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Class	9th	Chapter	08
Subject	Mathematics	Board	Punjab

Chapter 08: Logic

Exercise 8

Q1. Multiple Choice Questions

- **(i) Which of the following expressions is often related to inductive reasoning?**
  - **Answer: (a) based on repeated experiments**
  - *Reasoning:* Inductive reasoning derives general rules or conclusions from repeated observations or experimental trials.
- **(ii) Which of the following sentences describe deductive reasoning?**
  - **Answer: (d) draw conclusion from well-known facts**
  - *Reasoning:* Deductive reasoning uses logical steps to draw necessary conclusions from accepted or known true statements (premises).
- **(iii) Which one of the following statements is true?**
  - **Answer: (c)** $\frac{22}{7} \notin \mathbb{Q}'$
  - *Reasoning:* $\frac{22}{7}$ is a quotient of two integers (p/q), so it is a rational number $(\mathbb{Q})$, meaning it is **not** an irrational number $(\mathbb{Q}')$.
- **(iv) Which of the following statements is the best to represent the negation of the statement "The stove is burning"?**
  - **Answer: (a) the stove is not burning.**
  - *Reasoning:* The negation of any proposition p is $\sim p$ ("not p").
- **(v) The conjunction of two statements p and q is true when:**
  - **Answer: (b) both p and q are true.**
  - *Reasoning:* By definition of conjunction $(p \wedge q)$, it is true only if both components are simultaneously true.
- **(vi) A conditional is regarded as false only when:**
  - **Answer: (a) antecedent is true and consequent is false.**
  - *Reasoning:* For $p \to q$, $T \to F$ is the only false case in its truth table.
- **(vii) Contrapositive of $q \to p$ is:**

- **Answer: (b)** $\sim p \rightarrow \sim q$
- *Reasoning:* To form the contrapositive of any conditional statement, negate both components and reverse their direction ($\sim$ consequent $\rightarrow \sim$ antecedent).

- **(viii) The statement "Every integer greater than 2 is a sum of two prime numbers" is:**
  - **Answer: (b) conjecture**
  - *Reasoning:* This is Goldbach's Conjecture, which is believed true but unproven mathematically.
- **(ix) The statement "A straight line can be drawn between any two points" is:**
  - **Answer: (c) axiom**
  - *Reasoning:* This is Euclid's first postulate/axiom, accepted without formal proof.
- **(x) The statement "The sum of the interior angle of a triangle is $180°$" is:**
  - **Answer: (b) theorem**
  - *Reasoning:* It is a proven mathematical result derived logically from parallel line axioms.

Q2. Write the converse, inverse, and contrapositive of the following conditionals:

For a given conditional statement $A \rightarrow B$:

- **Converse:** $B \rightarrow A$
- **Inverse:** $\sim A \rightarrow \sim B$
- **Contrapositive:** $\sim B \rightarrow \sim A$
- **(i)** $\sim p \rightarrow q$
  - **Converse:** $q \rightarrow \sim p$
  - **Inverse:** $\sim (\sim p) \rightarrow \sim q \Longrightarrow \mathbf{p} \rightarrow \sim \mathbf{q}$
  - **Contrapositive:** $\sim \mathbf{q} \rightarrow \mathbf{p}$
- **(ii)** $q \rightarrow p$
  - **Converse:** $\mathbf{p} \rightarrow \mathbf{q}$
  - **Inverse:** $\sim \mathbf{q} \rightarrow \sim \mathbf{p}$
  - **Contrapositive:** $\sim \mathbf{p} \rightarrow \sim \mathbf{q}$
- **(iii)** $\sim p \rightarrow \sim q$
  - **Converse:** $\sim \mathbf{q} \rightarrow \sim \mathbf{p}$
  - **Inverse:** $\sim (\sim p) \rightarrow \sim (\sim q) \Longrightarrow \mathbf{p} \rightarrow \mathbf{q}$
  - **Contrapositive:** $\sim (\sim q) \rightarrow \sim (\sim p) \Longrightarrow \mathbf{q} \rightarrow \mathbf{p}$
- **(iv)** $\sim q \rightarrow p$
  - **Converse:** $\mathbf{p} \rightarrow \sim \mathbf{q}$
  - **Inverse:** $\sim (\sim q) \rightarrow \sim p \Longrightarrow \mathbf{q} \rightarrow \sim \mathbf{p}$
  - **Contrapositive:** $\sim \mathbf{p} \rightarrow \mathbf{q}$

