بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

Quran Education Point

QuranEducationPoint.Com

Class	9th	Chapter	10
Subject	Mathematics	Board	Punjab

Chapter 10: Graphs of Functions

Exercise 10.1

Q1. Sketch the graph of the following linear functions:

(i) $y = 3x - 5$

- **Step 1: Find Intercepts**
  - y-intercept (put $x = 0$): $y = 3(0) - 5 = -5 \Longrightarrow (0, -5)$.
  - x-intercept (put $y = 0$): $0 = 3x - 5 \Longrightarrow 3x = 5 \Longrightarrow x = \frac{5}{3} \approx 1.67 \Longrightarrow (1.67,0)$.
- **Step 2: Table of Values**

$$\begin{array}{lccc} \backslash hline x & -1 & 0 & 2 \\ \backslash hline y & -8 & -5 & 1 \\ & & & \backslash hline \end{array}$$

- **Step 3: Graph Characteristics**

Plot the points $(0, -5)$, $(1.67,0)$, and $(2,1)$ and connect them with a straight line rising to the right with a slope $m = 3$.

(ii) $y = -2x + 8$

- **Step 1: Find Intercepts**
  - y-intercept (put $x = 0$): $y = -2(0) + 8 = 8 \Longrightarrow (0,8)$.
  - x-intercept (put $y = 0$): $0 = -2x + 8 \Longrightarrow 2x = 8 \Longrightarrow x = 4 \Longrightarrow (4,0)$.
- **Step 2: Table of Values**

$$\begin{array}{lccc} \backslash hline x & 0 & 2 & 4 \\ \backslash hline y & 8 & 4 & 0 \\ & & & \backslash hline \end{array}$$

- **Step 3: Graph Characteristics**

Plot the points $(0,8)$, $(2,4)$, and $(4,0)$ and connect them with a straight line falling to the right with slope $m = -2$.

(iii) $y = 0.5x - 1$

- **Step 1: Find Intercepts**

- y-intercept (put $x = 0$): $y = 0.5(0) - 1 = -1 \Longrightarrow (0, -1)$.
- x-intercept (put $y = 0$): $0 = 0.5x - 1 \Longrightarrow 0.5x = 1 \Longrightarrow x = 2 \Longrightarrow (2,0)$.

- **Step 2: Table of Values**

$$\begin{array}{lccc} \backslash hline x & -2 & 0 & 4 \\ \backslash hline y & -2 & -1 & 1 \\ & & & \backslash hline \end{array}$$

- **Step 3: Graph Characteristics**

Plot the points $(-2, -2)$, $(0, -1)$, and $(2,0)$ and draw a straight line through them.

Q2. Plot the graph of the following quadratic and cubic functions:

(i) $y = x^3 + 2x^2 - 5x - 6$ for $-3.5 \leq x \leq 2.5$

- **Step 1: Table of Values**
  - $x = -3.5 \Longrightarrow y = (-3.5)^3 + 2(-3.5)^2 - 5(-3.5) - 6 = -42.875 + 24.5 + 17.5 - 6 = -6.875$
  - $x = -3 \Longrightarrow y = (-3)^3 + 2(-3)^2 - 5(-3) - 6 = -27 + 18 + 15 - 6 = 0$
  - $x = -2 \Longrightarrow y = (-2)^3 + 2(-2)^2 - 5(-2) - 6 = -8 + 8 + 10 - 6 = 4$
  - $x = -1 \Longrightarrow y = (-1)^3 + 2(-1)^2 - 5(-1) - 6 = -1 + 2 + 5 - 6 = 0$
  - $x = 0 \Longrightarrow y = -6$
  - $x = 1 \Longrightarrow y = (1)^3 + 2(1)^2 - 5(1) - 6 = 1 + 2 - 5 - 6 = -8$
  - $x = 2 \Longrightarrow y = (2)^3 + 2(2)^2 - 5(2) - 6 = 8 + 8 - 10 - 6 = 0$
  - $x = 2.5 \Longrightarrow y = (2.5)^3 + 2(2.5)^2 - 5(2.5) - 6 = 15.625 + 12.5 - 12.5 - 6 = 9.625$
- **Step 2: Summary Table**

$$\begin{array}{lcccccccc} \backslash hline x & -3.5 & -3 & -2 & -1 & 0 & 1 & 2 & 2.5 \\ \backslash hline y & -6.88 & 0 & 4 & 0 & -6 & -8 & 0 & 9.63 \\ & & & & & & & & \backslash hline \end{array}$$

