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Class	9th	Chapter	05
Subject	Mathematics	Board	Punjab

Chapter 5: Linear Equations and Inequalities

Exercise 5.1

Q1. Solve and represent the solution on a real line:

- **(i)** $12x + 30 = -6$
  - *Step 1:* Subtract 30 from both sides:

$$12x = -6 - 30 \implies 12x = -36$$

  - *Step 2:* Divide both sides by 12:

$$x = \frac{-36}{12} \implies \mathbf{x = -3}.$$

  - *Solution Set:* $\{-3\}$
  - *Real Line Representation:* A solid dot marked at -3 on the number line.
- **(ii)** $\frac{x}{3} + 6 = -12$
  - *Step 1:* Subtract 6 from both sides:

$$\frac{x}{3} = -12 - 6 \implies \frac{x}{3} = -18$$

  - *Step 2:* Multiply both sides by 3:

$$x = -18 \times 3 \implies \mathbf{x = -54}$$

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- **(iii)** $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$
  - *Step 1:* Find the LCM of denominators (2,4,12), which is 12. Multiply the entire equation by 12:

$$12\left(\frac{x}{2}\right) - 12\left(\frac{3x}{4}\right) = 12\left(\frac{1}{12}\right)$$
$$6x - 9x = 1 \implies -3x = 1$$

  - *Step 2:* Solve for x:

$$\mathbf{x = -\frac{1}{3}}$$

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- **(iv)** $2 = 7(2x + 4) + 12x$
  - *Step 1:* Expand the parenthesis:

$$2 = 14x + 28 + 12x$$

  - *Step 2:* Combine like terms:

$$2 = 26x + 28$$

  - *Step 3:* Rearrange terms:

$$2 - 28 = 26x \Longrightarrow -26 = 26x \Longrightarrow \mathbf{x = -1}$$

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- **(v)** $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$
  - *Step 1:* Multiply the entire equation by the LCM (12):

$$12\left(\frac{2x-1}{3}\right) - 12\left(\frac{3x}{4}\right) = 12\left(\frac{5}{6}\right)$$
$$4(2x-1) - 3(3x) = 2(5)$$
$$8x - 4 - 9x = 10$$

  - *Step 2:* Simplify and solve:

$$-x - 4 = 10 \Longrightarrow -x = 14 \Longrightarrow \mathbf{x = -14}$$

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- **(vi)** $\frac{-5x}{10} = 9 - \frac{10}{5}x$
  - *Step 1:* Simplify fractions:

$$-\frac{1}{2}x = 9 - 2x$$

  - *Step 2:* Add $2x$ to both sides:

$$-0.5x + 2x = 9 \Longrightarrow 1.5x = 9 \Longrightarrow \frac{3}{2}x = 9$$

  - *Step 3:* Solve for x:

$$x = 9 \times \frac{2}{3} = \mathbf{6}$$

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Q2. Solve each inequality and represent the solution on a real line:

- **(i)** $x - 6 \leq -2$
  - *Step:* Add 6 to both sides: $x \leq -2 + 6 \Longrightarrow \mathbf{x \leq 4}$ (Interval: $(-\infty, 4]$).
- **(ii)** $-9 > -16 + x$
  - *Step:* Add 16 to both sides: $-9 + 16 > x \Longrightarrow \mathbf{x < 7}$ (Interval: $(-\infty, 7)$).
- **(iii)** $3 + 2x \geq 3$
  - *Step:* Subtract 3: $2x \geq 0 \Longrightarrow \mathbf{x \geq 0}$ (Interval: $[0, \infty)$).
- **(iv)** $6(x + 10) \leq 0$
  - *Step:* Divide by 6: $x + 10 \leq 0 \Longrightarrow \mathbf{x \leq -10}$ (Interval: $(-\infty, -10]$).

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- **(v)** $\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$
  - *Step 1:* Multiply by 12: $20x - 9 < -1$
  - *Step 2:* $20x < 8 \Longrightarrow x < \frac{8}{20} \Longrightarrow \mathbf{x < \frac{2}{5}}$ (Interval: $\left(-\infty, \frac{2}{5}\right)$).
- **(vi)** $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$
  - *Step 1:* Multiply by 4: $x - 2 \leq -4 + 2x$
  - *Step 2:* Rearrange: $-2 + 4 \leq 2x - x \Longrightarrow \mathbf{x \geq 2}$ (Interval: $[2, \infty)$).

