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Class	10th	Chapter	12
Subject	Mathematics	Board	Punjab

CHAPTER 12: Probability

EXERCISE 12.1

Question 1

Find the sample space for tossing three coins using tree diagram.

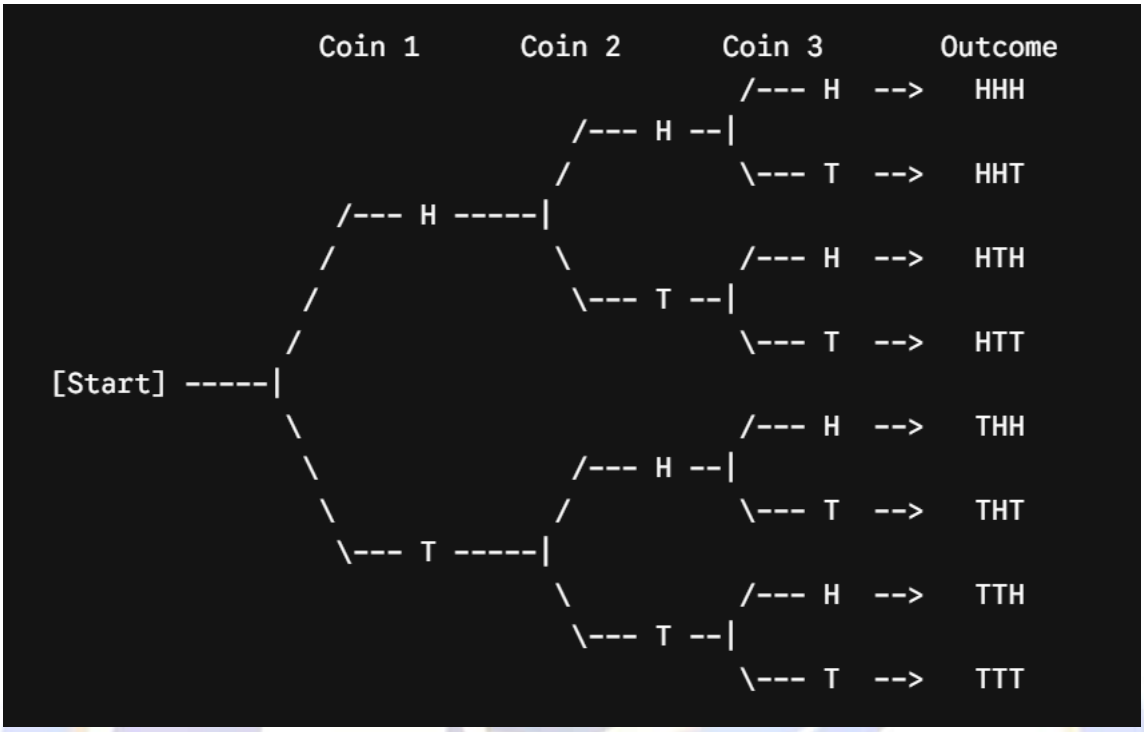
Solution:

Each coin has 2 possible outcomes: Heads (H) or Tails (T).

Tree Diagram Representation:

- **Coin 1:** Branch into H and T.
- **Coin 2:** From each outcome of Coin 1, branch into H and T.
- **Coin 3:** From each outcome of Coin 2, branch into H and T.

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Sample Space (S):

$$S = \{\text{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}\}$$

- Total number of outcomes: $n(S) = 2^3 = 8$

Question 2

A teacher randomly selects one boy and one girl from a group of 2 boys (Hamid, Ahmad) and 2 girls (Zainab, Samia). Draw a tree diagram and list the sample space for all possible outcomes.

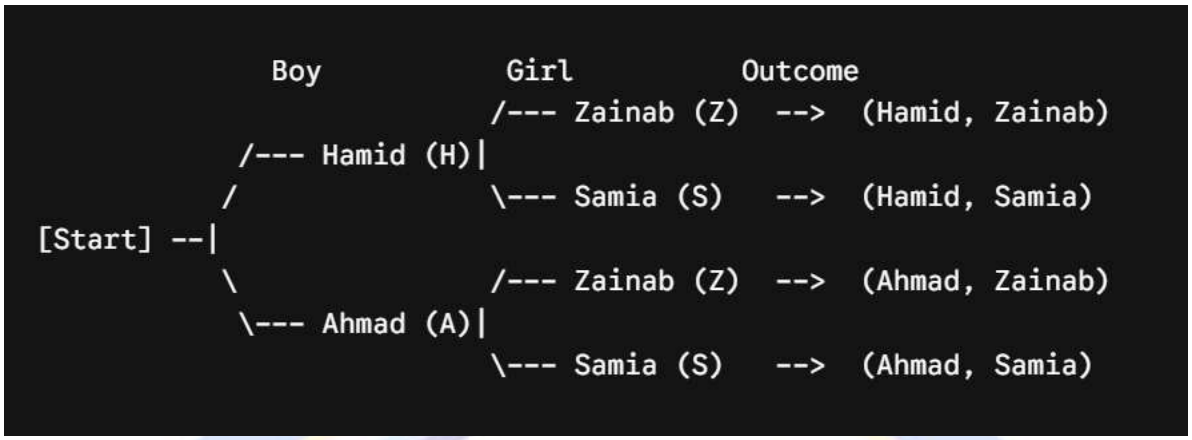
Solution:

Let:

- Boys = {Hamid (H),Ahmad (A)}
- Girls = {Zainab (Z),Samia (S)}

Tree Diagram:

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Sample Space (S):

$$S = \{(\text{Hamid,Zainab}), (\text{Hamid,Samia}), (\text{Ahmad,Zainab}), (\text{Ahmad,Samia})\}$$

- Total number of outcomes: $n(S) = 2 \times 2 = 4$

Question 3

If A is an event of a random experiment such that $P(A) : P(\overline{A}) = 17 : 15$ and $n(S) = 640$, then find (i) $P(\overline{A})$ (ii) $n(A)$.

