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Class	10th	Chapter	11
Subject	Mathematics	Board	Punjab

CHAPTER 11: Information Handling

EXERCISE 11.1.

Question 1

Data Given:

- **Scores:** $44-48, 49-53, 54-58, 59-63, 64-68, 69-73, 74-78$
- **Frequencies (f):** $3, 12, 15, 23, 12, 8, 7$
- **Step 1: Construct the Frequency Table**

To draw the cumulative frequency polygon (ogive) and determine values graphically, calculate the **Upper Class Boundaries (UCB)** and **Cumulative Frequencies (cf)**.

- Class Boundary Adjustment Unit $= \frac{49-48}{2} = 0.5$

Class Interval	Frequency (f)	Upper Class Boundary (UCB)	Cumulative Frequency (cf)
$44-48$	3	48.5	3
$49-53$	12	53.5	15
$54-58$	15	58.5	30
$59-63$	23	63.5	53

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Class Interval	Frequency (f)	Upper Class Boundary (UCB)	Cumulative Frequency (cf)
$64-68$	12	68.5	65
$69-73$	8	73.5	73
$74-78$	7	78.5	80

- **Total Number of Students (N):** 80

Part (i): Plot Cumulative Frequency Graph (Ogive)

1. **X-axis:** Upper Class Boundaries $(43.5,48.5,53.5,58.5,63.5,68.5,73.5,78.5)$.
2. **Y-axis:** Cumulative Frequency $(0$ to $80)$.
3. **Points to plot:**
   $(43.5,0), (48.5,3), (53.5,15), (58.5,30), (63.5,53), (68.5,65), (73.5,73), (78.5,80)$.
4. Join the points with a smooth curve.

Part (ii): Graphical & Formula Calculations

(Note: As per question instructions, values are rounded to whole numbers).

1. **Lower Quartile (Q_1):**
   - Position $= \frac{N}{4} = \frac{80}{4} = 20$th position
   - Value $\approx 55.17 \approx \mathbf{55}$
2. **Median (Q_2):**
   - Position $= \frac{N}{2} = \frac{80}{2} = 40$th position
   - Value $\approx 60.67 \approx \mathbf{61}$
3. **Upper Quartile (Q_3):**
   - Position $= \frac{3N}{4} = \frac{3\times80}{4} = 60$th position
   - Value $\approx 66.42 \approx \mathbf{66}$
4. **Interquartile Range (IQR):**

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- $IQR = Q_3 - Q_1 = 66.42 - 55.17 = 11.25 \approx \mathbf{11}$

5. **3rd Decile (D_3):**
   - Position $= \frac{3N}{10} = \frac{3\times80}{10} = 24$th position
   - Value $\approx 56.5 \approx \mathbf{57}$
6. **90th Percentile (P_{90}):**
   - Position $= \frac{90N}{100} = \frac{90\times80}{100} = 72$nd position
   - Value $\approx 72.88 \approx \mathbf{73}$

Question 2

Data Given:

- **Scores:** $69-79, 79-89, 89-99, 99-109, 109-119$
- **Frequencies (f):** $4, 10, 13, 8, 5$

Step 1: Construct the Frequency Table

Class Interval	Frequency (f)	Upper Boundary	Cumulative Frequency (cf)
$69-79$	4	79	4
$79-89$	10	89	14
$89-99$	13	99	27
$99-109$	8	109	35
$109-119$	5	119	40

- **Total Frequency (N):** 40

Calculations:

1. **Lower Quartile (Q_1):**
   - Position $= \frac{40}{4} = 10$th position

- Class: $79 - 89 \Longrightarrow Q_1 = 79 + \frac{10}{10}(10 - 4) = \mathbf{85}$

2. **Median (Q_2):**
   - Position = $\frac{40}{2} = 20$th position
   - Class: $89 - 99 \Longrightarrow$ Median $= 89 + \frac{10}{13}(20 - 14) = 89 + 4.62 = \mathbf{93.62}$
3. **Upper Quartile (Q_3):**
   - Position = $\frac{3\times40}{4} = 30$th position
   - Class: $99 - 109 \Longrightarrow Q_3 = 99 + \frac{10}{8}(30 - 27) = 99 + 3.75 = \mathbf{102.75}$
4. **Interquartile Range (IQR):**
   - $IQR = Q_3 - Q_1 = 102.75 - 85 = \mathbf{17.75}$
5. **6th Decile (D_6):**
   - Position = $\frac{6\times40}{10} = 24$th position
   - Class: $89 - 99 \Longrightarrow D_6 = 89 + \frac{10}{13}(24 - 14) = 89 + 7.69 = \mathbf{96.69}$
6. **90th Percentile (P_{90}):**
   - Position = $\frac{90\times40}{100} = 36$th position
   - Class: $109 - 119 \Longrightarrow P_{90} = 109 + \frac{10}{5}(36 - 35) = 109 + 2 = \mathbf{111}$