- **Step 3: Plotting**

Plot the coordinates and draw a smooth S-shaped cubic curve passing through the x-intercepts at $x = -3, -1, 2$.

(ii) $y = x^2 + x - 2$

- **Step 1: Table of Values**

$$\begin{array}{lccccccc} \backslash hline x & -3 & -2 & -0.5 & 0 & 1 & 2 \\ \backslash hline y & 4 & 0 & -2.25 & -2 & 0 & 4 \\ & & & & & & \backslash hline \end{array}$$

- **Step 2: Key Features**
  - Vertex at $x = -\frac{b}{2a} = -\frac{1}{2} = -0.5 \Longrightarrow y = -2.25$.
  - x-intercepts at $x = -2$ and $x = 1$.
  - y-intercept at $(0, -2)$.
- **Step 3: Plotting**

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

Draw a smooth upward-opening parabolic curve.

(iii) $y = x^3 + 3x^2 + 2x$ for $-2.5 \leq x \leq 0.5$

- **Step 1: Table of Values**
  - $x = -2.5 \Rightarrow y = (-2.5)^3 + 3(-2.5)^2 + 2(-2.5) = -15.625 + 18.75 - 5 = -1.875$
  - $x = -2 \Rightarrow y = (-2)^3 + 3(-2)^2 + 2(-2) = -8 + 12 - 4 = 0$
  - $x = -1.5 \Rightarrow y = (-1.5)^3 + 3(-1.5)^2 + 2(-1.5) = -3.375 + 6.75 - 3 = 0.375$
  - $x = -1 \Rightarrow y = (-1)^3 + 3(-1)^2 + 2(-1) = -1 + 3 - 2 = 0$
  - $x = -0.5 \Rightarrow y = (-0.5)^3 + 3(-0.5)^2 + 2(-0.5) = -0.125 + 0.75 - 1 = -0.375$
  - $x = 0 \Rightarrow y = 0$
  - $x = 0.5 \Rightarrow y = (0.5)^3 + 3(0.5)^2 + 2(0.5) = 0.125 + 0.75 + 1 = 1.875$
- **Step 2: Plotting**

Plot points and draw a smooth cubic curve with turning points between -2 and -1.

(iv) $y = 5x^2 - 2x - 3$

- **Step 1: Table of Values**

$$\begin{array}{lccccc} \hline x & -1 & 0 & 0.2 & 1 & 2 \\ \hline y & 4 & -3 & -3.2 & 0 & 13 \\ \hline \end{array}$$

- **Step 2: Plotting**

Draw a U-shaped parabola opening upwards with x-intercepts at $x = 1$ and $x = -0.6$.

Q3. Plot the graph of the following functions:

(i) $y = 4^x$

- **Exponential Growth Function**
- **Table of Values:**

$$\begin{array}{lccccc} \hline x & -2 & -1 & 0 & 1 & 2 \\ \hline y & 0.0625 & 0.25 & 1 & 4 & 16 \\ \hline \end{array}$$

- **Plotting:** Curve passes through $(0,1)$ and increases rapidly for $x > 0$, approaching $y = 0$ as an asymptote for $x < 0$.

(ii) $y = 5^{-x}$

- **Exponential Decay Function**
- **Table of Values:**

$$\begin{array}{lccccc} \hline x & -2 & -1 & 0 & 1 & 2 \\ \hline y & 25 & 5 & 1 & 0.2 & 0.04 \\ \hline \end{array}$$

- **Plotting:** Curve falls steeply from left to right, passing through $(0,1)$.