Solution:

Given:

- Ratio of probabilities: $P(A) : P(\overline{A}) = 17 : 15$
- Total outcomes in sample space: $n(S) = 640$

We know that:

$$P(A) + P(\overline{A}) = 1$$

Let $P(A) = 17x$ and $P(\overline{A}) = 15x$.

$$17x + 15x = 1$$

$$32x = 1 \Rightarrow x = \frac{1}{32}$$

(i) Find $P(\overline{A})$

$$P(\overline{A}) = 15x = 15 \times \frac{1}{32} = \mathbf{\frac{15}{32}}$$

(ii) Find $n(A)$

First, find $P(A)$:

$$P(A) = 17x = \frac{17}{32}$$

Using the formula $P(A) = \frac{n(A)}{n(S)}$:

$$\frac{17}{32} = \frac{n(A)}{640}$$

$$n(A) = \frac{17 \times 640}{32} = 17 \times 20 = \mathbf{340}$$

Question 4

A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution:

When a coin is tossed three times, the sample space is:

$$S = \{\text{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}\}$$

$$n(S) = 8$$

Let event E be "getting **two consecutive tails**".

- Outcomes with consecutive tails (TT):
  - HTT (tails on 2nd and 3rd toss)
  - TTH (tails on 1st and 2nd toss)
  - TTT (contains two consecutive tails as well)

$$E = \{\text{HTT,TTH,TTT}\}$$

$$n(E) = 3$$

$$\text{Probability } P(E) = \frac{n(E)}{n(S)} = \mathbf{\frac{3}{8}}$$

Question 5

A dice is rolled and coin is tossed together. (i) Find sample space by drawing possibility diagram. (ii) Find sample space by sketching tree diagram. (iii) What is a probability of getting a tail and an even number?

Solution:

(i) Possibility Diagram (Grid Table):

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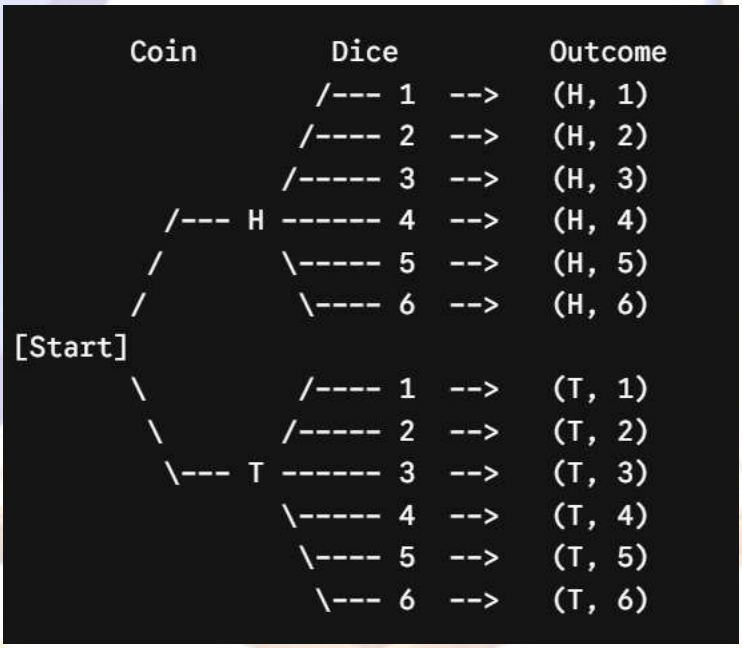
Coin / Dice	1	2	3	4	5	6
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

Sample Space (S):

$$S = \{(\mathrm{H}, 1), (\mathrm{H}, 2), (\mathrm{H}, 3), (\mathrm{H}, 4), (\mathrm{H}, 5), (\mathrm{H}, 6), (\mathrm{T}, 1), (\mathrm{T}, 2), (\mathrm{T}, 3), (\mathrm{T}, 4), (\mathrm{T}, 5), (\mathrm{T}, 6)\}$$

$$n(S) = 12$$

(ii) Tree Diagram:



(iii) Probability of getting a tail and an even number:

Let E be the event of getting a tail (T) and an even number ($\{2,4,6\}$).

$$E = \{(\mathrm{T}, 2), (\mathrm{T}, 4), (\mathrm{T}, 6)\}$$

$$n(E) = 3$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{12} = \mathbf{\frac{1}{4}}$$

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Question 6

Two unbiased dice are rolled once. (i) Find sample space by sketching tree diagram. (ii) Find sample space by drawing possibility diagram. (iii) Find the probability of getting: *(a) same number on both dice. (b) the product as a prime number. (c) the sum as an even number. (d) the sum as 13.*

Solution:

(ii) Possibility Diagram (Grid Table):

(Showing this first for clear reference)

Die 1 \ Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Total sample space size: $n(S) = 6 \times 6 = 36$.