Question 3

Given Data:

3,4,5,6,7,8,9,10,11,12,13,14,15,16,17

- Number of observations (n) = 15

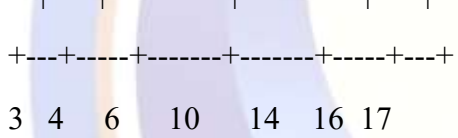
Calculations:

1. **Minimum:** 3
2. **Maximum:** 17
3. **Median (Q_2):**
   - $\frac{15+1}{2} = 8$th term $\Longrightarrow$ Median $= \mathbf{10}$

4. **Lower Quartile (Q_1):**
   - Lower half: 3,4,5,6,7,8,9 (7 terms)
   - Median of lower half = 4th term $\Longrightarrow Q_1 = \mathbf{6}$
5. **Upper Quartile (Q_3):**
   - Upper half: 11,12,13,14,15,16,17 (7 terms)
   - Median of upper half = 4th term $\Longrightarrow Q_3 = \mathbf{14}$
6. **Range:**
   - Range $=$ Max $-$ Min $= 17 - 3 = \mathbf{14}$
7. **Interquartile Range (IQR):**
   - $IQR = Q_3 - Q_1 = 14 - 6 = \mathbf{8}$

Box-and-Whisker Plot Five-Number Summary:

- **Min:** 3, Q_1**:** 6, **Median:** 10, Q_3**:** 14, **Max:** 17



Question 4

Given Data:

92,93,95,96,97,98,99,100,101,102,104,105,107,108,110

- Number of observations (n) = 15

Calculations:

1. **Minimum:** 92
2. **Maximum:** 110
3. **Median (Q_2):**
   - $\frac{15+1}{2} = 8$th term $\Longrightarrow$ Median $= \mathbf{100}$
4. **Lower Quartile (Q_1):**
   - Lower half: 92,93,95,96,97,98,99
   - 4th term $\Longrightarrow Q_1 = \mathbf{96}$

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5. **Upper Quartile (Q_3):**
    - Upper half: 101,102,104,105,107,108,110
    - 4th term $\Longrightarrow Q_3 = \mathbf{105}$
6. **Range:**
    - Range $= 110 - 92 = \mathbf{18}$
7. **Interquartile Range (IQR):**
    - $IQR = 105 - 96 = \mathbf{9}$

Check for Outliers / Extreme Values:

- **Lower Fence:** $Q_1 - 1.5 \times IQR = 96 - 1.5(9) = 82.5$
- **Upper Fence:** $Q_3 + 1.5 \times IQR = 105 + 1.5(9) = 118.5$
- Since all values lie between 82.5 and 118.5, **there are no extreme values/outliers**.

Question 5

Reading directly from the provided box plot image:

- **Minimum value: 12**
- **Lower Quartile (Q_1): 22**
- **Median (Q_2): 25**
- **Upper Quartile (Q_3): 34**
- **Maximum value: 48**

Question 6

Given Data:

- Minimum $= 12$
- $Q_1 = 14$
- Median $= 19$
- $Q_3 = 24$
- Maximum $= 26$

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Box-and-Whisker Plot:

|---|=======|=======|---|

| | | | |

12 14 19 24 26

Question 7

Given Data Table:

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1st **Quiz** (x)	14	28	34	36	43	45	46	49
2nd **Quiz** (y)	16	20	33	35	38	39	42	43

Step 1: Calculations for Line of Best Fit ($y = mx + c$)

$$\sum x = 295, \quad \sum y = 266, \quad n = 8$$

$$\bar{x} = \frac{295}{8} = 36.875, \quad \bar{y} = \frac{266}{8} = 33.25$$

$$\sum x^2 = 14^2 + 28^2 + 34^2 + 36^2 + 43^2 + 45^2 + 46^2 + 49^2 = 11843$$

$$\sum xy = (14 \cdot 16) + (28 \cdot 20) + (34 \cdot 33) + (36 \cdot 35) + (43 \cdot 38) + (45 \cdot 39) + (46 \cdot 42) + (49 \cdot 43) = 10568$$

- **Slope (m):**

$$m = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{8(10568) - (295)(266)}{8(11843) - (295)^2} = \frac{84544 - 78470}{94744 - 87025} = \frac{6074}{7719} \approx \mathbf{0.787}$$

- **Y-intercept (c):**

$$c = \bar{y} - m\bar{x} = 33.25 - 0.787(36.875) \approx \mathbf{4.23}$$

- **Equation of Line of Best Fit:**

$$y = 0.787x + 4.23$$

Description of Correlation:

There is a **strong positive linear correlation** between the 1st and 2nd quiz scores. Students who score higher in the first quiz generally score higher in the second quiz as well.

