mathbf{7+22i}$$

(ii) $\frac{z_1}{z_2}$

$$\frac{5+4i}{3+2i}\times\frac{3-2i}{3-2i} = \frac{15-10i+12i-8i^2}{3^2-(2i)^2} = \frac{15+2i+8}{9+4} = \frac{23}{13}+\frac{2}{13}i$$

$$\frac{5+4i}{3+2i}\times\frac{3-2i}{3-2i} = \frac{15-10i+12i-8i^2}{3^2-(2i)^2} = \frac{15+2i+8}{9+4} = \mathbf{\frac{23}{13}+\frac{2}{13}i}$$

(iii) $\overline{z_1}\cdot\overline{z_2}$

$$\bar{z_1}\cdot\bar{z_2} = (5-4i)(3-2i) = 15-10i-12i+8i^2 = 15-22i-8 = 7-22i$$

$$\overline{z_1}\cdot\overline{z_2} = (5-4i)(3-2i) = 15-10i-12i+8i^2 = 15-22i-8 = \mathbf{7-22i}$$

(iv) $|z_1 z_2|$

$$|\, z_1 z_2 \,| = |\, 7+22i \,| = \sqrt{7^2+22^2} = \sqrt{49+484} = \sqrt{533}$$

$$|z_1 z_2| = |7+22i| = \sqrt{7^2+22^2} = \sqrt{49+484} = \mathbf{\sqrt{533}}$$

7. Find real and imaginary parts of $z = (2+7i)^{-1}$:

$$z = \frac{1}{2+7i} = \frac{2-7i}{(2+7i)(2-7i)} = \frac{2-7i}{2^2-(7i)^2} = \frac{2-7i}{4+49} = \frac{2-7i}{53} = \frac{2}{53}-\frac{7}{53}i$$

$$z = \frac{1}{2+7i} = \frac{2-7i}{(2+7i)(2-7i)} = \frac{2-7i}{2^2-(7i)^2} = \frac{2-7i}{4+49} = \frac{2-7i}{53} = \frac{2}{53}-\frac{7}{53}i$$

- **Real part:** $\frac{2}{53}$
- **Imaginary part:** $-\frac{7}{53}$

8. Solve the given simultaneous linear equations with complex coefficients for z and w:

1. $iz+(2-i)w = 4+i$--- (Eq. 1)
2. $iz+(3+i)w = 3+3i$--- (Eq. 2)

Subtract Eq. 1 from Eq. 2:

$$[iz+(3+i)w]-[iz+(2-i)w] = (3+3i)-(4+i)$$

$$[iz+(3+i)w]-[iz+(2-i)w] = (3+3i)-(4+i)$$
$$[(3+i)-(2-i)]w = -1+2i$$

$$[(3+i)-(2-i)]w = -1+2i$$
$$(1+2i)w = -1+2i$$

$$(1+2i)w = -1+2i$$

$$w = \frac{-1+2i}{1+2i} = \frac{(-1+2i)(1-2i)}{(1+2i)(1-2i)} = \frac{-1+2i+2i-4i^2}{1^2-(2i)^2} = \frac{-1+4i+4}{1+4} = \frac{3+4i}{5} = \frac{3}{5}+\frac{4}{5}i$$

$$w = \frac{-1+2i}{1+2i} = \frac{(-1+2i)(1-2i)}{(1+2i)(1-2i)} = \frac{-1+2i+2i-4i^2}{1^2-(2i)^2} = \frac{-1+4i+4}{1+4} = \frac{3+4i}{5} = \mathbf{\frac{3}{5}+\frac{4}{5}i}$$

Substitute $w$into Eq. 1:

$$iz = (4+i)-(2-i)\left(\frac{3+4i}{5}\right)$$

$$iz = (4+i)-(2-i)\left(\frac{3+4i}{5}\right)$$

$$(2-i)(3+4i) = 6+8i-3i-4i^2 = 10+5i$$

$$(2-i)(3+4i) = 6+8i-3i-4i^2 = 10+5i$$

$$iz = 4+i-\left(\frac{10+5i}{5}\right) = 4+i-(2+i) = 2$$

$$iz = 4+i-\left(\frac{10+5i}{5}\right) = 4+i-(2+i) = 2$$

$$z = \frac{2}{i} = \frac{2(-i)}{-i^2} = -2i$$

$$z = \frac{2}{i} = \frac{2(-i)}{-i^2} = \mathbf{-2i}$$

Solution: $\mathbf{z = -2i}$, $\mathbf{w = \frac{3}{5}+\frac{4}{5}i}$

9. Solve $(3-4i)(a+bi) = 1+0i$and find the values of $a$and b:

$$a+bi = \frac{1}{3-4i}$$

$$a+bi = \frac{1}{3-4i}$$

Multiply numerator and denominator by $(3+4i)$:

$$a+bi = \frac{3+4i}{3^2-(4i)^2} = \frac{3+4i}{9+16} = \frac{3+4i}{25} = \frac{3}{25}+\frac{4}{25}i$$

$$a+bi = \frac{3+4i}{3^2-(4i)^2} = \frac{3+4i}{9+16} = \frac{3+4i}{25} = \frac{3}{25}+\frac{4}{25}i$$

Equating real and imaginary parts:

$$a = \frac{3}{25},\ b = \frac{4}{25}$$

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$$\mathbf{a} = \frac{\mathbf{3}}{\mathbf{25}}, \quad \mathbf{b} = \frac{\mathbf{4}}{\mathbf{25}}$$

10. Solve the equation for $x$and y:

$$(3-2i)(x+yi) = 2(x-2yi) + 2i - 1$$

$$(3-2i)(x+yi) = 2(x-2yi) + 2i - 1$$

Expand both sides:

$$3x + 3yi - 2xi - 2yi^2 = 2x - 4yi + 2i - 1$$

$$3x + 3yi - 2xi - 2yi^2 = 2x - 4yi + 2i - 1$$

$$(3x + 2y) + (3y - 2x)i = (2x - 1) + (2 - 4y)i$$

$$(3x + 2y) + (3y - 2x)i = (2x - 1) + (2 - 4y)i$$

Equate real parts and imaginary parts:

1. **Real Parts:**

$$3x + 2y = 2x - 1 \Longrightarrow x + 2y = -1 \Longrightarrow x = -1 - 2y$$

$$3x + 2y = 2x - 1 \Longrightarrow x + 2y = -1 \Longrightarrow x = -1 - 2y$$

--- (Equation 1)

2. **Imaginary Parts:**

$$3y - 2x = 2 - 4y \Longrightarrow -2x + 7y = 2$$

$$3y - 2x = 2 - 4y \Longrightarrow -2x + 7y = 2$$

--- (Equation 2)

Substitute Eq. 1 into Eq. 2:

$$-2(-1 - 2y) + 7y = 2$$

$$-2(-1 - 2y) + 7y = 2$$

$$2 + 4y + 7y = 2$$

$$2 + 4y + 7y = 2$$

$$11y = 0 \Longrightarrow y = 0$$

$$11y = 0 \Longrightarrow \mathbf{y = 0}$$

Substitute $y = 0$back into Eq. 1:

$$x = -1 - 2(0) \Longrightarrow x = -1$$

$$x = -1 - 2(0) \Longrightarrow \mathbf{x = -1}$$

Solution: $\mathbf{x = -1}$, $\mathbf{y = 0}$

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