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

(iii) $y = \frac{1}{x-3}, x \neq 3$

- **Reciprocal Function with Asymptotes**
  - Vertical Asymptote at $x = 3$.
  - Horizontal Asymptote at $y = 0$.
- **Table of Values:**

x	1	2	2.5	3.5	4	5
y	-0.5	-1	-2	2	1	0.5

- **Plotting:** Two rectangular hyperbola branches—one in the lower left of $x = 3$, one in the upper right of $x = 3$.

(iv) $y = \frac{2}{x} + 3, x \neq 0$

- **Reciprocal Function**
  - Vertical Asymptote at $x = 0$ (y-axis).
  - Horizontal Asymptote shifted up to $y = 3$.
- **Table of Values:**

x	-2	-1	-0.5	0.5	1	2
y	2	1	-1	7	5	4

(v) $y = x^{\frac{1}{2}} = \sqrt{x}, x \geq 0$

- **Radical / Power Function**
- **Table of Values:**

x	0	1	4	9	16
y	0	1	2	3	4

- **Plotting:** Curve starts at the origin $(0,0)$ and curves smoothly into the first quadrant.

(vi) $y = 3x^{\frac{1}{3}} = 3\sqrt[3]{x}$

- **Cube Root Power Function**
- **Table of Values:**

x	-8	-1	0	1	8
y	-6	-3	0	3	6

- **Plotting:** Smooth S-shaped curve passing through origin $(0,0)$.

(vii) $y = 2x^{-2} = \frac{2}{x^2}, x \neq 0$

- **Reciprocal Power Function**
- **Table of Values:**

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

x	-2	-1	-0.5	0.5	1	2
y	0.5	2	8	8	2	0.5

- **Plotting:** Symmetric curve about the y-axis with both branches above the x-axis.

Exercise 10.2

Q1. Plot $y = 2x^2 - 4x + 3$ from $x = -1$ to 3. Draw tangent at $(2, 3)$ and find gradient.

- **Step 1: Calculate Table of Values**

x	-1	0	1	2	3
y	9	3	1	3	9

- **Step 2: Draw Curve**

Plot points $(-1,9), (0,3), (1,1), (2,3), (3,9)$ and join with a parabola.

- **Step 3: Gradient at** $(2,3)$
  - Tangent line slope using derivative concept / geometric tangent line: $\frac{dy}{dx} = 4x - 4$.
  - At $x = 2$: Gradient $= 4(2) - 4 = \mathbf{4}$.

Q2. Plot $y = 3x^2 + x + 1$ and draw tangent at $(1, 5)$. Find gradient.

- **Step 1: Table of Values**

x	-2	-1	0	1	2
y	11	3	1	5	15

- **Step 2: Gradient at** $(1,5)$
  - Tangent slope formula: $\frac{dy}{dx} = 6x + 1$.
  - At $x = 1$: Gradient $= 6(1) + 1 = \mathbf{7}$.

Q3. Student Strength Equation: $S = 1000e^{-t}$

- **(a) Graph equation for $t = 0$ (2016) to $t = 9$ (2025):**

t	0 (2016)	3 (2019)	7 (2023)	9 (2025)
$S = 1000e^{-t}$	1000	$1000(0.0498) \approx 50$	$1000(0.0009) \approx 1$	≈ 0

- **(b) Estimate strength in 2019 ($t = 3$) and 2023 ($t = 7$):**
  - In 2019 ($t = 3$): $S = 1000e^{-3} = 1000(0.049787) \approx \mathbf{50}$ students.
  - In 2023 ($t = 7$): $S = 1000e^{-7} = 1000(0.000912) \approx \mathbf{1}$ student.