(i) Tree Diagram Structure:

- Branch 1st die into outcomes 1,2,3,4,5,6.
- From each outcome of 1st die, branch into outcomes 1,2,3,4,5,6 for the 2nd die, yielding 36 branches leading to pairs $(1,1), (1,2), \ldots, (6,6)$.

(iii) Probabilities:

- **(a) Same number on both dice (Doublets):**
  - Event $E_a = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$
  - $n(E_a) = 6$

- $P(E_a) = \frac{6}{36} = \mathbf{\frac{1}{6}}$

- **(b) Product as a prime number:**
  - Prime numbers possible: 2,3,5.
  - Pairs: $(1,2), (2,1)$ [product = 2]; $(1,3), (3,1)$ [product = 3]; $(1,5), (5,1)$ [product = 5].
  - Event $E_b = \{(1,2), (2,1), (1,3), (3,1), (1,5), (5,1)\}$
  - $n(E_b) = 6$
  - $P(E_b) = \frac{6}{36} = \mathbf{\frac{1}{6}}$
- **(c) Sum as an even number:**
  - Even sums occur when both numbers are even or both are odd.
  - Number of outcomes with even sum = 18
  - $n(E_c) = 18$
  - $P(E_c) = \frac{18}{36} = \mathbf{\frac{1}{2}}$
- **(d) Sum as 13:**
  - Maximum possible sum with two dice is $6 + 6 = 12$.
  - Event $E_d = \emptyset$ (Impossible event)
  - $n(E_d) = 0$
  - $P(E_d) = \frac{0}{36} = \mathbf{0}$

Question 7

Three fair coins are tossed together. Find the probability of getting: (i) all tails (ii) at least one head (iii) at most two tails (iv) 2 heads (v) at most 2 heads (vi) no head

Solution:

Total sample space $S = \{\text{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}\}$, $n(S) = 8$.

1. **(i) All tails:**
   - $E = \{\text{TTT}\} \Longrightarrow n(E) = 1$
   - $P(E) = \mathbf{\frac{1}{8}}$

2. **(ii) At least one head (≥ 1 Head):**
   - $E = \{\text{HHH,HHT,HTH,HTT,THH,THT,TTH}\} \Longrightarrow n(E) = 7$
   - $P(E) = \frac{7}{8}$
3. **(iii) At most two tails (≤ 2 Tails):**
   - Means 0, 1, or 2 tails (all outcomes except TTT).
   - $n(E) = 7$
   - $P(E) = \frac{7}{8}$
4. **(iv) 2 heads:**
   - $E = \{\text{HHT,HTH,THH}\} \Longrightarrow n(E) = 3$
   - $P(E) = \frac{3}{8}$
5. **(v) At most 2 heads (≤ 2 Heads):**
   - Means 0, 1, or 2 heads (all outcomes except HHH).
   - $n(E) = 7$
   - $P(E) = \frac{7}{8}$
6. **(vi) No head (0 Heads):**
   - $E = \{\text{TTT}\} \Longrightarrow n(E) = 1$
   - $P(E) = \frac{1}{8}$

Question 8

A bag contains 4 red balls, 5 white balls, 6 green balls and 3 black balls. Ali draws a ball at random from the bag. Find the probability that the ball drawn is: (i) white (ii) red (iii) not white (iv) not black

Solution:

Total number of balls:

$$n(S) = 4 + 5 + 6 + 3 = 18$$

- **(i) Probability of getting a white ball:**
  - Number of white balls $n(W) = 5$

- $P(W) = \frac{5}{18}$

- **(ii) Probability of getting a red ball:**
  - Number of red balls $n(R) = 4$
  - $P(R) = \frac{4}{18} = \frac{2}{9}$
- **(iii) Probability of getting a ball that is NOT white:**
  - $P(\text{not white}) = 1 - P(W) = 1 - \frac{5}{18} = \frac{13}{18}$
- **(iv) Probability of getting a ball that is NOT black:**
  - Number of black balls = 3
  - Number of non-black balls = $18 - 3 = 15$
  - $P(\text{not black}) = \frac{15}{18} = \frac{5}{6}$

Question 9

A number is selected at random from the set of whole numbers 1 to 15, both inclusive. Find the probability that the number selected is: (i) odd (ii) a multiple of 5 (iii) the square of 2 (iv) prime (v) 20

Solution:

Sample space $S = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$ Total outcomes $n(S) = 15$

- **(i) Odd number:**
  - $E_1 = \{1,3,5,7,9,11,13,15\} \Longrightarrow n(E_1) = 8$
  - $P(E_1) = \frac{8}{15}$
- **(ii) A multiple of 5:**
  - $E_2 = \{5,10,15\} \Longrightarrow n(E_2) = 3$
  - $P(E_2) = \frac{3}{15} = \frac{1}{5}$
- **(iii) The square of 2:**
  - Square of $2 = 2^2 = 4$
  - $E_3 = \{4\} \Longrightarrow n(E_3) = 1$
  - $P(E_3) = \frac{1}{15}$

- **(iv) Prime number:**
  - $E_4 = \{2,3,5,7,11,13\} \Longrightarrow n(E_4) = 6$
  - $P(E_4) = \frac{6}{15} = \mathbf{\frac{2}{5}}$
- **(v) Number 20:**
  - Number 20 is not in the set $\{1, \ldots, 15\}$.
  - $E_5 = \emptyset \Longrightarrow n(E_5) = 0$
  - $P(E_5) = \frac{0}{15} = \mathbf{0}$

Question 10

If the probability of an event A is $\frac{7}{10}$, then find the probability of the event "not A".

