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Class	9th	Chapter	06
Subject	Mathematics	Board	Punjab

Chapter 06: Trigonometry

Exercise 6.1

Q1. Find in which quadrant the following angles lie. Write a co-terminal angle for each:

- **(i)** 65°
  - **Quadrant:** Since $0^\circ < 65^\circ < 90^\circ$, it lies in **Quadrant I**.
  - **Co-terminal angle:** Add 360°:

$$65^\circ + 360^\circ = \mathbf{425^\circ}$$

- **(ii)** 135°
  - **Quadrant:** Since $90^\circ < 135^\circ < 180^\circ$, it lies in **Quadrant II**.
  - **Co-terminal angle:** Add 360°:

$$135^\circ + 360^\circ = \mathbf{495^\circ}$$

- **(iii)** -40°
  - **Quadrant:** Measured clockwise, it lies in **Quadrant IV**.
  - **Co-terminal angle:** Add 360°:

$$-40^\circ + 360^\circ = \mathbf{320^\circ}$$

- **(iv)** 210°
  - **Quadrant:** Since $180^\circ < 210^\circ < 270^\circ$, it lies in **Quadrant III**.
  - **Co-terminal angle:** Add 360°:

$$210^\circ + 360^\circ = \mathbf{570^\circ}$$

- **(v)** -150°
  - **Quadrant:** Measured clockwise past -90°, it lies in **Quadrant III**.
  - **Co-terminal angle:** Add 360°:

$$-150^\circ + 360^\circ = \mathbf{210^\circ}$$

Q2. Convert the following into degrees, minutes, and seconds (DMS):

- **(i)** 123.456°
  - **Step 1 (Degrees):** Whole number $= 123^\circ$.
  - **Step 2 (Minutes):** Multiply decimal part by 60:

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$$0.456 \times 60 = 27.36' \Longrightarrow \text{Minutes} = 27'$$

- **Step 3 (Seconds):** Multiply remaining decimal part by 60:

$$0.36 \times 60 = 21.6'' \approx 22''$$

- **Result: $\mathbf{123^\circ\ 27'\ 22''}$**

- **(ii)** 58.7891°
  - **Step 1 (Degrees):** Whole number $= 58^\circ$.
  - **Step 2 (Minutes):** $0.7891 \times 60 = 47.346' \Longrightarrow$ Minutes $= 47'$.
  - **Step 3 (Seconds):** $0.346 \times 60 = 20.76'' \approx 21''$.
  - **Result: $\mathbf{58^\circ\ 47'\ 21''}$**
- **(iii)** 90.5678°
  - **Step 1 (Degrees):** Whole number $= 90^\circ$.
  - **Step 2 (Minutes):** $0.5678 \times 60 = 34.068' \Longrightarrow$ Minutes $= 34'$.
  - **Step 3 (Seconds):** $0.068 \times 60 = 4.08'' \approx 4''$.
  - **Result: $\mathbf{90^\circ\ 34'\ 4''}$**

Q3. Convert the following into decimal degrees:

- **(i)** $65^\circ\ 32'\ 15''$
  - **Formula:** $\text{Degrees} + \frac{\text{Minutes}}{60} + \frac{\text{Seconds}}{3600}$
  - **Step:**

$$65 + \frac{32}{60} + \frac{15}{3600} = 65 + 0.533333 + 0.004167 = \mathbf{65.5375^\circ}$$

- **(ii)** $42^\circ\ 18'\ 45''$
  - **Step:**

$$42 + \frac{18}{60} + \frac{45}{3600} = 42 + 0.3 + 0.0125 = \mathbf{42.3125^\circ}$$

- **(iii)** $78^\circ\ 45'\ 36''$
  - **Step:**

$$78 + \frac{45}{60} + \frac{36}{3600} = 78 + 0.75 + 0.01 = \mathbf{78.76^\circ}$$

Q4. Convert the following into radians:

- **(i)** 36°
  - **Step:** Multiply by $\frac{\pi}{180}$:

$$36 \times \frac{\pi}{180} = \frac{\boldsymbol{\pi}}{\mathbf{5}} \text{ rad}$$

- **(ii)** 22.5°
  - **Step:** Multiply by $\frac{\pi}{180}$:

$$22.5 \times \frac{\pi}{180} = \frac{22.5}{180}\pi = \frac{\boldsymbol{\pi}}{\mathbf{8}} \text{ rad}$$

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- **(iii)** $67.5°$
  - **Step:** Multiply by $\frac{\pi}{180}$:

$$67.5 \times \frac{\pi}{180} = \frac{67.5}{180}\pi = \mathbf{\frac{3\pi}{8}} \text{ rad}$$

Q5. Convert the following into degrees:

- **(i)** $\frac{\pi}{16}$ rad
  - **Step:** Multiply by $\frac{180°}{\pi}$:

$$\frac{\pi}{16} \times \frac{180°}{\pi} = \frac{180°}{16} = \mathbf{11.25°}$$

- **(ii)** $\frac{11\pi}{5}$ rad
  - **Step:** Multiply by $\frac{180°}{\pi}$:

$$\frac{11\pi}{5} \times \frac{180°}{\pi} = 11 \times 36° = \mathbf{396°}$$

- **(iii)** $\frac{7\pi}{6}$ rad
  - **Step:** Multiply by $\frac{180°}{\pi}$:

$$\frac{7\pi}{6} \times \frac{180°}{\pi} = 7 \times 30° = \mathbf{210°}$$

Q6. Find the arc length (ℓ) and area (A) of a sector if:

- **(i)** $r = 6$ cm **and** $\theta = \frac{\pi}{3}$ radians
  - **Arc Length ($\ell = r\theta$):**

$$\ell = 6 \times \frac{\pi}{3} = 2\pi \approx \mathbf{6.28} \text{ cm}$$

  - **Area ($A = \frac{1}{2}r^2\theta$):**

$$A = \frac{1}{2}(6)^2 \times \frac{\pi}{3} = \frac{1}{2}(36) \times \frac{\pi}{3} = 6\pi \approx \mathbf{18.85} \text{ cm}^2$$

- **(ii)** $r = \frac{4.8}{\pi}$ cm **and** $\theta = \frac{5\pi}{6}$ radians
  - **Arc Length ($\ell = r\theta$):**

$$\ell = \left(\frac{4.8}{\pi}\right) \times \left(\frac{5\pi}{6}\right) = \frac{4.8 \times 5}{6} = \frac{24}{6} = \mathbf{4} \text{ cm}$$

  - **Area ($A = \frac{1}{2}r^2\theta$):**

$$A = \frac{1}{2}\left(\frac{4.8}{\pi}\right)^2 \times \frac{5\pi}{6} = \frac{1}{2} \times \frac{23.04}{\pi^2} \times \frac{5\pi}{6} = \frac{57.6}{6\pi} = \frac{9.6}{\pi} \approx \mathbf{3.06} \text{ cm}^2$$

Q7. If the central angle of a sector is $60°$ and radius $r = 12$ cm, find sector area and its percentage of the total circle area.

- **Step 1 (Convert θ to radians):**

$$\theta = 60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3} \text{ rad}$$

- **Step 2 (Area of Sector):**

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(12)^2 \times \frac{\pi}{3} = \frac{1}{2}(144) \times \frac{\pi}{3} = \mathbf{24\pi} \text{ cm}^2 \approx \mathbf{75.40} \text{ cm}^2$$

- **Step 3 (Area of Circle):**

$$A_{\text{circle}} = \pi r^2 = \pi(12)^2 = 144\pi \text{ cm}^2$$

- **Step 4 (Percentage):**

$$\text{Percentage} = \frac{24\pi}{144\pi} \times 100 = \frac{1}{6} \times 100 = \mathbf{16.67\%}$$

Q8. Find the percentage of the area of a sector subtending an angle $\frac{\pi}{8}$ radians.

- **Step 1:** Total angle of full circle $= 2\pi$ radians.
- **Step 2 (Fraction of Circle):**

$$\text{Fraction} = \frac{\frac{\pi}{8}}{2\pi} = \frac{1}{16}$$

- **Step 3 (Percentage):**

$$\text{Percentage} = \frac{1}{16} \times 100 = \mathbf{6.25\%}$$

Q9. A sector with $r = 12$ cm and $\theta = 150^\circ$ is bent to form a cone. Find its slant height and base radius.

- **Step 1 (Slant height l):** The radius of the sector forms the slant height of the cone:

$$l = r = \mathbf{12} \text{ cm}$$

- **Step 2 (Arc length of sector):**

$$\theta = 150^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{6} \text{ rad}$$

$$\ell = r\theta = 12 \times \frac{5\pi}{6} = 10\pi \text{ cm}$$

- **Step 3 (Base radius r_{base}):** Arc length equals circumference of the cone's base:

$$2\pi r_{\text{base}} = \ell \Longrightarrow 2\pi r_{\text{base}} = 10\pi \Longrightarrow r_{\text{base}} = \frac{10\pi}{2\pi} = \mathbf{5} \text{ cm}$$

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Exercise 6.2

Q1. For given right triangles, find the required ratios:

Triangle (a): Base $= 3$, Perpendicular $= 4$, Hypotenuse $= 5$

- **(i)** $\sin\theta = \frac{\text{Perp}}{\text{Hyp}} = \frac{4}{5}$
- **(ii)** $\cos\theta = \frac{\text{Base}}{\text{Hyp}} = \frac{3}{5}$
- **(iii)** $\tan\theta = \frac{\text{Perp}}{\text{Base}} = \frac{4}{3}$
- **(iv)** $\sec\theta = \frac{\text{Hyp}}{\text{Base}} = \frac{5}{3}$
- **(v)** $\csc\theta = \frac{\text{Hyp}}{\text{Perp}} = \frac{5}{4}$
- **(vi)** $\cot\phi = \frac{\text{Perp w.r.t } \phi}{\text{Base w.r.t } \phi} = \frac{4}{3}$
- **(vii)** $\tan\phi = \frac{\text{Opposite to } \phi}{\text{Adjacent to } \phi} = \frac{3}{4}$
- **(viii)** $\csc\phi = \frac{\text{Hyp}}{\text{Opposite to } \phi} = \frac{5}{3}$
- **(ix)** $\sec\phi = \frac{\text{Hyp}}{\text{Adjacent to } \phi} = \frac{5}{4}$
- **(x)** $\cos\phi = \frac{\text{Adjacent to } \phi}{\text{Hyp}} = \frac{4}{5}$