Question 8

Given Data Table:

1st Test (x)	**20**	**38**	**70**	**40**	**100**	**110**	**50**	**136**	**150**	**80**
2nd Test (y)	160	142	130	120	70	50	140	44	36	100

Part (i): Line of Best Fit Equation

$$\sum x = 794, \quad \sum y = 992, \quad n = 10$$

$$\bar{x} = 79.4, \quad \bar{y} = 99.2$$

$$\sum x^2 = 80440, \quad \sum xy = 62716$$

- **Slope (m):**

$$m = \frac{10(62716) - (794)(992)}{10(80440) - (794)^2} = \frac{627160 - 787648}{804400 - 630436} = \frac{-160488}{173964} \approx \mathbf{-0.923}$$

- **Y-intercept (c):**

$$c = 99.2 - (-0.923 \times 79.4) = 99.2 + 73.28 = \mathbf{172.48}$$

- **Equation of Line of Best Fit:**

$$y = -0.923x + 172.48$$

Part (ii): Describe Correlation

There is a **strong negative linear correlation** between the 1st and 2nd tests.

Part (iii): Estimation

Given 2nd test score $y = 60$:

$$60 = -0.923x + 172.48$$

$$0.923x = 172.48 - 60 = 112.48$$

$$x = \frac{112.48}{0.923} \approx \mathbf{121.87} \quad (\approx 122)$$

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- **Estimated score in the 1st test: $\mathbf{122}$**

Question 9

Given Data Table:

Hours (x)	**0.5**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
GPA (y)	4.0	3.9	3.7	3.5	3.3	3.1	2.9	2.8	2.6	2.5

Part (i): Line of Best Fit Equation

$$\sum x = 45.5, \quad \sum y = 32.3, \quad n = 10$$

$$\bar{x} = 4.55, \quad \bar{y} = 3.23$$

$$\sum x^2 = 285.25, \quad \sum xy = 132.85$$

- **Slope (m):**

$$m = \frac{10(132.85) - (45.5)(32.3)}{10(285.25) - (45.5)^2} = \frac{1328.5 - 1469.65}{2852.5 - 2070.25} = \frac{-141.15}{782.25} \approx \mathbf{-0.180}$$

- **Y-intercept (c):**

$$c = 3.23 - (-0.180 \times 4.55) = 3.23 + 0.819 = \mathbf{4.05}$$

- **Equation of Line of Best Fit:**

$$\text{GPA} = -0.180(\text{Hours}) + 4.05$$

Part (ii): Nature of Correlation

There is a **strong negative linear correlation** between social media hours and GPA. As social media usage increases, the GPA tends to decrease.

Part (iii): Estimation for $x = 6.5$ hours

$$\text{GPA} = -0.180(6.5) + 4.05 = -1.17 + 4.05 = \mathbf{2.88}$$

- **Estimated GPA: $\mathbf{2.88}$**

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EXERCISE 11.2.

Question 1

Find the range of the following data sets:

(i) Data: $63, 89, 98, 125, 79, 108, 117, 60$

- **Formula:** Range $= X_{\max} - X_{\min}$
- **Maximum value ($X_{\max}$):** 125
- **Minimum value ($X_{\min}$):** 60

$$\text{Range} = 125 - 60 = 65$$

(ii) Data: $43.5, 13.6, 18.9, 38.4, 61.4, 29.4$

- **Maximum value ($X_{\max}$):** 61.4
- **Minimum value ($X_{\min}$):** 13.6

$$\text{Range} = 61.4 - 13.6 = 47.8$$

Question 2

If the range and the lowest value of a set of data are 46.7 and 13.4 respectively, then find the highest value.

Step-by-step Solution:

- Given:
  - Range $= 46.7$
  - $X_{\min} = 13.4$
- **Formula:** Range $= X_{\max} - X_{\min}$
- $46.7 = X_{\max} - 13.4$
- $X_{\max} = 46.7 + 13.4 = 60.1$

Answer: The highest value is 60.1.

Question 3

Calculate the range of the following data:

Income (in Rs.)	$4000 - 4500$	$4500 - 5000$	$5000 - 5500$	$5500 - 6000$	$6000 - 6500$
No. of workers	8	12	30	21	6

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Step-by-step Solution:

For grouped data, Range is calculated as:

$$\text{Range} = \text{Upper Boundary of Highest Class} - \text{Lower Boundary of Lowest Class}$$

- Upper boundary of highest class $(6000 - 6500) = 6500$
- Lower boundary of lowest class $(4000 - 4500) = 4000$

$$\text{Range} = 6500 - 4000 = 2500 \text{ Rs.}$$

Question 4

A group of 7 workers reported the number of items they assembled in a day as: 52,55,50,53,54,56,52

Find the standard deviation and variance of the items assembled.

