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Class	9th	Chapter	04
Subject	Mathematics	Board	Punjab

Chapter 4: Factorization and Algebraic Manipulation

Exercise 4.1

Q1. Factorize by identifying common factors:

- **(i)** $6x + 12 \rightarrow 6(x + 2)$
- **(ii)** $15y^2 + 20y \rightarrow 5y(3y + 4)$
- **(iii)** $-12x^2 - 3x \rightarrow -3x(4x + 1)$
- **(iv)** $4a^2b + 8ab^2 \rightarrow 4ab(a + 2b)$
- **(v)** $xy - 3x^2 + 2x \rightarrow x(y - 3x + 2)$
- **(vi)** $3a^2b - 9ab^2 + 15ab \rightarrow 3ab(a - 3b + 5)$

Q2. Factorize and represent pictorially:

- **(i)** $5x + 15 \rightarrow 5(x + 3)$
- **(ii)** $x^2 + 4x + 3 \rightarrow (x + 1)(x + 3)$
- **(iii)** $x^2 + 6x + 8 \rightarrow (x + 2)(x + 4)$
- **(iv)** $x^2 + 4x + 4 \rightarrow (x + 2)^2$

Q3. Factorize:

- **(i)** $x^2 + x - 12 \rightarrow x^2 + 4x - 3x - 12 = x(x + 4) - 3(x + 4) = \mathbf{(x + 4)(x - 3)}$
- **(ii)** $x^2 + 7x + 10 \rightarrow \mathbf{(x + 5)(x + 2)}$
- **(iii)** $x^2 - 6x + 8 \rightarrow \mathbf{(x - 4)(x - 2)}$
- **(iv)** $x^2 - x - 56 \rightarrow \mathbf{(x - 8)(x + 7)}$
- **(v)** $x^2 - 10x - 24 \rightarrow \mathbf{(x - 12)(x + 2)}$
- **(vi)** $y^2 + 4y - 12 \rightarrow \mathbf{(y + 6)(y - 2)}$
- **(vii)** $y^2 + 13y + 36 \rightarrow \mathbf{(y + 9)(y + 4)}$
- **(viii)** $x^2 - x - 2 \rightarrow \mathbf{(x - 2)(x + 1)}$

Q4. Factorize:

- **(i)** $2x^2 + 7x + 3 \rightarrow 2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = \mathbf{(x + 3)(2x + 1)}$
- **(ii)** $2x^2 + 11x + 15 \rightarrow 2x^2 + 6x + 5x + 15 = 2x(x + 3) + 5(x + 3) = \mathbf{(x + 3)(2x + 5)}$
- **(iii)** $4x^2 + 13x + 3 \rightarrow 4x^2 + 12x + x + 3 = 4x(x + 3) + 1(x + 3) = \mathbf{(x + 3)(4x + 1)}$

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- **(iv)** $3x^2 + 5x + 2 \rightarrow \mathbf{(3x + 2)(x + 1)}$
- **(v)** $3y^2 - 11y + 6 \rightarrow \mathbf{(3y - 2)(y - 3)}$
- **(vi)** $2y^2 - 5y + 2 \rightarrow \mathbf{(2y - 1)(y - 2)}$
- **(vii)** $4z^2 - 11z + 6 \rightarrow \mathbf{(4z - 3)(z - 2)}$
- **(viii)** $6 + 7x - 3x^2 \rightarrow \mathbf{(3 + x)(2 - 3x)}$

Exercise 4.2

Q1. Factorize each of the following expressions:

- **(i)** $4x^4 + 81y^4 \rightarrow (2x^2)^2 + (9y^2)^2 + 2(2x^2)(9y^2) - 36x^2y^2 = (2x^2 + 9y^2)^2 - (6xy)^2 = \mathbf{(2x^2 - 6xy + 9y^2)(2x^2 + 6xy + 9y^2)}$
- **(ii)** $a^4 + 64b^4 \rightarrow (a^2 + 8b^2)^2 - 16a^2b^2 = \mathbf{(a^2 - 4ab + 8b^2)(a^2 + 4ab + 8b^2)}$
- **(iii)** $x^4 + 4x^2 + 16 \rightarrow (x^2 + 4)^2 - 4x^2 = \mathbf{(x^2 - 2x + 4)(x^2 + 2x + 4)}$
- **(iv)** $x^4 - 14x^2 + 1 \rightarrow (x^2 - 1)^2 - 12x^2 = \mathbf{\left(x^2 - 2\sqrt{3}x - 1\right)\left(x^2 + 2\sqrt{3}x - 1\right)}$
- **(v)** $x^4 - 30x^2y^2 + 9y^4 \rightarrow \mathbf{(x^2 - 3xy - 3y^2)(x^2 + 3xy - 3y^2)}$ *(or equivalent completion)*
- **(vi)** $x^4 + 11x^2y^2 + y^4 \rightarrow \mathbf{(x^2 - 3xy + y^2)(x^2 + 3xy + y^2)}$

Q2. Factorize each of the following expressions:

- **(i)** $(x + 1)(x + 2)(x + 3)(x + 4) + 1 \rightarrow [(x + 1)(x + 4)][(x + 2)(x + 3)] + 1 = (x^2 + 5x + 4)(x^2 + 5x + 6) + 1 = \mathbf{(x^2 + 5x + 1)^2}$
- **(ii)** $(x + 2)(x - 7)(x - 4)(x - 1) + 17 \rightarrow \mathbf{(x^2 - 5x - 5)(x^2 - 5x - 3)}$
- **(iii)** $(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1 \rightarrow \mathbf{(2x^2 + 7x + 4)^2}$
- **(iv)** $(3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3 \rightarrow \mathbf{(3x^2 + 5x + 6)(3x^2 + 5x + 2)}$
- **(v)** $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2 \rightarrow \mathbf{(x^2 + 4x + 6)(x^2 + 4x + 3)}$
- **(vi)** $(x + 1)(x - 1)(x + 2)(x - 2) + 13x^2 \rightarrow \mathbf{(x^2 + 3x + 2)(x^2 - 3x + 2)}$

Q3. Factorize:

- **(i)** $8x^3 + 12x^2 + 6x + 1 \rightarrow \mathbf{(2x + 1)^3}$
- **(ii)** $27a^3 + 108a^2b + 144ab^2 + 64b^3 \rightarrow \mathbf{(3a + 4b)^3}$
- **(iii)** $x^3 + 48x^2y + 108xy^2 + 216y^3 \rightarrow \mathbf{(x + 6y)^3}$
- **(iv)** $8x^3 - 125y^3 + 150xy^2 - 60x^2y \rightarrow \mathbf{(2x - 5y)^3}$

Q4. Factorize:

- **(i)** $125a^3 - 1 \rightarrow \mathbf{(5a - 1)(25a^2 + 5a + 1)}$
- **(ii)** $64x^3 + 125 \rightarrow \mathbf{(4x + 5)(16x^2 - 20x + 25)}$
- **(iii)** $x^6 - 27 \rightarrow \mathbf{(x^2 - 3)(x^4 + 3x^2 + 9)}$
- **(iv)** $1000a^3 + 1 \rightarrow \mathbf{(10a + 1)(100a^2 - 10a + 1)}$
- **(v)** $343x^3 + 216 \rightarrow \mathbf{(7x + 6)(49x^2 - 42x + 36)}$
- **(vi)** $27 - 512y^3 \rightarrow \mathbf{(3 - 8y)(9 + 24y + 64y^2)}$

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Exercise 4.3

Q1. Find HCF by factorization method:

- **(i)** $21x^2y, 35xy^2 \rightarrow \mathbf{7xy}$
- **(ii)** $4x^2 - 9y^2, 2x^2 - 3xy \rightarrow \mathbf{2x - 3y}$
- **(iii)** $x^3 - 1, x^2 + x + 1 \rightarrow \mathbf{x^2 + x + 1}$
- **(iv)** $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a \rightarrow \mathbf{a(a + 3)}$
- **(v)** $t^2 + 3t - 4, t^2 + 5t + 4, t^2 - 1 \rightarrow \mathbf{t + 1}$
- **(vi)** $x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x \rightarrow \mathbf{x + 8}$

Q2. Find HCF of the following expressions by using division method:

- **(i)** $27x^3 + 9x^2 - 3x - 9, 3x - 2 \rightarrow \mathbf{3x - 2}$
- **(ii)** $x^3 - 9x^2 + 21x - 15, x^2 - 4x + 3 \rightarrow \mathbf{x - 3}$
- **(iii)** $2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12 \rightarrow \mathbf{x^2 + 1}$
- **(iv)** $2x^3 - 4x^2 + 6x, x^3 - 2x, 3x^2 - 6x \rightarrow \mathbf{x(x - 2)}$

Q3. Find LCM of the following expressions by using prime factorization method:

- **(i)** $2a^2b, 4ab^2, 6ab \rightarrow \mathbf{12a^2b^2}$
- **(ii)** $x^2 + x, x^3 + x^2 \rightarrow \mathbf{x^2(x + 1)}$
- **(iii)** $a^2 - 4a + 4, a^2 - 2a \rightarrow \mathbf{a(a - 2)^2}$
- **(iv)** $x^4 - 16, x^3 - 4x \rightarrow \mathbf{x(x - 2)(x + 2)(x^2 + 4)}$
- **(v)** $16 - 4x^2, x^2 + x - 6, 4 - x^2 \rightarrow \mathbf{4(2 - x)(2 + x)(x + 3)}$

Q4. The HCF of two polynomials is $y - 7$ and their LCM is $y^3 - 10y^2 + 11y + 70$. If one of the polynomials is $y^2 - 5y - 14$, find the other.

- *Step:* Other polynomial $= \frac{\text{LCM} \times \text{HCF}}{\text{Given Polynomial}} = \frac{(y^3 - 10y^2 + 11y + 70)(y - 7)}{(y - 7)(y + 2)} = \frac{y^3 - 10y^2 + 11y + 70}{y + 2}$.
- *Result:* $\mathbf{y^2 - 12y + 35}$.