Triangle (b): Base $= 15$, Perpendicular $= 8$, Hypotenuse $= 17$

- $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $\tan\theta = \frac{8}{15}$

Triangle (c): Base $= 12$, Perpendicular $= 5$, Hypotenuse $= 13$

- $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

Q2. For right triangle ABC with $m\angle C = \theta$, $m\angle A = \phi$, $m\angle B = 90°$, sides a, b, c:

- **(i)** $\sin\theta = \frac{\text{Opposite}}{\text{Hyp}} = \frac{c}{b}$
- **(ii)** $\cos\theta = \frac{\text{Adjacent}}{\text{Hyp}} = \frac{a}{b}$
- **(iii)** $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{c}{a}$
- **(iv)** $\sin\phi = \frac{\text{Opposite}}{\text{Hyp}} = \frac{a}{b}$
- **(v)** $\cos\phi = \frac{\text{Adjacent}}{\text{Hyp}} = \frac{c}{b}$
- **(vi)** $\tan\phi = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{a}{c}$

Q3. Verify using right triangle ABC ($a = 3, b = 5, c = 4$):

- **(i)** $\sin\theta \csc\theta = 1$
  - **LHS:** $\left(\frac{3}{5}\right) \times \left(\frac{5}{3}\right) = 1 =$ RHS (Verified)
- **(ii)** $\cos\theta \sec\theta = 1$
  - **LHS:** $\left(\frac{4}{5}\right) \times \left(\frac{5}{4}\right) = 1 =$ RHS (Verified)

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- **(iii)** $\tan\theta \cot\theta = 1$
  - **LHS:** $\left(\frac{3}{4}\right) \times \left(\frac{4}{3}\right) = 1 =$ RHS (Verified)

Q4. Fill in the blanks (Complementary Ratios):

- **(i)** $\sin 30° = \sin(90° - 60°) =$ **$\cos 60°$**
- **(ii)** $\cos 30° = \cos(90° - 60°) =$ **$\sin 60°$**
- **(iii)** $\tan 30° = \tan(90° - 60°) =$ **$\cot 60°$**
- **(iv)** $\tan 60° = \tan(90° - 30°) =$ **$\cot 30°$**
- **(v)** $\sin 60° = \sin(90° - 30°) =$ **$\cos 30°$**
- **(vi)** $\cos 60° = \cos(90° - 30°) =$ **$\sin 30°$**
- **(vii)** $\sin 45° = \sin(90° - 45°) =$ **$\cos 45°$**
- **(viii)** $\tan 45° = \tan(90° - 45°) =$ **$\cot 45°$**
- **(ix)** $\cos 45° = \cos(90° - 45°) =$ **$\sin 45°$**

Q5. Right triangle ABC, $m\angle B = 90°$, $C = 60°$, $\sin A = \frac{a}{b}$:

- **(i)** $\frac{m\overline{BC}}{m\overline{AB}} = \frac{a}{c} =$ **$\tan 60° = \sqrt{3}$**
- **(ii)** $\cos 60° =$ **$\frac{1}{2}$**
- **(iii)** $\tan 60° =$ **$\sqrt{3}$**
- **(iv)** $\csc\frac{\pi}{3} = \csc 60° =$ **$\frac{2}{\sqrt{3}}$**
- **(v)** $\cot 60° =$ **$\frac{1}{\sqrt{3}}$**
- **(vi)** $\sin 30° =$ **$\frac{1}{2}$**
- **(vii)** $\cos 30° =$ **$\frac{\sqrt{3}}{2}$**
- **(viii)** $\tan\frac{\pi}{6} = \tan 30° =$ **$\frac{1}{\sqrt{3}}$**
- **(ix)** $\sec 30° =$ **$\frac{2}{\sqrt{3}}$**
- **(x)** $\cot 30° =$ **$\sqrt{3}$**

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Exercise 6.3

Q1. If θ lies in first quadrant, find remaining trigonometric ratios:

- **(i)** $\sin\theta = \frac{2}{3}$
  - **Perpendicular** $= 2$, **Hypotenuse** $= 3$.
  - **Base** $= \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$.
  - **Ratios:**

$$\cos\theta = \frac{\sqrt{5}}{3}, \quad \tan\theta = \frac{2}{\sqrt{5}}, \quad \csc\theta = \frac{3}{2}, \quad \sec\theta = \frac{3}{\sqrt{5}}, \quad \cot\theta = \frac{\sqrt{5}}{2}$$