Step-by-step Solution:

- Number of observations $(n) = 7$

1. **Calculate the Mean ($\bar{x}$):**

$$\bar{x} = \frac{\sum x}{n} = \frac{52 + 55 + 50 + 53 + 54 + 56 + 52}{7} = \frac{372}{7} \approx 53.143$$

2. **Calculate $(x - \bar{x})^2$ for each observation:**

x	$x - \bar{x}$	$(x - \bar{x})^2$
52	$52 - 53.143 = -1.143$	1.306
55	$55 - 53.143 = 1.857$	3.448
50	$50 - 53.143 = -3.143$	9.878
53	$53 - 53.143 = -0.143$	0.020
54	$54 - 53.143 = 0.857$	0.734
56	$56 - 53.143 = 2.857$	8.162

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x	$x-\bar{x}$	$(x-\bar{x})^2$
52	$52-53.143=-1.143$	1.306
Sum		$\sum(x-\bar{x})^2=24.854$

3. **Variance (S^2):**

$$S^2=\frac{\sum(x-\bar{x})^2}{n}=\frac{24.854}{7}\approx 3.55$$

4. **Standard Deviation (S):**

$$S=\sqrt{3.55}\approx 1.88$$

Question 5

A librarian recorded the number of visitors during 5 days of a week: 120,135,130,125,140

Calculate the variance and standard deviation of visitors.

Step-by-step Solution:

- $n=5$

1. **Mean ($\bar{x}$):**

$$\bar{x}=\frac{120+135+130+125+140}{5}=\frac{650}{5}=130$$

2. **Table of Deviations:**

x	$x-\bar{x}$	$(x-\bar{x})^2$
120	$120-130=-10$	100
135	$135-130=5$	25
130	$130-130=0$	0
125	$125-130=-5$	25

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x	$x-\bar{x}$	$(x-\bar{x})2$
140	$140-130=10$	100
Total		$\sum(x-\bar{x})^2=250$

3. **Variance (S^2):**

$$S^2=\frac{\sum(x-\bar{x})^2}{n}=\frac{250}{5}=50$$

4. **Standard Deviation (S):**

$$S=\sqrt{50}\approx 7.07$$

Question 6

Find the range, variance and standard deviation of first 23 odd numbers.

Step-by-step Solution:

The first 23 odd numbers are: 1,3,5,7, ... ,45

1. **Range:**

$$\text{Range}=X_{\max}-X_{\min}=45-1=44$$

2. **Mean ($\bar{x}$):**

$$\text{Sum of first } n \text{ odd numbers}=n^2=23^2=529$$

$$\bar{x}=\frac{529}{23}=23$$

3. **Variance (S^2):** Using the shortcut formula for variance of the first n odd numbers:

$$S^2=\frac{n^2-1}{3}=\frac{23^2-1}{3}=\frac{529-1}{3}=\frac{528}{3}=176$$

4. **Standard Deviation (S):**

$$S=\sqrt{176}\approx 13.27$$

Question 7

The rainfall recorded in various places of five districts in a week is given below. Find its variance and standard deviation.

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| Rainfall (x in mm) | 42 | 51 | 54 | 61 | 63 | 71 | | :--- | :---: | :---: | :---: | :---: | :---: | :---: | | **Number of places (f)** | 5 | 13 | 4 | 9 | 5 | 4 |

Step-by-step Solution:

x	f	f·x	x2	f·x2
42	5	210	1764	8820
51	13	663	2601	33813
54	4	216	2916	11664
61	9	549	3721	33489
63	5	315	3969	19845
71	4	284	5041	20164
Total	$\sum \mathbf{f = 40}$	$\sum \mathbf{f x = 2237}$		$\sum \mathbf{f x^2 = 127795}$

1. **Mean ($\bar{x}$):**

$$\bar{x} = \frac{\sum f x}{\sum f} = \frac{2237}{40} = 55.925$$

2. **Variance (S^2):**

$$S^2 = \frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f}\right)^2 = \frac{127795}{40} - (55.925)^2 = 3194.875 - 3127.6056 = 67.27$$

3. **Standard Deviation (S):**

$$S = \sqrt{67.27} \approx 8.20 \text{ mm}$$

Question 8

Machine A Output (units): 98,100,102,101,99

Machine B Output (units): 95,100,105,90,110

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(i) Which machine has better performance?

(ii) Which machine is more consistent?

Step-by-step Solution:

Machine A:

- Mean $= \bar{x}_A = \frac{98+100+102+101+99}{5} = \frac{500}{5} = 100$
- $\sum(x - \bar{x}_A)^2 = (-2)^2 + (0)^2 + (2)^2 + (1)^2 + (-1)^2 = 4 + 0 + 4 + 1 + 1 = 10$
- Standard Deviation $(S_A) = \sqrt{\frac{10}{5}} = \sqrt{2} \approx 1.414$

Machine B:

- Mean $= \bar{x}_B = \frac{95+100+105+90+110}{5} = \frac{500}{5} = 100$
- $\sum(x - \bar{x}_B)^2 = (-5)^2 + (0)^2 + (5)^2 + (-10)^2 + (10)^2 = 25 + 0 + 25 + 100 + 100 = 250$
- Standard Deviation $(S_B) = \sqrt{\frac{250}{5}} = \sqrt{50} \approx 7.071$

Answers:

- **(i) Performance:** Since both machines have the exact same mean average output ($\bar{x} = 100$), **both machines have equal average performance**.
- **(ii) Consistency: Machine A** is more consistent because it has a significantly smaller standard deviation ($1.414 < 7.071$), showing less variability in output.