Solution:

Given:

- $P(A) = \frac{7}{10}$

Formula for complementary event:

$$P(\text{not } A) = P(\overline{A}) = 1 - P(A)$$

$$P(\text{not } A) = P(\overline{A}) = 1 - P(A)$$

Question 11

A dice is rolled twice. Find the probability of having a number greater than 4 on each roll.

Solution:

When a die is rolled twice:

- Total possible outcomes: $n(S) = 6 \times 6 = 36$

Numbers greater than 4 on a single die roll are $\{5,6\}$.

Let event E be "getting a number greater than 4 on **each** roll":

- Outcomes for roll 1: $\{5,6\}$
- Outcomes for roll 2: $\{5,6\}$

$$E = \{(5,5), (5,6), (6,5), (6,6)\}$$

$$E = \{(5,5), (5,6), (6,5), (6,6)\}$$

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$$n(E) = 4$$

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EXERCISE 12.2

Question 1

Given:

$P(A) = \frac{1}{4}$

$P(B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{8}$

To find: $P(A \cup B)$

Formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step-by-step Solution:

1. Substitute the given values into the formula:

$$P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

2. Find a common denominator, which is 8:

$$P(A \cup B) = \frac{2}{8} + \frac{4}{8} - \frac{1}{8}$$

3. Simplify:

$$P(A \cup B) = \frac{2 + 4 - 1}{8} = \frac{5}{8}$$

Answer:

$$P(A \cup B) = \frac{5}{8}$$

Question 2

Given:

Door numbers from 1 to 50.

Total sample space, $n(S) = 50$.

Let A = event of getting an **even number**.

Let B = event of getting a **perfect square number**.

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Step-by-step Solution:

1. **Find $n(A)$ and $P(A)$:**
   - Even numbers between 1 and 50: $\{2,4,6,8, \ldots, 50\}$
   - Number of even numbers, $n(A) = 25$
   - $P(A) = \frac{n(A)}{n(S)} = \frac{25}{50}$
2. **Find $n(B)$ and $P(B)$:**
   - Perfect squares between 1 and 50: $\{1,4,9,16,25,36,49\}$
   - Number of perfect squares, $n(B) = 7$
   - $P(B) = \frac{n(B)}{n(S)} = \frac{7}{50}$
3. **Find $n(A \cap B)$ and $P(A \cap B)$:**
   - Numbers that are both **even** and **perfect squares**: $\{4,16,36\}$
   - $n(A \cap B) = 3$
   - $P(A \cap B) = \frac{3}{50}$
4. **Apply the Addition Theorem of Probability:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{25}{50} + \frac{7}{50} - \frac{3}{50} = \frac{25 + 7 - 3}{50} = \frac{29}{50}$$

Answer:

$$\text{Probability} = \frac{29}{50}$$

Question 3

Given:

$P(\text{winning}) = P(W) = \frac{3}{10}$

$P(\text{losing}) = P(L) = \frac{2}{10}$

Since winning and losing are mutually exclusive events ($P(W \cap L) = 0$):

(i) What is the probability that the team wins or loses a particular match?

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$$P(W \cup L) = P(W) + P(L)$$

$$P(W \cup L) = \frac{3}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$$

(ii) What is the probability that the team neither wins nor loses a match?

(Note: This represents a draw or tie)

$$P(\text{neither } W \text{ nor } L) = 1 - P(W \cup L)$$

$$P(\text{neither } W \text{ nor } L) = 1 - \frac{1}{2} = \frac{1}{2}$$

Answer:

(i) $\frac{1}{2}$

(ii) $\frac{1}{2}$

Question 4

Given:

In a single throw of two dice, total possible outcomes $n(S) = 6 \times 6 = 36$.

Let A = sum is 7.

Let B = sum is 11.

Step-by-step Solution:

1. Outcomes for A (sum = 7): $\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$
   - $n(A) = 6 \Longrightarrow P(A) = \frac{6}{36}$
2. Outcomes for B (sum = 11): $\{(5,6), (6,5)\}$
   - $n(B) = 2 \Longrightarrow P(B) = \frac{2}{36}$
3. Since a sum cannot be both 7 and 11 at the same time, A and B are **mutually exclusive** ($A \cap B = \emptyset$).

$$P(A \cup B) = P(A) + P(B) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$$

Answer:

$$\text{Probability} = \frac{2}{9}$$

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Question 5

Given:

Two dice are thrown, total outcomes $n(S) = 36$.

Let A = sum is 5.

Let B = sum is 7.

Step-by-step Solution:

1. Outcomes for A (sum = 5): $\{(1,4), (2,3), (3,2), (4,1)\}$
   - $n(A) = 4 \Longrightarrow P(A) = \frac{4}{36}$
2. Outcomes for B (sum = 7): $\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$
   - $n(B) = 6 \Longrightarrow P(B) = \frac{6}{36}$
3. Since A and B are **mutually exclusive**:

$$P(A \cup B) = P(A) + P(B) = \frac{4}{36} + \frac{6}{36} = \frac{10}{36} = \frac{5}{18}$$

Answer:

$$\text{Probability} = \frac{5}{18}$$

Question 6

Given:

Standard pack of 52 cards, $n(S) = 52$.