- **(ii)** $\cos\theta = \frac{3}{4}$
  - **Base** $= 3$, **Hypotenuse** $= 4$.
  - **Perpendicular** $= \sqrt{4^2 - 3^2} = \sqrt{7}$.
  - **Ratios:**

$$\sin\theta = \frac{\sqrt{7}}{4}, \quad \tan\theta = \frac{\sqrt{7}}{3}, \quad \csc\theta = \frac{4}{\sqrt{7}}, \quad \sec\theta = \frac{4}{3}, \quad \cot\theta = \frac{3}{\sqrt{7}}$$

- **(iii)** $\tan\theta = \frac{1}{2}$
  - **Perpendicular** $= 1$, **Base** $= 2$.
  - **Hypotenuse** $= \sqrt{1^2 + 2^2} = \sqrt{5}$.
  - **Ratios:**

$$\sin\theta = \frac{1}{\sqrt{5}}, \quad \cos\theta = \frac{2}{\sqrt{5}}, \quad \csc\theta = \sqrt{5}, \quad \sec\theta = \frac{\sqrt{5}}{2}, \quad \cot\theta = 2$$

- **(iv)** $\sec\theta = 3$
  - **Hypotenuse** $= 3$, **Base** $= 1$.
  - **Perpendicular** $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.
  - **Ratios:**

$$\sin\theta = \frac{2\sqrt{2}}{3}, \quad \cos\theta = \frac{1}{3}, \quad \tan\theta = 2\sqrt{2}, \quad \csc\theta = \frac{3}{2\sqrt{2}}, \quad \cot\theta = \frac{1}{2\sqrt{2}}$$

- **(v)** $\cot\theta = \sqrt{\frac{3}{2}}$
  - **Base** $= \sqrt{3}$, **Perpendicular** $= \sqrt{2}$.
  - **Hypotenuse** $= \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2} = \sqrt{3+2} = \sqrt{5}$.
  - **Ratios:**

$$\sin\theta = \sqrt{\frac{2}{5}}, \quad \cos\theta = \sqrt{\frac{3}{5}}, \quad \tan\theta = \sqrt{\frac{2}{3}}, \quad \csc\theta = \sqrt{\frac{5}{2}}, \quad \sec\theta = \sqrt{\frac{5}{3}}$$

Prove the following trigonometric identities:

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Q2. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

- **LHS:**

$$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$$

Since $\sin^2\theta + \cos^2\theta = 1$:

$$= 1 + 2\sin\theta\cos\theta = \text{RHS} \quad \text{(Proved)}$$

Q3. $\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

- **LHS:**

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{1}{\tan\theta} = \text{RHS} \quad \text{(Proved)}$$

Q4. $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} = 1$

- **LHS:**

$$\sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta = \sin^2\theta + \cos^2\theta = 1 = \text{RHS} \quad \text{(Proved)}$$

Q5. $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

- **LHS:**

$$\cos^2\theta - (1 - \cos^2\theta) = \cos^2\theta - 1 + \cos^2\theta = 2\cos^2\theta - 1 = \text{RHS} \quad \text{(Proved)}$$

Q6. $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

- **LHS:**

$$(1 - \sin^2\theta) - \sin^2\theta = 1 - 2\sin^2\theta = \text{RHS} \quad \text{(Proved)}$$

Q7. $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

- **LHS:** Multiply numerator and denominator by $(1 + \sin\theta)$:

$$\frac{(1 - \sin\theta)(1 + \sin\theta)}{\cos\theta\,(1 + \sin\theta)} = \frac{1 - \sin^2\theta}{\cos\theta\,(1 + \sin\theta)} = \frac{\cos^2\theta}{\cos\theta\,(1 + \sin\theta)} = \frac{\cos\theta}{1 + \sin\theta} = \text{RHS} \quad \text{(Proved)}$$

Q8. $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

- **LHS:**

$$\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2 = \frac{(1 - \sin\theta)^2}{\cos^2\theta} = \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}$$

$$= \frac{(1 - \sin\theta)^2}{(1 - \sin\theta)(1 + \sin\theta)} = \frac{1 - \sin\theta}{1 + \sin\theta} = \text{RHS} \quad \text{(Proved)}$$

Q9. $(\tan\theta + \cot\theta)^2 = \sec^2\theta\csc^2\theta$

- **LHS:**

$$\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)^2 = \left(\frac{1}{\sin\theta\cos\theta}\right)^2 = \frac{1}{\sin^2\theta\cos^2\theta} = \sec^2\theta\csc^2\theta$$

$$= \text{RHS} \quad \text{(Proved)}$$

Q10. $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \tan\theta + \sec\theta$

- **LHS:** Replace $1 = \sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$ in numerator:

$$\frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}=\frac{(\tan\theta+\sec\theta)[1-(\sec\theta-\tan\theta)]}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)(1-\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}=\tan\theta+\sec\theta=\text{RHS}\quad\text{(Proved)}$$

Q11. $\boldsymbol{\sin^3\theta-\cos^3\theta=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}$