Question 9

The monthly sales (rupees in lacs) for two salespersons over 6 **months are:**

Person A: 5.5,5.7,5.4,5.6,5.8,5.6

Person B: 5.5,6.5,4.5,6.0,5.0,6.0

Compare their performance and consistency.

Step-by-step Solution:

Person A:

- $\bar{x}_A = \frac{5.5+5.7+5.4+5.6+5.8+5.6}{6} = \frac{33.6}{6} = 5.6$

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- Deviations squared $\sum(x - \bar{x}_A)^2 = (-0.1)^2 + (0.1)^2 + (-0.2)^2 + (0.0)^2 + (0.2)^2 + (0.0)^2 = 0.01 + 0.01 + 0.04 + 0.00 + 0.04 + 0.00 = 0.10$
- Standard Deviation $S_A = \sqrt{\frac{0.10}{6}} \approx 0.129$

Person B:

- $\bar{x}_B = \frac{5.5+6.5+4.5+6.0+5.0+6.0}{6} = \frac{33.5}{6} \approx 5.583$
- Deviations squared $\sum(x - \bar{x}_B)^2 = (-0.083)^2 + (0.917)^2 + (-1.083)^2 + (0.417)^2 + (-0.583)^2 + (0.417)^2 \approx 2.708$
- Standard Deviation $S_B = \sqrt{\frac{2.708}{6}} \approx 0.672$

Comparison:

- **Performance: Person A** has a slightly higher average monthly sales (5.6 lacs vs 5.583 lacs).
- **Consistency: Person A** is much more consistent than Person B because Person A has a much lower standard deviation ($0.129 < 0.672$).

Question 10

Daily wages of workers in a textile mill, grouped into six income brackets:

| Daily Wage (Rs) | $800 - 1000$ | $1000 - 1200$ | $1200 - 1400$ | $1400 - 1600$ | $1600 - 1800$ | $1800 - 2000$ | | :--- | :---: | :---: | :---: | :---: | :---: | :---: | | **Frequency (f)** | 2 | 4 | 6 | 8 | 2 | 1 |

Calculate the mean, variance and standard deviation of the wages.

Step-by-step Solution:

Midpoints (x) are calculated as $\frac{\text{Lower Limit}+\text{Upper Limit}}{2}$.

Class	x	f	f·x	x2	f·x2
$800 - 1000$	900	2	1800	810,000	1,620,000
$1000 - 1200$	1100	4	4400	1,210,000	4,840,000

Class	x	f	f·x	x2	f·x2
1200 – 1400	1300	6	7800	1,690,000	10,140,000
1400 – 1600	1500	8	12000	2,250,000	18,000,000
1600 – 1800	1700	2	3400	2,890,000	5,780,000
1800 – 2000	1900	1	1900	3,610,000	3,610,000
Total		$\sum \mathbf{f} = \mathbf{23}$	$\sum \mathbf{fx} = \mathbf{31300}$		$\sum \mathbf{fx^2} = \mathbf{43,990,000}$

1. **Mean ($\bar{x}$):**

$$\bar{x} = \frac{\sum f\,x}{\sum f} = \frac{31300}{23} \approx 1360.87 \text{ Rs.}$$

2. **Variance (S^2):**

$$S^2 = \frac{\sum f\,x^2}{\sum f} - (\bar{x})^2 = \frac{43990000}{23} - (1360.87)^2 = 1912608.7 - 1851967.16 \approx 60641.54$$

3. **Standard Deviation (S):**

$$S = \sqrt{60641.54} \approx 246.25 \text{ Rs.}$$

Question 11

A company forecasts monthly sales (rupees in millions): 15,18,14,20,13.

Find variability in sales predictions.

Step-by-step Solution:

Variability is measured by standard deviation (or variance).

- $n = 5$

- **Mean ($\bar{x}$):**

$$\bar{x} = \frac{15 + 18 + 14 + 20 + 13}{5} = \frac{80}{5} = 16$$

- **Deviations and Squares:**

x	$x - \bar{x}$	$(x - \bar{x})^2$
15	-1	1
18	2	4
14	-2	4
20	4	16
13	-3	9
Total		$\sum (x - \bar{x})^2 = 34$

- **Variance (S^2):**

$$S^2 = \frac{34}{5} = 6.8$$

- **Standard Deviation (S):**

$$S = \sqrt{6.8} \approx 2.61 \text{ million rupees}$$

Answer: The variability (standard deviation) in sales predictions is 2.61 **million rupees** (variance = 6.8).