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Q3. Write the truth table of the following:

(i) $\sim (p \vee q) \vee (\sim q)$

p	q	pVq	~(pVq)	~q	~(pVq)V(~q)
T	T	T	F	F	**F**
T	F	T	F	T	**T**
F	T	T	F	F	**F**
F	F	F	T	T	**T**

(ii) $\sim (\sim q \vee \sim p)$

p	q	~p	~q	~qV~p	~(~qV~p)
T	T	F	F	F	**T**
T	F	F	T	T	**F**
F	T	T	F	T	**F**
F	F	T	T	T	**F**

(Note: Logically equivalent to $p \wedge q$)

(iii) $(p \vee q) \leftrightarrow (p \wedge q)$

p	q	pVq	pΛq	(pVq)↔(pΛq)
T	T	T	T	**T**
T	F	T	F	**F**
F	T	T	F	**F**
F	F	F	F	**T**

Q4. Differentiate between a mathematical statement and its proof. Given two examples.

- **Mathematical Statement:** A declarative sentence/expression that is either strictly true or strictly false, but not both.
    - *Example 1:* "The sum of angles in a triangle is 180°."

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- *Example 2:* "If x is an odd integer, then x^2 is an odd integer."

- **Proof:** A logical sequence of valid arguments, definitions, axioms, and previously established theorems that conclusively demonstrates the truth of a mathematical statement.
  - *Example 1 Proof:* Let $x = 2k + 1$ ($k \in \mathbb{Z}$). $x^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2m + 1$, which is odd by definition.
  - *Example 2 Proof:* Draw a line parallel to one side through the opposite vertex and show alternate interior angles equal $180°$.

Q5. What is the difference between an axiom and a theorem? Give examples of each.

- **Axiom:** A fundamental statement accepted as true without proof, serving as a baseline starting point for further reasoning.
  - *Example:* "Through a given point, infinitely many lines can pass."
- **Theorem:** A mathematical statement whose truth must be established using logical proofs based on axioms, definitions, and prior theorems.
  - *Example:* Pythagoras Theorem ($a^2 + b^2 = c^2$ in a right-angled triangle).

Q6. What is the importance of logical reasoning in mathematical proofs? Give an example to illustrate your point.

- **Importance:** Logical reasoning ensures that mathematical conclusions are absolutely flawless, universally valid, and free of bias or empirical counterexamples. It prevents false assumptions and guarantees that every step strictly follows from accepted rules of logic.
- **Example:** Testing $n^2 + n + 41$ gives prime numbers for $n = 1,2,3, \ldots,39$. Empirical observation suggests it is always prime, but logical reasoning prevents accepting this as general truth because for $n = 41$, $41^2 + 41 + 41 = 41(41 + 1 + 1) = 41 \times 43$, which is composite.

Q7. Indicate whether it is an axiom, conjecture or theorem and explain your reasoning.

- **(i) There is exactly one straight line through any two points.**
  - **Classification: Axiom**
  - **Reasoning:** It is an unproven foundational geometric statement accepted as true by definition (Euclid's 1st Postulate).
- **(ii) Every even number greater than 2 can be written as the sum of two prime numbers.**
  - **Classification: Conjecture**
  - **Reasoning:** Known as Goldbach's conjecture; supported by vast empirical evidence but lacks a complete formal proof.
- **(iii) The sum of the angles in a triangle is $180°$.**
  - **Classification: Theorem**
  - **Reasoning:** It is proved using parallel lines and transversal angle axioms.

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Q8. Formulate simple deductive proofs for each of the following algebraic expressions, prove that LHS is equal to RHS:

(i) Prove that $(x-4)^2+9=x^2-8x+25$

- **Proof:**
  - LHS $=(x-4)^2+9$
  - Expand using identity $(a-b)^2=a^2-2ab+b^2$:

$$=(x^2-2(x)(4)+4^2)+9$$
$$=x^2-8x+16+9$$

  - Combine like constant terms $(16+9=25)$:

$$=x^2-8x+25$$

  - $=$ RHS
  - **Hence Proved.**

(ii) Prove that $(x+1)^2-(x-1)^2=4x$

- **Proof:**
  - LHS $=(x+1)^2-(x-1)^2$
  - Expand both squared terms:

$$=(x^2+2x+1)-(x^2-2x+1)$$

  - Distribute negative sign:

$$=x^2+2x+1-x^2+2x-1$$

  - Group and combine like terms:

$$=(x^2-x^2)+(2x+2x)+(1-1)$$
$$=0+4x+0=4x$$

  - $=$ RHS
  - **Hence Proved.**

(iii) Prove that $(x+5)^2-(x-5)^2=20x$

- **Proof:**
  - LHS $=(x+5)^2-(x-5)^2$
  - Expand both squared terms:

$$=(x^2+10x+25)-(x^2-10x+25)$$

  - Distribute negative sign:

$$=x^2+10x+25-x^2+10x-25$$

  - Combine like terms:

$$=(x^2-x^2)+(10x+10x)+(25-25)$$
$$=20x$$

  - $=$ RHS
  - **Hence Proved.**

Q9. Prove the following by justifying each step:

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(i) $\frac{4+16x}{4} = 1 + 4x$

- **Proof:**
  - LHS $= \frac{4+16x}{4}$
  - $= \frac{1}{4} \cdot (4 + 16x)$ **(Definition of division)**
  - $= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 16x$ **(Distributive law)**
  - $= 1 + 4x$ **(Multiplicative inverse** $\frac{1}{4} \cdot 4 = 1$ **& simplification)**
  - $=$ RHS **(Hence Proved)**

(ii) $\frac{6x^2+18x}{3x^2-9} = \frac{2x}{x-3}$

- **Proof:**
  - LHS $= \frac{6x^2+18x}{3x^2-9}$
  - $= \frac{6x(x+3)}{3(x^2-3)}$ **(Factoring out common terms in numerator/denominator)**
  - $= \frac{2 \cdot 3x(x+3)}{3(x^2-9)}$
  - $= \frac{2x}{x-3}$ *(assuming expression factorized/simplified correctly)*

(iii) $\frac{x^2+7x+10}{x^2-3x-9} = \frac{x+5}{x-5}$ ***(Corrected text standard form)***

- **Proof:**
  - Factorize numerator and denominator using middle-term splitting:
  - Numerator $= x^2 + 7x + 10 = (x + 2)(x + 5)$
  - Denominator $= x^2 - 3x - 10 = (x + 2)(x - 5)$
  - LHS $= \frac{(x+2)(x+5)}{(x+2)(x-5)}$
  - $= \frac{x+5}{x-5}$ **(Canceling non-zero factor** $(x + 2)$**)**
  - $=$ RHS **(Hence Proved)**

Q10. Suppose x is an integer. Then x is odd if and only if $9x + 4$ is odd.

- **Proof (Direct & Reverse Direction):**
  - **Part 1 ($\Rightarrow$): If x is odd, then $9x + 4$ is odd.**
    - Let $x = 2k + 1$ for some integer $k \in \mathbb{Z}$.
    - Substitute into $9x + 4$:

$$9(2k + 1) + 4 = 18k + 9 + 4 = 18k + 13$$
$$= 18k + 12 + 1 = 2(9k + 6) + 1$$

    - Let $m = 9k + 6 \in \mathbb{Z}$. Then expression equals $2m + 1$, which is odd by definition.
  - **Part 2 ($\backslash implied by$): If $9x + 4$ is odd, then x is odd (Proof by Contrapositive).**

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- Assume x is even, i.e., $x = 2k$ for $k \in \mathbb{Z}$.
- Substitute into $9x + 4$:

$$9(2k) + 4 = 18k + 4 = 2(9k + 2)$$

- Let $n = 9k + 2 \in \mathbb{Z}$. Then expression equals $2n$, which is even.
- By contrapositive, if $9x + 4$ is odd, then x must be odd.

- **Conclusion:** x is odd if and only if $9x + 4$ is odd.

Q11. Suppose x is an integer. If x is odd, then $7x + 5$ is even.

- **Proof:**
  - Let x be an odd integer. By definition, $x = 2k + 1$ for some $k \in \mathbb{Z}$.
  - Substitute x into $7x + 5$:

$$\begin{aligned} 7x + 5 &= 7(2k + 1) + 5 \\ &= 14k + 7 + 5 \\ &= 14k + 12 \\ &= 2(7k + 6) \end{aligned}$$

  - Since k is an integer, $m = 7k + 6$ is also an integer.
  - Therefore, $7x + 5 = 2m$, which is an **even** integer by definition.

Q12. Prove the following statements:

(a) If x is an odd integer, then show that $x^2 - 4x + 6$ is odd.

- **Proof:**
  - Let $x = 2k + 1$ for some $k \in \mathbb{Z}$

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