- **LHS:** Using $a^3-b^3=(a-b)(a^2+ab+b^2)$:

$$(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)$$

Since $\sin^2\theta+\cos^2\theta=1$:

$$=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)=\text{RHS}\quad\text{(Proved)}$$

Q12. $\boldsymbol{\sin^6\theta-\cos^6\theta=(\sin^2\theta-\cos^2\theta)(1-\sin^2\theta\cos^2\theta)}$

- **LHS:** Express as $(\sin^2\theta)^3-(\cos^2\theta)^3$:

$$=(\sin^2\theta-\cos^2\theta)(\sin^4\theta+\sin^2\theta\cos^2\theta+\cos^4\theta)$$

$$=(\sin^2\theta-\cos^2\theta)[(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta]$$

$$=(\sin^2\theta-\cos^2\theta)(1-\sin^2\theta\cos^2\theta)=\text{RHS}\quad\text{(Proved)}$$

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Exercise 6.4

Q1. Find values without using calculator:

- **(i)** $\sin 30^\circ = \frac{1}{2}$
- **(ii)** $\cos 30^\circ = \frac{\sqrt{3}}{2}$
- **(iii)** $\tan\frac{\pi}{6} = \tan 30^\circ = \frac{1}{\sqrt{3}}$
- **(iv)** $\tan 60^\circ = \sqrt{3}$
- **(v)** $\sec 60^\circ = 2$
- **(vi)** $\cos\frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}$
- **(vii)** $\cot 60^\circ = \frac{1}{\sqrt{3}}$
- **(viii)** $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- **(ix)** $\sec 30^\circ = \frac{2}{\sqrt{3}}$
- **(x)** $\csc 30^\circ = 2$
- **(xi)** $\sin 45^\circ = \frac{1}{\sqrt{2}}$
- **(xii)** $\cos\frac{\pi}{4} = \cos 45^\circ = \frac{1}{\sqrt{2}}$

Q2. Evaluate:

- **(i)** $2\sin 60^\circ \cos 60^\circ$

$$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

- **(ii)** $2\cos\frac{\pi}{3}\sin\frac{\pi}{3}$

$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$$

- **(iii)** $2\sin 45^\circ + 2\cos 45^\circ$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + 2\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

- **(iv)** $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = 1$$

- **(v)** $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

- **(vi)** $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

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- **(vii)** $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \mathbf{\frac{\sqrt{3}}{2}}$$

- **(viii)** $\tan\frac{\pi}{6}\cot\frac{\pi}{6} + 1$

$$= \left(\frac{1}{\sqrt{3}}\right)(\sqrt{3}) + 1 = 1 + 1 = \mathbf{2}$$

Q3. If $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, find:

- **(i)** $2\sin 45^\circ - 2\cos 45^\circ$

$$= 2\left(\frac{1}{\sqrt{2}}\right) - 2\left(\frac{1}{\sqrt{2}}\right) = \mathbf{0}$$

- **(ii)** $3\cos 45^\circ + 4\sin 45^\circ$

$$= 3\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{\sqrt{2}}\right) = \mathbf{\frac{7}{\sqrt{2}}}$$

- **(iii)** $5\cos 45^\circ - 3\sin 45^\circ$

$$= 5\left(\frac{1}{\sqrt{2}}\right) - 3\left(\frac{1}{\sqrt{2}}\right) = \frac{\mathbf{2}}{\sqrt{\mathbf{2}}} = \mathbf{\sqrt{2}}$$

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Exercise 6.5

Q1. Find values of x, y, z from right-angled triangles:

- **(i)** Given $\theta = 30°$, adjacent $y = 4$ cm:
  - $\tan 30° = \frac{x}{y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{4} \Rightarrow \mathbf{x = \frac{4}{\sqrt{3}}}$ cm
  - $\cos 30° = \frac{y}{z} \Rightarrow \frac{\sqrt{3}}{2} = \frac{4}{z} \Rightarrow \mathbf{z = \frac{8}{\sqrt{3}}}$ cm
- **(ii)** Given $\theta = \frac{\pi}{4} = 45°$, opposite $y = \sqrt{3}$ cm:
  - $\tan 45° = \frac{y}{x} \Rightarrow 1 = \frac{\sqrt{3}}{x} \Rightarrow \mathbf{x = \sqrt{3}}$ cm
  - $\sin 45° = \frac{y}{z} \Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{z} \Rightarrow \mathbf{z = \sqrt{6}}$ cm
- **(iii)** Given $\theta = 60°$, hypotenuse $z = 2$ cm:
  - $\sin 60° = \frac{y}{z} \Rightarrow \frac{\sqrt{3}}{2} = \frac{y}{2} \Rightarrow \mathbf{y = \sqrt{3}}$ cm
  - $\cos 60° = \frac{x}{z} \Rightarrow \frac{1}{2} = \frac{x}{2} \Rightarrow \mathbf{x = 1}$ cm
- **(iv)** Given $\theta = 45°$, adjacent $y = 4$ cm:
  - $\tan 45° = \frac{x}{y} \Rightarrow 1 = \frac{x}{4} \Rightarrow \mathbf{x = 4}$ cm
  - $\cos 45° = \frac{y}{z} \Rightarrow \frac{1}{\sqrt{2}} = \frac{4}{z} \Rightarrow \mathbf{z = 4\sqrt{2}}$ cm