Question 12

Unemployment rates (%) in five provinces are 5.2,6.0,4.8,5.5,6.2**. Calculate standard deviation and describe is there balanced unemployment rate?**

Step-by-step Solution:

- $n = 5$

- **Mean ($\bar{x}$):**

$$\bar{x} = \frac{5.2 + 6.0 + 4.8 + 5.5 + 6.2}{5} = \frac{27.7}{5} = 5.54\%$$

- **Deviations:**

x	x−5.54	(x−5.54)2
5.2	−0.34	0.1156
6.0	0.46	0.2116
4.8	−0.74	0.5476
5.5	−0.04	0.0016
6.2	0.66	0.4356
Total		**1.312**

- **Standard Deviation (S):**

$$S = \sqrt{\frac{1.312}{5}} = \sqrt{0.2624} \approx 0.512\%$$

- **Interpretation:** Since the standard deviation (0.512%) is very small relative to the mean (5.54%), the variation among provinces is low. Therefore, the unemployment rate is **relatively balanced** across the provinces.

Question 13

Find variance and standard deviation:

(i) $\sum x = 45, \sum x^2 = 421, n = 5$

- **Variance (S^2):**

$$S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{421}{5} - \left(\frac{45}{5}\right)^2 = 84.2 - 9^2 = 84.2 - 81 = 3.2$$

- **Standard Deviation (S):**

$$S = \sqrt{3.2} \approx 1.79$$

(ii) $\sum f x = 210, \sum f x^2 = 7560, \sum f = 6$

- **Variance (S^2):**

$$S^2 = \frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f}\right)^2 = \frac{7560}{6} - \left(\frac{210}{6}\right)^2 = 1260 - (35)^2 = 1260 - 1225 = 35$$

- **Standard Deviation (S):**

$$S = \sqrt{35} \approx 5.92$$

(iii) $\bar{x} = 18, \sum f x^2 = 1670, \sum f = 5$

- **Variance (S^2):**

$$S^2 = \frac{\sum f x^2}{\sum f} - (\bar{x})^2 = \frac{1670}{5} - (18)^2 = 334 - 324 = 10$$

- **Standard Deviation (S):**

$$S = \sqrt{10} \approx 3.16$$

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REVIEW EXERCISE 11

(i) _____ *is used to get the cumulative frequencies.*

- **Options:** (a) Addition (b) square root (c) multiplication (d) division
- **Answer: (a) Addition**

(ii) Second quartile represents:

- **Options:** (a) mean (b) mode (c) median (d) variance
- **Answer: (c) median**

(iii) First quartile divides the data into _____ *equal parts.*

- **Options:** (a) Two (b) Three (c) four (d) ten
- **Answer: (c) four** *(Note: A quartile divides data into four equal parts, and the first quartile represents the 25th percentile/lower quartile value).*

(iv) Difference between the highest and the lowest values is called:

- **Options:** (a) mean (b) variance (c) range (d) standard deviation
- **Answer: (c) range**

(v) Scatter diagram represents a relationship between _____ *variables.*

- **Options:** (a) five (b) two (c) three (d) four
- **Answer: (b) two**

(vi) _____ *is measure of dispersion:*

- **Options:** (a) mean (b) median (c) mode (d) variance
- **Answer: (d) variance**

(vii) Positive square root of variance is called:

- **Options:** (a) mean (b) median (c) standard deviation (d) range
- **Answer: (c) standard deviation**

(viii) _____ *is not measure of dispersion.*

- **Options:** (a) range (b) arithmetic mean (c) variance (d) standard deviation
- **Answer: (b) arithmetic mean**

(ix) Variance of the data 8,8,8,8,8,8 *is:*

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- **Options:** (a) 0 (b) 16 (c) 8 (d) 48
- **Answer: (a) 0** *(Since all values are identical, there is no dispersion/deviation from the mean).*

(x) Range of first 20 natural numbers is:

- **Options:** (a) 20 (b) 10 (c) 19 (d) 30
- **Answer: (c) 19** *(Highest value* $= 20$*, Lowest value* $= 1$*, Range* $= 20 - 1 = 19$*).*

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Question 2

Data Table & Cumulative Frequency Construction:

Total frequency $N = \sum f = 2 + 6 + 9 + 12 + 8 + 3 = 40$.

Class boundaries and cumulative frequency (cf) table:

Class Interval	Frequency (fff)	Upper Class Boundary (UCB)	Cumulative Frequency (cfcfcf)
170 – 180	2	180	2
180 – 190	6	190	8
190 – 200	9	200	17
200 – 210	12	210	29
210 – 220	8	220	37
220 – 230	3	230	40

1. Constructing Cumulative Frequency Polygon (Ogive)

- **X-axis:** Upper Class Boundaries $(170,180,190,200,210,220,230)$
- **Y-axis:** Cumulative Frequency $(0$ to $40)$
- Plot points: $(170,0), (180,2), (190,8), (200,17), (210,29), (220,37), (230,40)$ and connect with straight lines.