Q3. Shade the solution region for the following linear inequalities in xy-plane:

- **(i)** $2x + y \leq 6$
  - *Associated Equation:* $2x + y = 6$.
  - *Intercepts:* x-intercept at $(3,0)$, y-intercept at $(0,6)$.
  - *Test Point* $(0,0)$*:* $0 + 0 \leq 6$ (True).
  - *Shading:* Solid boundary line passing through $(3,0)$ and $(0,6)$, with shading towards the origin side.
- **(ii)** $3x + 7y \geq 21$
  - *Associated Equation:* $3x + 7y = 21$.
  - *Intercepts:* $(7,0)$ and $(0,3)$.
  - *Test Point* $(0,0)$*:* $0 \geq 21$ (False).
  - *Shading:* Solid line; shade the half-plane **not** containing the origin.
- **(iii)** $3x - 2y \geq 6$
  - *Associated Equation:* $3x - 2y = 6$. Intercepts: $(2,0)$ and $(0,-3)$.
  - *Test Point* $(0,0)$*:* $0 \geq 6$ (False).
  - *Shading:* Solid line; shade the half-plane away from the origin.
- **(iv)** $5x - 4y \leq 20$
  - *Associated Equation:* $5x - 4y = 20$. Intercepts: $(4,0)$ and $(0,-5)$.
  - *Test Point* $(0,0)$*:* $0 \leq 20$ (True).
  - *Shading:* Solid line; shade towards the origin side.
- **(v)** $2x + 1 \geq 0$
  - *Associated Line:* $x = -\frac{1}{2}$.
  - *Shading:* Solid vertical line at $x = -\frac{1}{2}$; shade the region to the right $(x \geq -\frac{1}{2})$.
- **(vi)** $3y - 4 \leq 0$
  - *Associated Line:* $y = \frac{4}{3}$.
  - *Shading:* Solid horizontal line at $y = \frac{4}{3}$; shade the region below the line $(y \leq \frac{4}{3})$.

Q4. Indicate the solution region of the following linear inequalities by shading:

- *(For systems of inequalities, find the intersection of the individual half-plane regions for each pair).*

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- **(i)** $2x - 3y \leq 6$ **and** $2x + 3y \leq 12$ → Shaded common intersection region bounded by both lines below the origin side.
- **(ii)** $x + y \geq 5$ **and** $-y + x \leq 1$ → Common region above $x + y = 5$ and right/above $-y + x \leq 1$.
- **(iii)** $3x + 7y \geq 21$ **and** $x - y \leq 2$ → Intersection of half-planes away from origin for (1) and towards origin for (2).
- **(iv)** $4x - 3y \leq 12$ **and** $x \geq -\frac{3}{2}$ → Region bounded by $4x - 3y = 12$ and right of vertical line $x = -1.5$.
- **(v)** $3x + 7y \geq 21$ **and** $y \leq 4$ → Region above $3x + 7y = 21$ and below horizontal line $y = 4$.
- **(vi)** $5x + 7y \leq 35$ **and** $x - 2y \leq 2$ → Common region satisfying both inequalities simultaneously.

Exercise 5.2

Q1. Maximize $f(x, y) = 2x + 5y$ subject to constraints: $2y - x \leq 8, x - y \leq 4, x \geq 0, y \geq 0$.

- *Step 1 (Find corner points):* Intersection of constraint boundaries and non-negativity axes yields corner points: $(0,0)$, $(4,0)$, $(0,4)$, and intersection of $2y - x = 8$ and $x - y = 4$ at $(16,12)$ [or respective boundary bounds].
- *Step 2 (Evaluate $f(x, y) = 2x + 5y$ at corner points):*
  - $f(0,0) = 2(0) + 5(0) = 0$
  - $f(4,0) = 2(4) + 5(0) = 8$
  - $f(0,4) = 2(0) + 5(4) = 20$
- *Result:* Maximum value is 20 at $(0,4)$.

Q2. Maximize $f(x, y) = x + 3y$ subject to: $2x + 5y \leq 30, 5x + 4y \leq 20, x \geq 0, y \geq 0$.

- *Step 1 (Corner points):* $(0,0)$, $(4,0)$ [since $5(4) + 4(0) = 20$], $(0,6)$ [since $2(0) + 5(6) = 30$].
- *Step 2 (Evaluate objective function):*
  - $f(0,0) = 0$
  - $f(4,0) = 4 + 0 = 4$
  - $f(0,6) = 0 + 3(6) = 18$
- *Result:* Maximum value is 18 at $(0,6)$.

Q3. Maximize $z = 2x + 3y$ subject to: $2x + y \leq 4, 4x - y \leq 4, x \geq 0, y \geq 0$.

- *Corner Points:* $(0,0)$, $(2,0)$, $(0,4)$, and intersection of $2x + y = 4$ and $4x - y = 4 \Longrightarrow 6x = 8 \Longrightarrow x = \frac{4}{3}, y = \frac{4}{3}$.
- *Evaluation:*
  - $f(0,0) = 0$

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- $f(2,0) = 4$
- $f(0,4) = 12$
- $f\left(\frac{4}{3},\frac{4}{3}\right) = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right) = \frac{20}{3} \approx 6.67$

- *Result:* Maximum value is 12 at $(0,4)$.

Q4. Minimize $z = 2x + y$ subject to: $x + y \geq 3, 7x + 5y \leq 35, x \geq 0, y \geq 0$.

- *Corner Points:* (3,0), (0,3), (5,0), (0,7) [feasible bounds].
- *Evaluation:*
  - $f(3,0) = 2(3) + 0 = 6$
  - $f(0,3) = 2(0) + 3 = 3$
  - $f(5,0) = 2(5) + 0 = 10$
- *Result:* Minimum value is 3 at $(0,3)$.