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

Q4. Demand $P_d = 400 - 5Q$ and Supply $P_s = 3Q + 24$ for $Q = 0$ to 300

- **Step 1: Table of Values**

$\hline Q$	0	47	100	300
$\hline P_d = 400 - 5Q$	400	165	−100	−1100
$\hline P_s = 3Q + 24$	24	165	324	924
				$\hline$

- **Step 2: Market Equilibrium Point**

$$P_d = P_s \Longrightarrow 400 - 5Q = 3Q + 24 \Longrightarrow 8Q = 376 \Longrightarrow Q = 47, P = 165$$

Q5. Salary function $S(x) = 45000 + 4500x$ for $0 \leq x \leq 5$

- **Step 1: Table of Values**

$\hline x$ (years)	0	1	2	3	4	5
$\hline S(x)$ (Rs.)	45000	49500	54000	58500	63000	67500
						$\hline$

- **Step 2: Interpretation**

The graph is a straight line with initial starting salary of **Rs. 45,000** at $x = 0$ and an annual linear increment of **Rs. 4,500** per year.

Q6. School Bags Production: $C(x) = 1200 + 800x$ and $R(x) = 2500x$ *(Note: $R(x) = 2500x$ based on textbook standard linear revenue format)*

- **(a) Find Break-even Point:**

$$\text{Revenue} = \text{Cost} \Longrightarrow 2500x = 1200 + 800x$$

$$1700x = 1200 \Longrightarrow x = \frac{1200}{1700} \approx 0.705 \Longrightarrow \mathbf{1} \text{ bag (approx)}$$

(If revenue is $R(x) = 2500x$, break-even occurs virtually at 1 unit).

- **(b) Profit or Loss when 2500 bags are sold:**

$$\text{Revenue } R(250) = 2500(250) = 625{,}000$$

$$\text{Cost } C(250) = 1200 + 800(250) = 1200 + 200000 = 201{,}200$$

$$\text{Profit} = R(250) - C(250) = 625{,}000 - 201{,}200 = \text{Rs. } \mathbf{423{,}800} \text{ (Profit)}$$

- **(c) Plot Graphs:**

Plot both straight lines $C(x)$ and $R(x)$ to visually show the intersection point.

Q7. Newspaper Agency Profit: $p(x) = 0.10x + 70$

- **Step 1: Plotting Graph**

$\hline x$ (newspapers)	0	250	500
$\hline p(x)$ (Rs.)	70	95	120
			$\hline$

- **Step 2: Profit for 500 newspapers:**

$$p(500) = 0.10(500) + 70 = 50 + 70 = \text{Rs. } \mathbf{120}$$

Q8. Shirt Cost Function: $C(x) = 1500 + 10x + 0.2x^2$ for $0 \leq x \leq 150$

- **Cost of 200 shirts (evaluating formula at $x = 200$):**

$$C(200) = 1500 + 10(200) + 0.2(200)^2$$

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

$$C(200) = 1500 + 2000 + 0.2(40000) = 1500 + 2000 + 8000 = \text{Rs. } \mathbf{11,500}$$

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

Review Exercise 10

Q1. Multiple Choice Questions

- **(i) $x = 5$ represents:**
  - **Correct Option: (d) line ∥ to y-axis**
- **(ii) Slope of the line $y = 5x + 3$ is:**
  - **Correct Option: (c) 5**
- **(iii) The y-intercepts of $y = -2x - 1$ is:**
  - **Correct Option: (c) -1**
- **(iv) The graph of $y = x^3$, cuts the x-axis at:**
  - **Correct Option: (a) $x = 0$**
- **(v) The graph of 3^x represents:**
  - **Correct Option: (a) growth**
- **(vi) The graph of $y = -x^2 + 5$ opens:**
  - **Correct Option: (b) downward**
- **(vii) The graph of $y = x^2 - 9$ opens:**
  - **Correct Option: (a) upward**
- **(viii) $y = 5^x$ is _____ function.**
  - **Correct Option: (d) exponential**
- **(ix) Reciprocal function is:**
  - **Correct Option: (b) $y = \frac{2}{x}$**
- **(x) $y = -3x^3 + 7$ is _____ function.**
  - **Correct Option: (b) cubic**

Q2. Plot the graph of the following functions:

(i) $y = 3^{-x}$ for x from -2 to 4

- **Table of Values:**

$\hlinex$	-2	-1	0	1	2	3	4
$\hliney$	9	3	1	0.33	0.11	0.037	0.012

$\hline$

- Exponential decay curve passing through $(0,1)$.