2. Calculating Quantiles with Steps

Using grouped data formula: Value $= L + \left(\frac{\text{Position} - cf_0}{f}\right) \times h$

a. First Quartile (Q_1)

Position $= \frac{1\times 40}{4} = 10^{\text{th}}$ item.

- Class containing 10^{th} item: **190 – 200** (since cf covers up to 17).

- $L = 190, cf_0 = 8, f = 9, h = 10.$

$$Q_1 = 190 + \left(\frac{10-8}{9}\right) \times 10 = 190 + \left(\frac{2}{9}\right) \times 10 = 190 + 2.22 = 192.22 \text{ cm}$$

b. Median (Q_2)

Position $= \frac{40}{2} = 20^{\text{th}}$ item.

- Class containing 20^{th} item: **200 – 210** (since cf covers up to 29).
- $L = 200, cf_0 = 17, f = 12, h = 10.$

$$\text{Median} = 200 + \left(\frac{20-17}{12}\right) \times 10 = 200 + \left(\frac{3}{12}\right) \times 10 = 200 + 2.5 = 202.5 \text{ cm}$$

c. Third Quartile (Q_3)

Position $= \frac{3\times40}{4} = 30^{\text{th}}$ item.

- Class containing 30^{th} item: **210 – 220** (since cf covers up to 37).
- $L = 210, cf_0 = 29, f = 8, h = 10.$

$$Q_3 = 210 + \left(\frac{30-29}{8}\right) \times 10 = 210 + \left(\frac{1}{8}\right) \times 10 = 210 + 1.25 = 211.25 \text{ cm}$$

d. 7th Decile (D_7)

Position $= \frac{7\times40}{10} = 28^{\text{th}}$ item.

- Class containing 28^{th} item: **200 – 210**.
- $L = 200, cf_0 = 17, f = 12, h = 10.$

$$D_7 = 200 + \left(\frac{28-17}{12}\right) \times 10 = 200 + \left(\frac{11}{12}\right) \times 10 = 200 + 9.17 = 209.17 \text{ cm}$$

e. 90th Percentile (P_{90})

Position $= \frac{90\times40}{100} = 36^{\text{th}}$ item.

- Class containing 36^{th} item: **210 – 220**.
- $L = 210, cf_0 = 29, f = 8, h = 10.$

$$P_{90} = 210 + \left(\frac{36-29}{8}\right) \times 10 = 210 + \left(\frac{7}{8}\right) \times 10 = 210 + 8.75 = 218.75 \text{ cm}$$

f. Interquartile Range (IQR)

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$$\text{IQR} = Q_3 - Q_1 = 211.25 - 192.22 = 19.03 \text{ cm}$$

Question 3

Given values: Minimum $= 5$, $Q_1 = 12.6$, Median $= 43$, $Q_3 = 55.6$, Maximum $= 70$.

Box-and-Whisker Plot Construction:

1. Draw a horizontal number line scaling from 0 to 80.
2. Draw a rectangular box extending from $Q_1 = 12.6$ to $Q_3 = 55.6$.
3. Draw a vertical line inside the box at Median $= 43$.
4. Draw left whisker extending from $Q_1 = 12.6$ to Min $= 5$.
5. Draw right whisker extending from $Q_3 = 55.6$ to Max $= 70$.

Question 4

(i) Plotting & Line of Best Fit

- Plot points (x, y) where $x =$ Exercise Hours, $y =$ Heart Rate:

$(0,90), (1,85), (2,83), (3,80), (4,76), (5,74), (6,72), (7,70), (8,68), (9,65)$.

- Draw a straight line passing through the center of the plotted scatter points sloping downward from left to right.

(ii) Type of Correlation

Negative correlation (As exercise hours increase, resting heart rate decreases).

(iii) Prediction for Sakeena ($x = 6.5$ hours)

Linear regression formula $y = a + bx$:

- $n = 10$
- $\sum x = 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$
- $\sum y = 90 + 85 + 83 + 80 + 76 + 74 + 72 + 70 + 68 + 65 = 763$
- $\sum xy = 0(90) + 1(85) + 2(83) + 3(80) + 4(76) + 5(74) + 6(72) + 7(70) + 8(68) + 9(65) = 3127$
- $\sum x^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 = 285$

$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{10(3127) - (45)(763)}{10(285) - (45)^2} = \frac{31270 - 34335}{2850 - 2025} = \frac{-3065}{825} \approx -3.715$$

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$$a = \frac{\sum y - b\sum x}{n} = \frac{763 - (-3.715 \times 45)}{10} = \frac{763 + 167.175}{10} = 93.02$$

For $x = 6.5$:

$$y = 93.02 - 3.715(6.5) = 93.02 - 24.15 = 68.87 \approx 69 \text{ beats per minute}$$