Q2. Find unknown side and angles of triangles:

- **(i)** Given $a = \sqrt{3}$ cm, $c = \sqrt{13}$ cm, $m\angle B = 90°$:
  - $b = \sqrt{a^2 + c^2} = \sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{13}\right)^2} = \sqrt{3 + 13} = \sqrt{16} = \mathbf{4}$ cm
  - $\tan A = \frac{a}{c} = \frac{\sqrt{3}}{\sqrt{13}} \Rightarrow m\angle A \approx \mathbf{25.66°}$
  - $m\angle C = 90° - 25.66° = \mathbf{64.34°}$
- **(ii)** Given $a = 4$ cm, $c = 4$ cm, $m\angle B = 90°$:
  - $b = \sqrt{4^2 + 4^2} = \sqrt{32} = \mathbf{4\sqrt{2}}$ cm
  - Since $a = c$, $m\angle A = m\angle C = \mathbf{45°}$

Q3. Side of square field = 60 m. Find length of diagonal.

- **Step:** $d = \sqrt{60^2 + 60^2} = \sqrt{3600 + 3600} = \sqrt{7200} = \mathbf{60\sqrt{2}}$ m $\approx \mathbf{84.85}$ m

Solve triangle ABC ($m\angle B = 90°$):

- **Q4.** $m\angle C = 60°, c = 3\sqrt{3}$ cm
  - $m\angle A = 90° - 60° = \mathbf{30°}$
  - $\sin 60° = \frac{c}{b} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{b} \Rightarrow \mathbf{b = 6}$ cm
  - $\tan 60° = \frac{c}{a} \Rightarrow \sqrt{3} = \frac{3\sqrt{3}}{a} \Rightarrow \mathbf{a = 3}$ cm
- **Q5.** $m\angle C = 45°, a = 8$ cm
  - $m\angle A = \mathbf{45°}, c = a = \mathbf{8}$ cm

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- $b = \sqrt{8^2 + 8^2} = \mathbf{8\sqrt{2}}$ cm

- **Q6.** $a = 12$ cm, $c = 6$ cm
  - $b = \sqrt{12^2 + 6^2} = \sqrt{144 + 36} = \sqrt{180} = \mathbf{6\sqrt{5}}$ cm
  - $\tan A = \frac{12}{6} = 2 \Longrightarrow m\angle A \approx \mathbf{63.43}^\circ$
  - $m\angle C = 90^\circ - 63.43^\circ = \mathbf{26.57}^\circ$
- **Q7.** $m\angle A = 60^\circ, c = 4$ cm
  - $m\angle C = \mathbf{30}^\circ$
  - $\tan 60^\circ = \frac{a}{c} \Longrightarrow \sqrt{3} = \frac{a}{4} \Longrightarrow \mathbf{a = 4\sqrt{3}}$ cm
  - $\cos 60^\circ = \frac{c}{b} \Longrightarrow \frac{1}{2} = \frac{4}{b} \Longrightarrow \mathbf{b = 8}$ cm
- **Q8.** $m\angle A = 60^\circ, c = 4$ cm *(Same as Q7)* $\Longrightarrow \mathbf{a = 4\sqrt{3}}$ cm, $\mathbf{b = 8}$ cm, $\mathbf{m\angle C = 30}^\circ$
- **Q9.** $b = 10$ cm, $a = 6$ cm
  - $c = \sqrt{10^2 - 6^2} = \sqrt{64} = \mathbf{8}$ cm
  - $\sin A = \frac{6}{10} = 0.6 \Longrightarrow m\angle A \approx \mathbf{36.87}^\circ$
  - $m\angle C = 90^\circ - 36.87^\circ = \mathbf{53.13}^\circ$

Q10. Canal with points Q, R on one side, P opposite to R. Given $QR = 5$, $QP = 13$:

- **Width of canal (PR):**

$$PR = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = \mathbf{12}$$

- **Angle PQR:**

$$\cos(\angle PQR) = \frac{5}{13} \Longrightarrow m\angle PQR = \arccos\left(\frac{5}{13}\right) \approx \mathbf{67.38}^\circ$$

Q11. Calculate length x in the adjoining figure:

- **Step 1:** In $\triangle ABD$, hypotenuse $AD = 17$, $AB = 10$.

$$BD = \sqrt{17^2 - 10^2} = \sqrt{289 - 100} = \sqrt{189} \text{ cm}$$

- **Step 2:** In $\triangle BCD$, hypotenuse $BD = \sqrt{189}$, $CD = 8$.

$$x = \sqrt{\left(\sqrt{189}\right)^2 - 8^2} = \sqrt{189 - 64} = \sqrt{125} = \mathbf{5\sqrt{5}} \text{ cm} \approx \mathbf{11.18} \text{ cm}$$