Q5. Maximize $f(x, y) = 2x + 3y$ subject to: $2x + y \leq 8, x + 2y \leq 14, x \geq 0, y \geq 0$.

- *Corner Points:* $(0,0)$, $(4,0)$, $(0,7)$, and intersection of $2x + y = 8$ and $x + 2y = 14 \Longrightarrow x = 2, y = 6$.
- *Evaluation:*
  - $f(0,0) = 0$
  - $f(4,0) = 8$
  - $f(0,7) = 21$
  - $f(2,6) = 2(2) + 3(6) = 22$
- *Result:* Maximum value is 22 at $(2,6)$.

Q6. Find minimum and maximum values of $z = 3x + y$ subject to: $3x + 5y \geq 15, x + 6y \geq 9, x \geq 0, y \geq 0$.

- *Corner Points:* Intersection vertices of boundary lines with axes: $(5,0)$, $(0,3)$, $(9,0)$, $(0,1.5)$ and intersection of $3x + 5y = 15$ and $x + 6y = 9 \Longrightarrow x = \frac{45}{13}, y = \frac{12}{13}$.
- *Evaluation:*
  - $f(5,0) = 15$
  - $f(9,0) = 27$
  - $f(0,3) = 3$
  - $f\left(\frac{45}{13},\frac{12}{13}\right) = 3\left(\frac{45}{13}\right) + \frac{12}{13} = \frac{147}{13} \approx 11.3$
- *Result:* Minimum value = 3 (at $(0,3)$); Maximum value = 27 (at $(9,0)$).

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Review Exercise 5

Q1. Multiple Choice Questions:

- i. In the following, linear equation is: **(c)** $2x + 1 = 1$
- ii. Solution of $5x - 10 = 10$ is: **(b)** 50 *(Wait,* $5x = 20 \Rightarrow x = 4$*, option (c))* → **(c)** 4
- iii. If $7x + 4 < 6x + 6$, then x belongs to the interval: **(c)** $(-\infty, 2)$
- iv. A vertical line divides the plane into: **(d) two half planes**
- v. The linear equation formed out of the linear inequality is called: **(b) associated equation**
- vi. $3x + 4 < 0$ is: **(b) inequality**
- vii. Corner point is also called: **(b) vertex**
- viii. $(0,0)$ is solution of inequality: **(c)** $-2x + 3y < 0$
- ix. The solution region restricted to the first quadrant is called: **(b) feasible region**
- x. A function that is to be maximized or minimized is called: **(b) objective function**

Q2. Solve and represent their solutions on real line:

- **(i)** $\frac{x+5}{3} = 1 - x$
  - *Step:* $x + 5 = 3(1 - x) \Rightarrow x + 5 = 3 - 3x \Rightarrow 4x = -2 \Rightarrow \mathbf{x = -\frac{1}{2}}$.
- **(ii)** $\frac{2x+1}{3} + \frac{1}{2} = 1 - \frac{x-1}{3}$
  - *Step:* Multiply by 6: $2(2x + 1) + 3 = 6 - 2(x - 1) \Rightarrow 4x + 2 + 3 = 6 - 2x + 2 \Rightarrow 4x + 5 = 8 - 2x \Rightarrow 6x = 3 \Rightarrow \mathbf{x = \frac{1}{2}}$.
- **(iii)** $3x + 7 < 16$
  - *Step:* $3x < 9 \Rightarrow \mathbf{x < 3}$.
- **(iv)** $5(x - 3) \geq 26x - (10x + 4)$
  - *Step:* $5x - 15 \geq 26x - 10x - 4 \Rightarrow 5x - 15 \geq 16x - 4 \Rightarrow -11 \geq 11x \Rightarrow \mathbf{x \leq -1}$.

Q3. Find the solution region of the following linear equalities:

- **(i)** $3x - 4y \leq 12$ **and** $3x + 2y \geq 3$ → Region representing the intersection of the two half-planes.
- **(ii)** $2x + y \leq 4$ **and** $x + 2y \leq 6$ → Feasible common shaded area bounded by both lines and axes.

Q4. Find the maximum value of $g(x, y) = x + 4y$ subject to constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

- *Corner Points:* $(0,0)$, $(4,0)$, $(0,4)$.
- *Evaluation:*
  - $g(0,0) = 0$
  - $g(4,0) = 4 + 0 = 4$
  - $g(0,4) = 0 + 4(4) = 16$
- *Result:* Maximum value is 16 at $(0,4)$.

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Q5. Find the minimum value of $f(x, y) = 3x + 5y$ subject to constraints $x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$.

- *Corner Points:* $(3,0)$, $(0,1)$, and intersection of $x + 3y = 3$ and $x + y = 2 \Longrightarrow y = \frac{1}{2}, x = \frac{3}{2}$.
- *Evaluation:*
  - $f(3,0) = 3(3) + 5(0) = 9$
  - $f(0,1) = 3(0) + 5(1) = 5$
  - $f\left(\frac{3}{2}, \frac{1}{2}\right) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = \frac{9+5}{2} = 7$
- *Result:* Minimum value is 5 at $(0,1)$.

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