Q5. The LCM and HCF of two polynomials $p(x)$ and $q(x)$ are $36x^3(x + a)(x^3 - a^3)$ and $x^2(x - a)$ respectively. If $p(x) = 4x^2(x^2 - a^2)$, find $q(x)$.

- *Step:* $q(x) = \frac{\text{LCM} \times \text{HCF}}{p(x)} = \frac{36x^3(x + a)(x - a)(x^2 + ax + a^2) \cdot x^2(x - a)}{4x^2(x + a)(x - a)}$.
- *Result:* $\mathbf{9x^3(x^2 + ax + a^2)}$.

Q6. The HCF and LCM of two polynomials is $(x + a)$ and $12x^2(x + a)(x^2 - a^2)$ respectively. Find the product of the two polynomials.

- *Step:* Product $=$ LCM $\times$ HCF $= (x + a) \cdot 12x^2(x + a)(x^2 - a^2)$.
- *Result:* $\mathbf{12x^2(x + a)^2(x^2 - a^2)}$.

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Exercise 4.4

Q1. Find the square root of the following polynomials by factorization method:

- **(i)** $x^2 - 8x + 16 \rightarrow \pm(\mathbf{x - 4})$
- **(ii)** $9x^2 + 12x + 4 \rightarrow \pm(\mathbf{3x + 2})$
- **(iii)** $36a^2 + 84a + 49 \rightarrow \pm(\mathbf{6a + 7})$
- **(iv)** $64y^2 - 32y + 4 \rightarrow \pm\mathbf{2}(\mathbf{4y - 1})$
- **(v)** $200t^2 - 120t + 18 \rightarrow \pm\mathbf{2}(\mathbf{10t - 3})$
- **(vi)** $40x^2 + 120x + 90 \rightarrow \pm\sqrt{\mathbf{10}}(\mathbf{2x + 3})$

Q2. Find the square root of the following polynomials by division method:

- **(i)** $4x^4 - 28x^3 + 37x^2 + 42x + 9 \rightarrow \pm(\mathbf{2x^2 - 7x - 3})$
- **(ii)** $121x^4 - 198x^3 - 183x^2 + 216x + 144 \rightarrow \pm(\mathbf{11x^2 - 9x - 12})$
- **(iii)** $x^4 - 10x^3 + 27x^2y^2 - 10xy^3 + y^4 \rightarrow \pm(\mathbf{x^2 - 5xy + y^2})$
- **(iv)** $4x^4 - 12x^3 + 37x^2 - 42x + 49 \rightarrow \pm(\mathbf{2x^2 - 3x + 7})$

Q3. An investor's return $R(x)$ in rupees after investing x thousand rupees is given by $R(x) = -x^2 + 6x - 8$. Find the investment levels that result in zero return.

- *Step:* $-(x^2 - 6x + 8) = 0 \Longrightarrow -(x - 4)(x - 2) = 0.$
- *Result:* $\mathbf{x = 2}$ or $\mathbf{x = 4}$ (i.e., **Rs. 2,000** or **Rs. 4,000**).

Q4. A company's profit $P(x)$ in rupees from selling x units is $P(x) = x^3 - 15x^2 + 75x - 125$. Find the break-even point(s).

- *Step:* $(x - 5)^3 = 0 \Longrightarrow x - 5 = 0.$
- *Result:* $\mathbf{5}$ units.

Q5. The potential energy $V(x)$ is given by $V(x) = 2x^3 - 6x^2 + 4x$. Determine where potential energy is zero.

- *Step:* $2x(x^2 - 3x + 2) = 0 \Longrightarrow 2x(x - 1)(x - 2) = 0.$
- *Result:* $\mathbf{x = 0, 1, 2}$.

Q6. The deflection $Y(x)$ of a beam is $Y(x) = 2x^2 - 8x + 6$. Find the points of zero deflection.

- *Step:* $2(x^2 - 4x + 3) = 0 \Longrightarrow 2(x - 3)(x - 1) = 0.$
- *Result:* $\mathbf{x = 1}$ and $\mathbf{x = 3}$.

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Review Exercise 4

Q1. Multiple Choice Questions:

- i. The factorization of $12x + 36$ is: **(a)** $12(x + 3)$
- ii. The factors of $4x^2 - 12x + 9$ are: **(b)** $(2x - 3)^2$
- iii. The HCF of a^3b^3 and ab^2 is: **(d)** ab^2
- iv. The LCM of $16x^2, 4x$ and $30xy$ is: **(a)** $240x^2y$
- v. Product of LCM and HCF = _____ of two polynomials. **(c) product**
- vi. The square root of $x^2 - 6x + 9$ is: **(b)** $\pm(x - 3)$
- vii. The LCM of $(a - b)^2$ and $(a - b)^4$ is: **(c)** $(a - b)^4$
- viii. Factorization of $x^3 + 3x^2 + 3x + 1$ is: **(a)** $(x + 1)^3$
- ix. Cubic polynomial has degree: **(c)** 3
- x. One of the factors of $x^3 - 27$ is: **(d) Both a and c**

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