Q12. Ladder foot 2 m from wall, length 8 m. Find height on wall:

- $h = \sqrt{8^2 - 2^2} = \sqrt{64 - 4} = \sqrt{60} = \mathbf{2\sqrt{15}}$ m $\approx \mathbf{7.75}$ m

Q13. Rectangle $ABCD$ with diagonal $(x + 9)$ m, sides $(x + 7)$ m and x m. Find x:

- **By Pythagoras Theorem:**

$$(x + 7)^2 + x^2 = (x + 9)^2$$
$$x^2 + 14x + 49 + x^2 = x^2 + 18x + 81$$
$$x^2 - 4x - 32 = 0$$
$$(x - 8)(x + 4) = 0$$

Since side length must be positive: $\mathbf{x = 8}$ m

Q14. Calculate x in each case:

- **Case 1:** Right triangle with hypotenuse 20 cm, base 12 cm, perpendicular x cm:

$$x = \sqrt{20^2 - 12^2} = \sqrt{400 - 144} = \sqrt{256} = \mathbf{16} \text{ cm}$$

- **Case 2:** $\triangle ADC$ split into two right triangles at B with $BC = 4$ cm, $CD = 5$ cm, and $AB = BC = 4$ cm:
  - In $\triangle BCD$: $BD = \sqrt{5^2 - 4^2} = 3$ cm
  - In $\triangle ABD$: $x = \sqrt{AB^2 + BD^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = \mathbf{5}$ cm

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Exercise 6.6

Q1. Angle of elevation to top of flag post from 40 m away is 60°. Find height:

- $\tan 60^\circ = \frac{h}{40} \Rightarrow \sqrt{3} = \frac{h}{40} \Rightarrow h = \mathbf{40\sqrt{3}}$ m $\approx \mathbf{69.28}$ m

Q2. Isosceles triangle with vertical angle 120° and base 10 cm. Find altitude:

- Altitude bisects base into 5 cm and vertical angle into 60°.
- In right triangle: $\tan 60^\circ = \frac{5}{h} \Rightarrow \sqrt{3} = \frac{5}{h} \Rightarrow h = \mathbf{\frac{5}{\sqrt{3}}}$ cm $\approx \mathbf{2.89}$ cm

Q3. Tree height = 72 m, distance = 100 m. Find angle of elevation θ:

- $\tan\theta = \frac{72}{100} = 0.72 \Rightarrow \theta = \arctan(0.72) \approx \mathbf{35.75^\circ}$

Q4. Ladder at 60° reaches height 10 m. Find ladder length L:

- $\sin 60^\circ = \frac{10}{L} \Rightarrow \frac{\sqrt{3}}{2} = \frac{10}{L} \Rightarrow L = \mathbf{\frac{20}{\sqrt{3}}}$ m $\approx \mathbf{11.55}$ m

Q5. Lighthouse height = 150 m, angle of depression = 60°. Find distance d:

- $\tan 60^\circ = \frac{150}{d} \Rightarrow \sqrt{3} = \frac{150}{d} \Rightarrow d = \mathbf{\frac{150}{\sqrt{3}}} = \mathbf{50\sqrt{3}}$ m $\approx \mathbf{86.60}$ m

Q6. Elevation increases from 15° to 30° upon walking 100 m towards pole. Find height h:

- Let initial distance be $x + 100$:
  - $\tan 30^\circ = \frac{h}{x} \Rightarrow x = h\sqrt{3}$
  - $\tan 15^\circ = \frac{h}{x+100} \Rightarrow h = \left(h\sqrt{3} + 100\right)\tan 15^\circ$
  - Using $\tan 15^\circ = 2 - \sqrt{3}$:

$$h = \mathbf{50}\text{ m}$$

Q7. Tower height = 300 m, shadow = 450 m. Find Sun's elevation θ:

- $\tan\theta = \frac{300}{450} = \frac{2}{3} \Rightarrow \theta = \arctan\left(\frac{2}{3}\right) \approx \mathbf{33.69^\circ}$

Q8. Cliff elevation changes from 25° to 45° after walking 100 m towards it. Find height h:

- At 45°, distance to cliff = h.
- At 25°, distance = $h + 100$.
- $\tan 25^\circ = \frac{h}{h+100} \Rightarrow 0.4663(h + 100) = h \Rightarrow 0.5337h = 46.63 \Rightarrow h \approx \mathbf{87.37}$ m

Q9. Hill height = 300 m. Angles of depression of river shores are 70° and 50°. Find river width and distance from hill:

- **Near shore distance (d_1):** $d_1 = \frac{300}{\tan 70^\circ} = \frac{300}{2.7475} = \mathbf{109.19}$ m
- **Far shore distance (d_2):** $d_2 = \frac{300}{\tan 50^\circ} = \frac{300}{1.1918} = \mathbf{251.73}$ m
- **Width of river:** $d_2 - d_1 = 251.73 - 109.19 = \mathbf{142.54}$ m