Question 5

Given data: 25,30,35,40,50,60,65,75

$$\text{Maximum value } (X_m) = 75$$

$$\text{Minimum value } (X_0) = 25$$

$$\text{Range} = X_m - X_0 = 75 - 25 = 50$$

Question 6

Class Interval	**Frequency (fff)**	**Midpoint (xxx)**	**f·xf \cdot xf·x**	**x2x^2x2**	**f·x2f \cdot x^2f·x2**
0 – 5	4	2.5	10	6.25	25
5 – 10	12	7.5	90	56.25	675
10 – 15	20	12.5	250	156.25	3125
15 – 20	24	17.5	420	306.25	7350
20 – 25	16	22.5	360	506.25	8100
25 – 30	4	27.5	110	756.25	3025
Total	$N = 80$		$\sum f x = 1240$		$\sum f x^2 = 22300$

1. Range

$$\text{Range} = \text{Upper limit of highest class} - \text{Lower limit of lowest class} = 30 - 0 = 30$$

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2. Variance (s^2)

$$s^2 = \frac{\sum f\,x^2}{N} - \left(\frac{\sum f\,x}{N}\right)^2 = \frac{22300}{80} - \left(\frac{1240}{80}\right)^2$$

$$s^2 = 278.75 - (15.5)^2 = 278.75 - 240.25 = 38.5$$

3. Standard Deviation (s)

$$s = \sqrt{38.5} \approx 6.20$$

Question 7

Given: $n = 5, \sum x = 45, \sum x^2 = 421$

1. Mean ($\bar{x}$)

$$\bar{x} = \frac{\sum x}{n} = \frac{45}{5} = 9$$

2. Standard Deviation (s)

$$s = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{421}{5} - (9)^2} = \sqrt{84.2 - 81} = \sqrt{3.2} \approx 1.79$$

Question 8

Family A Data

Values: 45,47,46,48,46,47

$n = 6$

- $\sum x_A = 45 + 47 + 46 + 48 + 46 + 47 = 279$
- $\sum x_A^2 = 45^2 + 47^2 + 46^2 + 48^2 + 46^2 + 47^2 = 2025 + 2209 + 2116 + 2304 + 2116 + 2209 = 12979$

Mean ($\bar{x}_A$):

$$\bar{x}_A = \frac{279}{6} = 46.5 \text{ (thousand rupees)}$$

Standard Deviation (s_A):

$$s_A = \sqrt{\frac{12979}{6} - (46.5)^2} = \sqrt{2163.17 - 2162.25} = \sqrt{0.92} \approx 0.96 \text{ (thousand rupees)}$$

Family B Data

Values: 38,52,40,50,42,49

$n = 6$

- $\sum x_B = 38 + 52 + 40 + 50 + 42 + 49 = 271$
- $\sum x_B^2 = 38^2 + 52^2 + 40^2 + 50^2 + 42^2 + 49^2 = 1444 + 2704 + 1600 + 2500 + 1764 + 2401 = 12413$

Mean ($\bar{x}_B$):

$$\bar{x}_B = \frac{271}{6} = 45.17 \text{ (thousand rupees)}$$

Standard Deviation (s_B):

$$s_B = \sqrt{\frac{12413}{6} - (45.17)^2} = \sqrt{2068.83 - 2040.33} = \sqrt{28.5} \approx 5.34 \text{ (thousand rupees)}$$

Conclusions

1. **Which family spends more on average?**

Family A spends more on average ($\bar{x}_A = 46.5$ vs $\bar{x}_B = 45.17$).

2. **Which family has more stable expenses?**

Family A has more stable expenses because it has a lower standard deviation ($s_A = 0.96$ vs $s_B = 5.34$).

Question 9

Daily Wages (Rs.)	**No. of Workers (fff)**	**Midpoint (xxx)**	**f·xf \cdot xf·x**	**x2x^2x2**	**f·x2f \cdot x^2f·x2**
1000 – 1200	6	1100	6600	1,210,000	7,260,000
1200 – 1400	10	1300	13000	1,690,000	16,900,000
1400 – 1600	12	1500	18000	2,250,000	27,000,000

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Daily Wages (Rs.)	No. of Workers (fff)	Midpoint (xxx)	f·xf \cdot xf·x	x2x^2x2	f·x2f \cdot x^2f·x2
1600 – 1800	8	1700	13600	2,890,000	23,120,000
1800 – 2000	4	1900	7600	3,610,000	14,440,000
Total	$N = 40$		$\sum f x = 58800$		$\sum f x^2 = 88,720,000$

1. Mean ($\bar{x}$)

$$\bar{x} = \frac{\sum f x}{N} = \frac{58800}{40} = 1470 \text{ Rs.}$$

2. Variance (s^2)

$$s^2 = \frac{\sum f x^2}{N} - \left(\frac{\sum f x}{N}\right)^2 = \frac{88720000}{40} - (1470)^2$$

$$s^2 = 2218000 - 2160900 = 57100$$

3. Standard Deviation (s)

$$s = \sqrt{57100} \approx 238.96 \text{ Rs.}$$

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