Number of Kings = 4

Number of Jacks = 4

Let K = card is a King, J = card is a Jack.

(i) Probability of taking out a King or a Jack:

Since a card cannot be both a King and a Jack, K and J are mutually exclusive.

$$P(K \cup J) = P(K) + P(J) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

(ii) Probability of taking out Neither a King nor a Jack:

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$$P(\text{Neither } K \text{ nor } J) = 1 - P(K \cup J) = 1 - \frac{2}{13} = \frac{11}{13}$$

Answer:

(i) $\frac{2}{13}$

(ii) $\frac{11}{13}$

Question 7

Given:

A die is thrown twice, $n(S) = 6 \times 6 = 36$.

Let E = event that **at least one** throw comes up with number 3.

Step-by-step Solution:

- Outcomes with a 3 on the first throw, second throw, or both:

$$\{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (1,3), (2,3), (4,3), (5,3), (6,3)\}$$

- $n(E) = 11$

$$P(E) = \frac{n(E)}{n(S)} = \frac{11}{36}$$

Answer:

$$\text{Probability} = \frac{11}{36}$$

Question 8

Given:

15 cards marked 1,2,3, … ,15. $n(S) = 15$.

Let A = event of getting a **multiple of 5**.

Let B = event of getting a **multiple of 7**.

Step-by-step Solution:

1. Multiples of 5 in $\{1, \ldots, 15\}$: $\{5,10,15\} \Longrightarrow n(A) = 3$
    - $P(A) = \frac{3}{15}$
2. Multiples of 7 in $\{1, \ldots, 15\}$: $\{7,14\} \Longrightarrow n(B) = 2$

- $P(B) = \frac{2}{15}$

3. There are no numbers between 1 and 15 that are multiples of both 5 and 7 ($A \cap B = \emptyset$).

$$P(A \cup B) = P(A) + P(B) = \frac{3}{15} + \frac{2}{15} = \frac{5}{15} = \frac{1}{3}$$

Answer:

$$\text{Probability} = \frac{1}{3}$$

Question 9

Given:

Two fair coins are tossed once.

Sample space, $S = \{HH, HT, TH, TT\}$, so $n(S) = 4$.

Let A = at least one head = $\{HT, TH, HH\}$

Let B = two heads = $\{HH\}$

Step-by-step Solution:

- $n(A) = 3 \Longrightarrow P(A) = \frac{3}{4}$
- $n(B) = 1 \Longrightarrow P(B) = \frac{1}{4}$
- $A \cap B = \{HH\} \Longrightarrow n(A \cap B) = 1 \Longrightarrow P(A \cap B) = \frac{1}{4}$

Using the formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{4} + \frac{1}{4} - \frac{1}{4} = \frac{3}{4}$$

Answer:

$$\text{Probability} = \frac{3}{4}$$

Question 10

Given:

$P(\text{Right}) = 50\% = 0.50$

$P(\text{Left}) = 30\% = 0.30$

$P(\text{Straight}) = 20\% = 0.20$

Step-by-step Solution:

Since turning left and turning right are mutually exclusive events:

$$P(\text{Left or Right}) = P(\text{Left}) + P(\text{Right})$$

$$P(\text{Left or Right}) = 30\% + 50\% = 80\% = 0.80 = \frac{4}{5}$$

Answer:

$$\text{Probability} = 80\% \text{ or } 0.8 \text{ or } \frac{4}{5}$$

Question 11

Given:

Two fair coins are tossed. $S = \{HH, HT, TH, TT\} \Longrightarrow n(S) = 4$.

Let A = getting two heads = $\{HH\} \Longrightarrow n(A) = 1$

Let B = getting two tails = $\{TT\} \Longrightarrow n(B) = 1$

Step-by-step Solution:

Since A and B cannot happen simultaneously, they are mutually exclusive.

$$P(A \cup B) = P(A) + P(B) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Answer:

$$\text{Probability} = \frac{1}{2}$$

Question 12

Given:

A and B are **mutually exclusive** events $\Longrightarrow P(A \cap B) = 0$.

$P(\text{not } A) = P(A') = 0.45$

$P(A \cup B) = 0.65$

Step-by-step Solution:

1. Find $P(A)$:

$$P(A) = 1 - P(A') = 1 - 0.45 = 0.55$$

2. Since A and B are mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

$$0.65 = 0.55 + P(B)$$

$$P(B) = 0.65 - 0.55 = 0.10$$

Answer:

$$P(B) = 0.10 \text{ or } \frac{1}{10}$$

Question 13

Given:

$P(A) = \frac{2}{3}$

$P(B) = \frac{2}{5}$

$P(A \cup B) = \frac{1}{3}$

(Note: Mathematically, $P(A \cup B)$ cannot be smaller than $P(A)$, but using the given formula values directly):

Formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Longrightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Step-by-step Solution:

$$P(A \cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$P(A \cap B) = \left(\frac{2}{3} - \frac{1}{3}\right) + \frac{2}{5} = \frac{1}{3} + \frac{2}{5}$$

Find a common denominator of 15:

$$P(A \cap B) = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

Answer:

$$P(A \cap B) = \frac{11}{15}$$

Question 14

Given:

$P(A) = 0.42$

$P(B) = 0.48$

$P(A \cap B) = 0.16$

(i) Find $P(\bar{A})$:

$$P(\bar{A}) = 1 - P(A) = 1 - 0.42 = 0.58$$

(ii) Find $P(\bar{B})$:

$$P(\bar{B}) = 1 - P(B) = 1 - 0.48 = 0.52$$

(iii) Find $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Answer:

(i) $P(\bar{A}) = 0.58$

(ii) $P(\bar{B}) = 0.52$

(iii) $P(A \cup B) = 0.74$

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EXERCISE 12.3

Question 1

Problem Statement:

Two numbers are randomly chosen from 1 to 10 with replacement. Find the probability that:

(i) both are prime

(ii) their product is even

Solution:

- **Total outcomes for one draw:** $\{1,2,3,4,5,6,7,8,9,10\} \Longrightarrow n(S_1) = 10$.
- Since the numbers are drawn **with replacement**, the total possible outcomes for two draws are:

$$n(S) = 10 \times 10 = 100$$

(i) Both are prime

1. **Prime numbers from 1 to 10:** $\{2,3,5,7\}$ (Total of 4 prime numbers).
2. **Probability of getting a prime number on the first draw:**

$$P(\text{Prime}_1) = \frac{4}{10} = \frac{2}{5}$$

3. **Probability of getting a prime number on the second draw:**

$$P(\text{Prime}_2) = \frac{4}{10} = \frac{2}{5}$$

4. Since the draws are independent:

$$P(\text{Both Prime}) = P(\text{Prime}_1) \times P(\text{Prime}_2) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

(ii) Their product is even

1. **Understanding even product:**

The product of two numbers is odd *only* when both numbers are odd. In all other cases, the product is even.

$$P(\text{Product is Even}) = 1 - P(\text{Both are Odd})$$

2. **Odd numbers from 1 to 10:** $\{1,3,5,7,9\}$ (Total of 5 odd numbers).
3. **Probability of drawing an odd number in one draw:**

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$$P(\text{Odd}) = \frac{5}{10} = \frac{1}{2}$$

4. **Probability that both drawn numbers are odd:**

$$P(\text{Both Odd}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

5. **Probability that product is even:**

$$P(\text{Product is Even}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Question 2

Problem Statement:

One letter is chosen from the word **PUNJAB** and another from **LAHORE**. What is the probability that:

(i) both are vowels

(ii) one is consonant and other is vowel

Solution:

- **Word 1: PUNJAB** (Total letters = 6)
  - Vowels: $\{U, A\} \Longrightarrow 2$
  - Consonants: $\{P, N, J, B\} \Longrightarrow 4$
- **Word 2: LAHORE** (Total letters = 6)
  - Vowels: $\{A, O, E\} \Longrightarrow 3$
  - Consonants: $\{L, H, R\} \Longrightarrow 3$

Total outcomes $n(S) = 6 \times 6 = 36$.

(i) Both are vowels

1. Probability of choosing a vowel from PUNJAB:

$$P(V_1) = \frac{2}{6}$$

2. Probability of choosing a vowel from LAHORE:

$$P(V_2) = \frac{3}{6}$$

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3. Combined probability:

$$P(\text{Both Vowels}) = \frac{2}{6} \times \frac{3}{6} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

(ii) One is consonant and other is vowel

There are two mutually exclusive cases:

- **Case 1:** Consonant from PUNJAB and Vowel from LAHORE (C_1 and V_2)

$$P(C_1 \cap V_2) = \frac{4}{6} \times \frac{3}{6} = \frac{12}{36}$$

- **Case 2:** Vowel from PUNJAB and Consonant from LAHORE (V_1 and C_2)

$$P(V_1 \cap C_2) = \frac{2}{6} \times \frac{3}{6} = \frac{6}{36}$$

Total probability:

$$P(\text{One Consonant, One Vowel}) = \frac{12}{36} + \frac{6}{36} = \frac{18}{36} = \frac{1}{2}$$

Question 3

Problem Statement:

A single dice is rolled twice. Find the probability that one roll is a multiple of 3 and the other is a 5.

Solution:

- **Sample space for rolling a die twice:** $n(S) = 6 \times 6 = 36$.
- **Multiples of 3 on a die:** {3,6} (2 outcomes).
- **Getting a 5:** {5} (1 outcome).

There are two possible arrangements for the two rolls:

1. **First roll is a multiple of 3, Second roll is 5:**

$$P(\text{Mult of 3 on 1st}) \times P(5 \text{ on 2nd}) = \frac{2}{6} \times \frac{1}{6} = \frac{2}{36}$$

2. **First roll is 5, Second roll is a multiple of 3:**

$$P(5 \text{ on 1st}) \times P(\text{Mult of 3 on 2nd}) = \frac{1}{6} \times \frac{2}{6} = \frac{2}{36}$$

Total probability:

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$$P(\text{One Mult of 3 and the other 5}) = \frac{2}{36} + \frac{2}{36} = \frac{4}{36} = \frac{1}{9}$$

Question 4

Problem Statement:

Two dice are rolled. Find the probability of getting an odd number on one and a multiple of 2 on other.

Solution:

- **Total outcomes for rolling 2 dice:** $n(S) = 6 \times 6 = 36$.
- **Odd numbers on a die:** $\{1,3,5\} \Longrightarrow 3$ outcomes.
- **Multiples of 2 (even numbers) on a die:** $\{2,4,6\} \Longrightarrow 3$ outcomes.