Q10. Kite string = 120 m, angle of elevation = 50°. Find height h:

- $\sin 50^\circ = \frac{h}{120} \Rightarrow h = 120 \times 0.7660 = \mathbf{91.92}$ m

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Review Exercise 6

Q1. Multiple Choice Questions:

- **(i)** Value of $\tan^{-1} 2$ in radians is: **(d) 1.107** *(or closest scalar numerical value)*
- **(ii)** In right triangle, hypotenuse $= 13$, $\theta = 30°$. Opposite side is: **(a) 6.5 units** ($\sin 30° = 0.5 \times 13$)
- **(iii)** Person 50 m away sees building top at 45°. Height is: **(a) 50 m**
- **(iv)** $\sec^2 \theta - \tan^2 \theta =$: **(b) 1**
- **(v)** If $\sin \theta = \frac{3}{5}$, $\cos^2 \theta =$: **(c)** $\frac{16}{25}\left(1 - \frac{9}{25}\right)$
- **(vi)** $\frac{5\pi}{24}$ rad $=$: **(b)** $37.5°$
- **(vii)** $292.5° =$: **(b)** $\frac{13\pi}{8}$ rad
- **(viii)** Valid identity is: **(a)** $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
- **(ix)** $\sin 60° =$: **(d)** $\frac{\sqrt{3}}{2}$
- **(x)** $\cos^2 100\,\pi + \sin^2 100\,\pi =$: **(a) 1**

Q2. Angle Conversions:

- **(a) Degrees to Radians (in terms of π):**
  - **(i)** $255°$**:** $255 \times \frac{\pi}{180} = \mathbf{\frac{17\pi}{12}}$ rad
  - **(ii)** $75°45'$**:** $75.75 \times \frac{\pi}{180} = \mathbf{\frac{101\pi}{240}}$ rad
  - **(iii)** $142.5°$**:** $142.5 \times \frac{\pi}{180} = \mathbf{\frac{19\pi}{24}}$ rad
- **(b) Radians to Degrees:**
  - **(i)** $\frac{17\pi}{24}$ rad**:** $\frac{17\times180°}{24} = \mathbf{127.5° = 127°30'}$
  - **(ii)** $\frac{7\pi}{12}$ rad**:** $\frac{7\times180°}{12} = \mathbf{105°}$
  - **(iii)** $\frac{11\pi}{16}$ rad**:** $\frac{11\times180°}{16} = \mathbf{123.75° = 123°45'}$

Q3. Prove identities:

- **(i)** $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$
  - **LHS:** Multiply numerator & denominator by $(1 + \cos\theta)$:

$$\frac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta} = \frac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta} = \text{RHS}$$

- **(ii)** $\sin\theta\,(\csc\theta - \sin\theta) = \frac{1}{\sec^2\theta}$
  - **LHS:**

$$\sin\theta\csc\theta - \sin^2\theta = 1 - \sin^2\theta = \cos^2\theta = \frac{1}{\sec^2\theta} = \text{RHS}$$

- **(iii)** $\frac{\csc\theta-\sec\theta}{\csc\theta+\sec\theta} = \frac{1-\tan\theta}{1+\tan\theta}$
  - **LHS:** Convert to sin and cos:

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$$\frac{\frac{1}{\sin\theta}-\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}+\frac{1}{\cos\theta}}=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

Divide numerator & denominator by $\cos\theta$:

$$=\frac{1-\tan\theta}{1+\tan\theta}=\text{RHS}$$

- **(iv)** $\tan\theta+\cot\theta=\frac{1}{\sin\theta\cos\theta}$
  - **LHS:**

$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta\cos\theta}=\text{RHS}$$

Q4. If $\tan\theta=\frac{3}{\sqrt{2}}$ in Quadrant I, find remaining ratios:

- **Perpendicular** $=3$, **Base** $=\sqrt{2}$.
- **Hypotenuse** $=\sqrt{3^2+\left(\sqrt{2}\right)^2}=\sqrt{9+2}=\sqrt{\mathbf{11}}$.
- **Ratios:**

$$\sin\theta=\frac{\mathbf{3}}{\sqrt{\mathbf{11}}},\quad \cos\theta=\sqrt{\frac{\mathbf{2}}{\mathbf{11}}},\quad \csc\theta=\frac{\sqrt{\mathbf{11}}}{\mathbf{3}},\quad \sec\theta=\sqrt{\frac{\mathbf{11}}{\mathbf{2}}},\quad \cot\theta=\frac{\sqrt{\mathbf{2}}}{\mathbf{3}}$$

Q5. Point on ground with angle of elevation 28° to top of 30 m building. Find distance d:

- $\tan 28^\circ=\frac{30}{d}\Rightarrow d=\frac{30}{0.5317}\approx \mathbf{56.42}$ m

Q6. Ladder length $=10$ m, angle with ground $=65^\circ$. Find height h on wall:

- $\sin 65^\circ=\frac{h}{10}\Rightarrow h=10\times 0.9063=\mathbf{9.06}$ m

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