(ii) $y = \frac{2}{x+7}, x \neq -7$

- **Reciprocal function:** Vertical asymptote at $x = -7$, horizontal asymptote at $y = 0$.
- **Table of Values:**

$\hlinex$	-9	-8	-7.5	-6.5	-6	-5
$\hliney$	-1	-2	-4	4	2	1

$\hline$

Q3. Magazine Sales Equation: $S = 200000\left(1 - e^{-0.05t}\right)$

- **(a) Plot graph for first 50 weeks:**

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

$\hline t$ (weeks)	0	5	20	35	50
$\hline S$	0	44,240	126,424	165,244	183,583
					$\hline$

- **(b) Calculate magazines sold when $t = 5$ and $t = 35$:**
  - For $t = 5$: $S = 200000(1 - e^{-0.25}) = 200000(1 - 0.7788) =$ **$44,240$** magazines.
  - For $t = 35$: $S = 200000(1 - e^{-1.75}) = 200000(1 - 0.17377) =$ **$165,246$** magazines.

Q4. Plot the graph of following for x from -5 to 5:

(i) $y = x^2 - 3$

- **Table of Values:**

$\hline x$	-5	-3	0	3	5
$\hline y$	22	6	-3	6	22
					$\hline$

- Upward opening parabola with vertex at $(0, -3)$.

(ii) $y = 15 - x^2$

- **Table of Values:**

$\hline x$	-5	-3	0	3	5
$\hline y$	-10	6	15	6	-10
					$\hline$

- Downward opening parabola with vertex at $(0,15)$.

Q5. Plot the graph of $y = \frac{1}{2}(x + 4)(x - 1)(x - 3)$ from -5 to 4

- **Table of Values:**
  - $x = -4 \Rightarrow y = 0$
  - $x = -2 \Rightarrow y = \frac{1}{2}(2)(-3)(-5) = 15$
  - $x = -1 \Rightarrow y = \frac{1}{2}(3)(-2)(-4) = 12$
  - $x = 0 \Rightarrow y = \frac{1}{2}(4)(-1)(-3) = 6$
  - $x = 1 \Rightarrow y = 0$
  - $x = 2 \Rightarrow y = \frac{1}{2}(6)(1)(-1) = -3$
  - $x = 3 \Rightarrow y = 0$
  - $x = 4 \Rightarrow y = \frac{1}{2}(8)(3)(1) = 12$
- Smooth cubic S-shaped curve crossing the x-axis at $x = -4, 1, 3$.

Q6. Market Functions: $P_s = Q^2 + 5$ and $P_d = Q^2 - 10Q$ for $Q = -20$ to 20

- **Table of Values:**

نوٹس حاصل کرنے کے لیے QuranEducationPoint.com لکھ کر سرچ کریں۔

$\hline Q$	-20	-10	0	10	20
$\hline P_s = Q^2 + 5$	405	105	5	105	405
$\hline P_d = Q^2 - 10Q$	600	200	0	0	200
					$\hline$

- Plot both quadratic curves on the same graph.

Q7. LED Manufacturer: $C(x) = 60000 + 250x$ and $R(x) = 1200x$

- **(a) Break-even Point:**

$$\text{Revenue} = \text{Cost} \Longrightarrow 1200x = 60000 + 250x$$

$$950x = 60000 \Longrightarrow x = \frac{60000}{950} \approx \mathbf{63.16} \text{ LEDs (approx 63 or 64 LEDs)}$$

- **(b) Profit or Loss when 100 LEDs are sold:**

$$\text{Revenue } R(100) = 1200(100) = 120{,}000$$

$$\text{Cost } C(100) = 60000 + 250(100) = 60000 + 25000 = 85{,}000$$

$$\text{Profit} = R(100) - C(100) = 120{,}000 - 85{,}000 = \text{Rs. } \mathbf{35{,}000} \text{ (Profit)}$$

QuranEducationPoint.Com