There are two cases:

1. **Odd on 1st die, Multiple of 2 on 2nd die:**

$$P(\text{Odd}_1 \cap \text{Even}_2) = \frac{3}{6} \times \frac{3}{6} = \frac{9}{36}$$

2. **Multiple of 2 on 1st die, Odd on 2nd die:**

$$P(\text{Even}_1 \cap \text{Odd}_2) = \frac{3}{6} \times \frac{3}{6} = \frac{9}{36}$$

Total probability:

$$P(\text{Odd on one and Multiple of 2 on other}) = \frac{9}{36} + \frac{9}{36} = \frac{18}{36} = \frac{1}{2}$$

Question 5

Problem Statement:

From a pack of well-shuffled cards, two cards are drawn at random one by one with replacement. Find the probability that the first is heart and second is king.

Solution:

- **Total cards in a deck:** 52.
- Number of **Hearts** = 13
- Number of **Kings** = 4
- Since drawing is **with replacement**, the second draw is independent of the first.

1. **Probability of drawing a Heart first:**

$$P(\text{Heart}) = \frac{13}{52} = \frac{1}{4}$$

2. **Probability of drawing a King second:**

$$P(\text{King}) = \frac{4}{52} = \frac{1}{13}$$

3. **Combined probability:**

$$P(\text{First Heart and Second King}) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

Question 6

Problem Statement:

If two cards are selected from a standard deck of 52 cards **without replacement**, find the probability that:

(i) Both are black

(ii) Both are queens

(iii) Both are spades

(iv) Both are diamonds

(i) Both are black

- Total black cards = 26
- Probability of 1st black card: $\frac{26}{52}$
- Probability of 2nd black card (without replacement): $\frac{25}{51}$

$$P(\text{Both Black}) = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

(ii) Both are queens

- Total queens = 4
- Probability of 1st queen: $\frac{4}{52}$
- Probability of 2nd queen (without replacement): $\frac{3}{51}$

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$$P(\text{Both Queens}) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

(iii) Both are spades

- Total spades = 13
- Probability of 1st spade: $\frac{13}{52}$
- Probability of 2nd spade (without replacement): $\frac{12}{51}$

$$P(\text{Both Spades}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$

(iv) Both are diamonds

- Total diamonds = 13
- Probability of 1st diamond: $\frac{13}{52}$
- Probability of 2nd diamond (without replacement): $\frac{12}{51}$

$$P(\text{Both Diamonds}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$

Question 7

Problem Statement:

Saleem draws two cards one by one **without replacement** from a well-shuffled pack of 52 playing cards. What is the probability that first card is jack and the second card is queen?

Solution:

- Total cards = 52
- Number of Jacks = 4
- Number of Queens = 4

1. **Probability that the first card is a Jack:**

$$P(\text{Jack}_1) = \frac{4}{52} = \frac{1}{13}$$

2. Since drawing is **without replacement**, there are now 51 cards left in the pack.
3. **Probability that the second card is a Queen:**

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$$P(\text{Queen}_2 \mid \text{Jack}_1) = \frac{4}{51}$$

4. **Combined probability:**

$$P(\text{First Jack and Second Queen}) = \frac{4}{52} \times \frac{4}{51} = \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$$

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REVIEW EXERCISE 12.

1. Multiple Choice Questions (MCQs)

- **(i) The probability of impossible event is:**

(a) 0 | (b) 1 | (c) 2 | (d) -1

Correct Answer: (a) 0

- **(ii) What is the probability of getting a head when a fair coin is tossed once?**

(a) 0 | (b) 0.25 | (c) 0.5 | (d) 1

Correct Answer: (c) 0.5

- **(iii) The sum of probabilities of all possible outcomes of an experiment is:**

(a) 0.5 | (b) 0.25 | (c) 1 | (d) 0.4

Correct Answer: (c) 1

- **(iv) If $P(A) = 0.6$, then the probability of event A not happening is:**

(a) 0.4 | (b) 0.6 | (c) 1 | (d) 1.6

Correct Answer: (a) 0.4

- **(v) Two events are said to be mutually exclusive if:**

(a) they can happen at the same time. | (b) one affects the other. | (c) they cannot happen together. | (d) they are always equal.

Correct Answer: (c) they cannot happen together.

- **(vi) If one coin is tossed and one dice is rolled, then the number of sample point are:**

(a) 3 | (b) 6 | (c) 2 | (d) 12

Correct Answer: (d) 12

- **(vii) What is the probability of sure event?**

(a) 1 | (b) 0 | (c) $\frac{2}{3}$ | (d) $\frac{4}{5}$

Correct Answer: (a) 1

- **(viii) The probability of getting a number greater than 6 on dice is:**

(a) 0.33 | (b) 1 | (c) 0 | (d) 0.5

Correct Answer: (c) 0

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- **(ix) _____ outcomes are possible when we draw a card from deck of cards.**

(a) 13 | (b) 1 | (c) 52 | (d) 26

Correct Answer: (c) 52

- **(x) If $P(E) = 0.07$, then the probability of 'not E' is:**

(a) 0.95 | (b) 0.89 | (c) 0.93 | (d) 0.90

Correct Answer: (c) 0.93

Subjective Questions (Full Solutions)

Question 2

Arshia selects a day from {Tuesday, Wednesday} and a time from {Morning, Evening}. Draw a possibility diagram. Also sketch a tree diagram.

Solution:

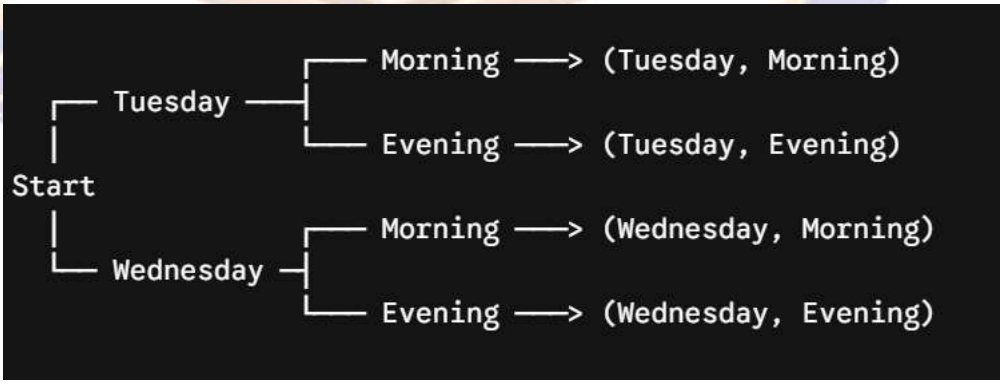
1. Possibility Diagram (Grid / Table):

Day / Time	**Morning (M)**	**Evening (E)**
Tuesday (T)	(Tuesday, Morning)	(Tuesday, Evening)
Wednesday (W)	(Wednesday, Morning)	(Wednesday, Evening)

Sample Space:

$$S = \{(\text{T, M}), (\text{T, E}), (\text{W, M}), (\text{W, E})\}$$

2. Tree Diagram:



Question 3

A card is drawn from a deck of cards. Find the probability of getting an ace or a spade card.

Solution:

Total number of cards in a deck, $n(S) = 52$.

- Let A be the event of getting an **Ace**.
  - Number of aces in a deck: $n(A) = 4$
  - $P(A) = \frac{4}{52}$
- Let B be the event of getting a **Spade**.
  - Number of spades in a deck: $n(B) = 13$
  - $P(B) = \frac{13}{52}$
- Let $A \cap B$ be the event of getting a card that is **both an Ace and a Spade** (i.e., Ace of Spades).
  - $n(A \cap B) = 1$
  - $P(A \cap B) = \frac{1}{52}$

Using the Addition Law of Probability for non-mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\text{Ace or Spade}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$P(\text{Ace or Spade}) = \frac{4 + 13 - 1}{52} = \frac{16}{52} = \frac{4}{13}$$

Answer: $\frac{4}{13}$

Question 4

Fatima dealt two cards successively (without replacement) from a shuffled deck of 52 playing cards. Find the probability that both cards are red.

Solution:

Total number of cards in the deck = 52

Total number of red cards = 26

- **Probability of drawing the 1st red card:**

$$P(\text{1st Red}) = \frac{26}{52} = \frac{1}{2}$$

- Since the card is **not replaced**, for the 2nd draw:
  - Remaining red cards = 25
  - Total remaining cards = 51
- **Probability of drawing the 2nd red card:**

$$P(\text{2nd Red} \mid \text{1st Red}) = \frac{25}{51}$$

- **Probability that both cards are red:**

$$P(\text{Both Red}) = P(\text{1st Red}) \times P(\text{2nd Red} \mid \text{1st Red})$$

$$P(\text{Both Red}) = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

Answer: $\frac{25}{102}$

Question 5

If two dice are rolled, then find the probability of getting the product of face values 6 or the difference of face values 5.

Solution:

When two dice are rolled, total possible outcomes:

$$n(S) = 6 \times 6 = 36$$

- **Event A:** Product of face values is 6.

Outcomes where product is 6:

$$A = \{(1,6), (2,3), (3,2), (6,1)\}$$

$$n(A) = 4 \Rightarrow P(A) = \frac{4}{36}$$

- **Event B:** Difference of face values is 5.

Outcomes where absolute difference is 5:

$$B = \{(1,6), (6,1)\}$$

$$n(B) = 2 \Longrightarrow P(B) = \frac{2}{36}$$

- **Event $A \cap B$:** Outcomes common to both events (product is 6 AND difference is 5):

$$A \cap B = \{(1,6), (6,1)\}$$

$$n(A \cap B) = 2 \Longrightarrow P(A \cap B) = \frac{2}{36}$$

Using the Addition Formula for Probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{4}{36} + \frac{2}{36} - \frac{2}{36} = \frac{4}{36} = \frac{1}{9}$$

Answer: $\frac{1}{9}$

Question 6

A bag contains 7 orange and 5 purple marbles. Two marbles are drawn one after the other without replacement. Find the probability that the first marble is orange and the second marble is also orange.

Solution:

- Number of orange marbles = 7
- Number of purple marbles = 5
- Total number of marbles = $7 + 5 = 12$
- **Probability of drawing 1st orange marble:**

$$P(\text{1st Orange}) = \frac{7}{12}$$

- Since the draw is **without replacement**:
  - Remaining orange marbles = 6
  - Total remaining marbles = 11
- **Probability of drawing 2nd orange marble:**

$$P(\text{2nd Orange} \mid \text{1st Orange}) = \frac{6}{11}$$

- **Probability that both marbles are orange:**

$$P(\text{Both Orange}) = P(\text{1st Orange}) \times P(\text{2nd Orange} \mid \text{1st Orange})$$

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$$P(\text{Both Orange}) = \frac{7}{12} \times \frac{6}{11} = \frac{42}{132} = \frac{7}{22}$$

Answer: $\frac{7}{22}$

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